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Shining a Light on Mathematics Learning

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Please Note:

RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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A STUDENT'S CONCEPTION OF NEGATIVE INTEGERS

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The purpose of this case study was to construct a model of one third grade student's conception of negative integers, and was situated within the theoretical framework of Central Conceptual Structure of Numbers (CCSN). Constructs within this framework were utilized to understand the student's mental model, and it was ultimately determined that the student had constructed a formal mental model with regard to the order and value of integers. This formal mental model implies a coordination between the negative and positive integers within his mental number line schema.

Introduction

Historically, negative numbers have been stigmatized as difficult; noted mathematicians such as Diophantus and Pascal have erroneously declared it impossible to subtract a larger number from a smaller, due to their inability to conceptualize negative integers (Bishop, Lamb, Philipp, Schappelle, & Whitacre, 2011). Within the frame of modern mathematics, a negative difference is no longer problematic; however, its history is interesting because developmentally immature students today may harbor the same misconception as did Diophantus and Pascal (Bishop, Lamb, Philipp, Schappelle, & Whitacre, 2011).

Despite the advances of modern mathematics, students' misconceptions regarding negative integers persist beyond the onset of instruction (Murray, 1985). Bofferding (2010) suggests that while the whole number system can be built by students through interaction with appropriate manipulatives, no such manipulatives exist for negative integers, therefore making the extension of the integer system to include negatives markedly more difficult. Existing manipulatives seem contrived to students and do not fit with their previously developed intuitions regarding negative integers (Peled, 1991). This can result in a disconnect between the students' intuitions and their school experiences, making the transfer of learning increasingly challenging.

Research indicates that common issues arise when students' intuitions and school experiences related to negatives are not in alignment. Altiparmak and Ozdogan (2010) succinctly categorized the difficulties that arise in the learning of negative integers into three distinct groups. These difficulties include "the meaning of the numerical system and the direction and magnitude of the number…, the difficulties experienced with regard to the meaning of arithmetic operations…, [and] the meaning of the minus sign" (p. 31).

As the introduction of negative integers is not as concrete as that of whole numbers, students struggle with the meaning of negatives. Bishop et al. (2011) found first graders to conceptualize negatives as either a position or movement, or a perceptible object; the former being more mathematically powerful. The inability of students to advance their conception of negative integers to include movement and position is the first noted struggle. Second is the difficulty extending arithmetic to appropriately include negatives. Peled (1991) found that an incomplete conception of negative integer operations led to misapplied commutativity and ignored minus signs by elementary students. The third difficulty involves the meaning of the minus sign. Vlassis (2008) recognized its three functions – binary (subtraction), symmetric (opposite), and unary (negative) – all of which students must differentiate among.

The opportunity to engage young children in an exploration of the negative integers is being missed, and is denying students the opportunity to deepen their understandings of mathematical concepts such as zero, the subtraction of a larger number from a smaller (Bofferding, 2014), and the meanings of the minus sign (Bofferding, 2010). Therefore, earlier exposure to the negative integers may benefit students' long term mathematical abilities. Further, research has identified that primary students (e.g., Behrend & Mohs, 2005/2006) may be capable of comprehending negative integers in increasingly sophisticated ways. Consequently, the purpose of this research is to develop an understanding of one third-grade student's mental model of negative integers in an attempt to better understand to what extent negative integers may be appropriate at this grade level. Moreover, how will this third grade student's mental model of negative integers fit within the theoretical framework of Central Conceptual Structure of Numbers (CCSN)?

Theoretical Framework

This paper reports on a study with a single third-grade student. In order to model his conception of negative integers, I adopted a CCSN framework. This neo-Piagetian framework indicates that at approximately six years of age, a child's *mental number line* (Figure 1) begins to emerge, following the reorganization of two distinct, lower-order schemas: the global quantity schema and the counting schema (Case, 1996). These initial mental models are hypothesized to emerge around four years of age and allow children to operate in two distinct ways. The global quantity schema allows children to make distinction such as more and less, or higher and lower. The counting schema allows for the coordination of the child's verbal counting sequence with the one-to-one activity of tagging objects, and ultimately encompassing the cardinality of the set

(Case, 1996). Initially, these two schemas operate separately. Once students reorganize and coordinate their schemas, however, the central numerical structure or mental number line can emerge. The central conceptual structure of numbers is termed such because the mental number line is considered to be at the center of the child's ability to reason quantitatively (Case, 1996).

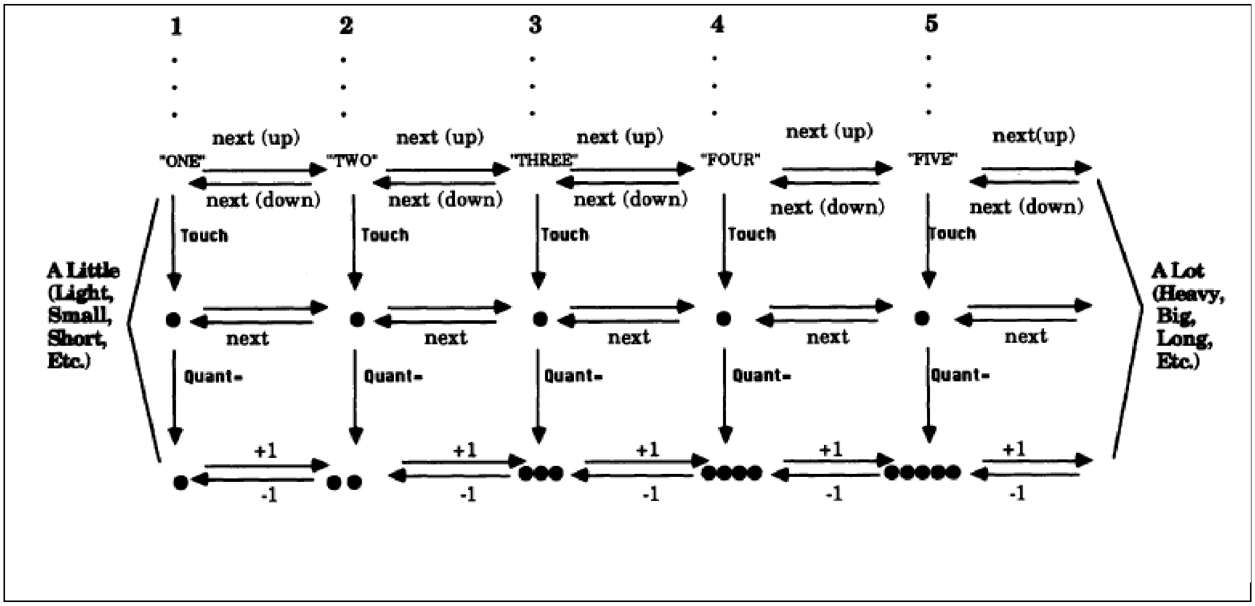

Figure 1: Case's (1996) depiction of a student's central numerical structure.

Encompassed within the theoretical framework of CCSN lie constructs for understanding students' mental models of negative integers. Broadly, these constructs can be identified as initial mental models, intermediary mental models, and formal mental models (Bofferding, 2012).

Initial mental models of negative integers represent the stages of understanding which exist prior to the integration of negative integers into the child's central conceptual structure (Bofferding, 2012). That is, when students initially begin to conceptualize negative integers, they do so in a way which is inconsistent with their existing mental number line. When students initially recognize the existence of negative integers, they will attempt to operate on them in a manner which is distinct from their mental number line.

Intermediary mental models represent a departure from the initial mental models because reliance upon intermediary mental models begins an extension of the CCSN to include negative integers, rather than an attempt to operate on negative integers separately, as with initial mental models (Bofferding, 2012). Thus, students demonstrating understanding in alignment with an intermediary mental model are able to conceptualize negative integers as existing to the left of zero on the mental number line. At this stage, however, the mental number line remains incomplete. Therefore, the student's understanding of more or less within the CCSN (Case, 1996) may not yet completely include the negative integers, thus impeding their ability to compare or order positive and negative integers consistently (Bofferding, 2012).

The formal mental model of negative integers represents a complete and final reorganization of the CCSN to include the entire integer system – both positive and negative (Bofferding, 2012). This reorganization is characterized by the ability of the student to not only to extend the number line to include negative integer symbols and words, but additionally to coordinate the quantitative concepts of more and less as they relate to the relationships between and among both positive and negative integers.

Methods

This study was conducted with one third-grade participant from a rural elementary school in the southeastern United States. The selection of the student, Christian (a pseudonym), was one of convenience; his teacher selected him due to his awareness of negative integers. Data for this study was collected through a series of three video recorded, semi-structured clinical interviews (Clement, 2000), lasting twenty minutes each. During each interview, the student completed mathematical tasks with the purpose of characterizing his conception of negative integers. Tasks were selected to address the three necessary constructions in conceptualizing negative integers: (a) conception of and symmetry of the number line, (b) ability to order integers, and (c) ability to compare integers.

To address these constructions, the student was engaged in three types of tasks. In the first and third interviews, the student completed a number line task, in which a number line marked only with a "1" and empty tick marks to the right and left was given, and the student filled in the missing integers (Bofferding, 2014). In the second type of task, ordering tasks, the student was provided with a set of eight cards, each containing integers between negative ten and ten, and was asked to order the cards from least to greatest (Schwarz, Kohn, & Resnick, 1993). The student completed ordering tasks in all three interviews. The last type of task, also given in all three interviews, were comparison tasks in which the student compared pairs of integers to identify the larger and smaller of the two (Peled et al., 1989). In each comparison task, the student compared one of four types of integers: (a) two positive integers, (b) two negative integers, (c) one positive and one negative integer (with the absolute value of the negative integer being greater than or equal to that of the positive), and (d) one negative integer and zero.

Analysis

To address the first construction, Christian's conception of the number line and its symmetry, he completed the number line task. On both occasions this task was given, he correctly completed the number line by first filling in the numbers two through six to the right of one and then beginning at zero and filling in negative one through negative four to the left of zero. All integers were correctly symbolized and ordered. This response indicates his awareness of the existence of negative integers and their placement on the number line. Christian also read aloud the integers beginning with negative four and increasing through six in a standard manner. His ability to complete a number line appropriately indicates some understanding of ordering integers. Therefore, based upon this task, I determined that Christian was aware of the notation and naming of negative integers, as well as their placement in relation to the whole numbers.

In addition to placing negative integers on the number line, the symmetry of the number line about zero is critical. In all three interviews, Christian used zero as a key point for determining the magnitude of integers in the comparison tasks. In the second interview, for instance, he indicated that all negative integers are "below zero." Later in the same interview he reasoned that negative seven is larger than negative ten because, "It's closer to zero." These responses were characteristic of Christian's reasoning throughout all three interviews, and I interpret them as demonstrating an understanding of zero as the point of symmetry on the number line.

The second mental construction, that of ordering integers, was briefly evaluated using the number line task, but was more fully evaluated using ordering tasks. On all occasions he correctly ordered all of the integers. In the third interview, he correctly determined negative nine to be the smallest integer in the set when he said, "I think because it's negative more than any of the others. … It's, like, further away from zero than the others." In many of these tasks, he used terminology such as "negative more" or "negative the most" to explain his reasoning for selecting the smallest integer. This type of language demonstrates an appropriate conceptualization of ordering integers based upon their distance from zero, and using zero as a critical point for determining the order of a set of integers. It furthermore demonstrates Christian's ability to appropriately conceptualize the value of an integer.

Lastly, Christian was engaged in comparison tasks to gauge his understanding of pairwise comparisons of integers. In all instances, Christian made correct pairwise comparisons by correctly determining the larger or smaller of the two integers, indicating understanding that the value of each integer is determined by its relative proximity to zero. Moreover, he was able to explain his selection of the larger integer in a manner which is consistent with a reorganized mental number line (Bofferding, 2012).

Conclusions

Based upon the evidence collected, I hypothesize that Christian has constructed a formal mental model, indicating a high level of reasoning about comparisons among and ordering of integers, and moreover, consistency with a completely reorganized mental number line schema. In other words, the previously constructed whole number schema (Figure 1) is superseded by a mental number line equipped to reason about the entire set of integers (Bofferding, 2012).

To address the reorganization of his mental number line to include the negative integers, Christian was asked to complete and describe a number line, and to explain his interpretation of the role of zero on the number line. I interpret his use of zero to determine the magnitude and order of integers on the number line as an understanding of the number line's symmetry. This provides sufficient evidence to support at least an intermediary mental model of negative integers.

However, further evidence was necessary to distinguish between an intermediary and a formal mental model; specifically, evidence of his ability to consistently order integers by engaging his mental number line. To address this, Christian ordered sets of integers ranging from negative ten to ten. He consistently ordered integers correctly, and explained the manner by which he selected the greatest and the least of the integers. His explanations of integers being "negative most" supports my hypothesis that Christian has at least an intermediary model of negative integers.

Lastly, Christian compared integers. The comparison tasks were selected to either identify or eliminate subcategories of an intermediary mental model. It was especially important to understand whether Christian's understanding of the value of negative integers would be dependent upon their relative proximity to zero or their absolute value. His ability to correctly and consistently determine the larger or smaller of two integers supports my hypothesis of Christian's reorganization of his mental number line for whole numbers to include the integers.

Considered separately, each of the outcomes of these tasks is only sufficient to assume Christian's reasoning is characteristic of an intermediary mental model. However, in total, it is my final hypothesis that Christian has a formal mental model of negative integers (Bofferding, 2012), thus representing a completed reorganization of his CCSN to include the entire integer system. According to Case (1996), this reorganization indicates a coordination of number symbols, number words, one-to-one correspondence, and the cardinality of sets for each integer, incorporated with understandings of more and less among the integers.

The results of this case study support the work of previous studies (e.g., Behrend & Mohs, 2005/2006) in outlining the ability of students to conceptualize the integer system in a sophisticated way prior to the onset of instruction in negative integers (NCTM, 2000). While NCTM calls for exposure in grades K-5 to the complete integer system through relatable situations, such as temperature, Christian's case study provides an existence of proof that students may be prepared for more. While the generalizability of this statement is limited due to the nature of a case study, the exploration of students' preparedness for conceptualizing integers is warranted. Furthermore, this conceptualization of negative integers within CCSN provides a powerful tool for teachers in interpreting the readiness of their students to receive instruction, enrichment, and remediation during the critical elementary mathematics stage.

References

Altiparmak, K. & Ozdogan, E. (2010). A study on the teaching of the concept of negative numbers. *International Journal of Mathematical Education in Science and Technology, 41*(1), 31-47.

Behrend, J. L. & Mohs, L. C. (2005/2006). From simple questions to powerful connections: A two-year conversation about negative numbers. *Teaching Children Mathematics*, *12*(5), 260-264.

Bishop, J. P., Lamb, L. L. C., Philipp, R. A., Schappelle, B. P. & Whitacre, I. (2011). First graders outwit a famous mathematician. *Teaching Children Mathematics*, *17*(6), 350-358.

Bofferding, L. (2010). Addition and subtraction with negatives: Acknowledging the multiple meanings of the minus sign. In *Proceedings of the 32nd annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 6, pp. 703-710).

Bofferding, L. (2012). Transitioning from whole numbers to integers. In Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* (pp. 935-942). Kalamazoo, MI: Western Michigan University.

Bofferding, L. (2014). Negative integer understanding: Characterizing first graders' mental models. *Journal for Research in Mathematics Education, 45*(2), 194-245.

Case, R. (1996). Introduction: The role of central conceptual structures in the development of children’s thought. *Monographs of the Society for Research in Child Development, 61*(1/2), 1-26.

Clement, J. J. (2000). Analysis of clinical interviews: Foundations and model viability. In R. Lesh & A. Kelley (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 547-589). Hillsdale, NJ: Lawrence Erlbaum.

Murray, J.C. (1985). Children's informal conceptions of integer arithmetic. In L. Streefland (Ed.), *Proceedings of the annual conference of the international group for the psychology of mathematics education* (pp. 147-153). The Netherlands.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Peled, I. (1991). Levels of knowledge about signed numbers: Effects of age and ability. In F. Furinghetti (Ed.), *Proceedings of the 15th Annual Meeting of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 145-152). Assisi, Italy.

Peled, I., Mukhopadhyay, S., & Resnick, L. B. (1989). Formal and informal sources of mental models for negative numbers. In G. Vergnaud, J. Rogalski, & M. Artique (Eds.), *Proceedings of the Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 106-110). Paris, France.

Schwarz, B. B., Kohn, A. S. & Resnick, L. B. (1993). Positives about negatives: A case study of an intermediate model for signed numbers. *Journal of the Learning Sciences, 3*(1), 37-92.

Vlassis, J. (2008). The role of mathematical symbols in the development of number conceptualization: The case of the minus sign. *Philosophical Psychology. Special Issue: Number as a Test Case for the Role of Language in Cognition, 21*(4), 555-570.

EXPLORING VALIDITY AND RELIABILITY FOR THE REVISED SMPS LOOK-FOR PROTOCOL[1]

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The Standards for Mathematical Practice (SMPs) describe mathematical behaviors and habits that students should express during mathematics instruction. Thus teachers should promote them during classroom-based mathematics instruction. The purpose of this manuscript is to discuss the validation process for an observation protocol called the Revised SMPs Look-for Protocol, which is meant to fill this gap. An implication of this study is that users with a robust understanding of the SMPs may feel confident using the protocol as a validated and reliable tool in research and school-based settings.

As of 2015, 42 of 50 states within the United States of America have adopted the Common Core State Standards for Mathematics (CCSSM) as their mathematics standards. The CCSSM has Standards for Mathematics Content and Standards for Mathematical Practice (SMPs; Common Core State Standards Initiative [CCSSI], 2010). SMPs are descriptions of mathematical habits and behaviors and are deeply connected to the National Council of Teachers of Mathematics' process standards (Kanold & Larson, 2012; Koestler, Felton, Bieda, & Otten, 2013). While the CCSSM have been in place for nearly five years, teachers are still struggling to make sense of them, especially the SMPs (Bostic & Matney, 2014). At times, it is unclear to teachers and observers what the SMPs look like during classroom mathematics instruction (Bostic, 2015; Bostic & Matney, 2014). For example, *modeling with mathematics* has a meaning distinct from *modeling as representation* discussed in the K-5 content standards and *mathematical modeling* as described in the high school content standards (Bostic, 2015; Bostic, Matney, & Sondergeld, 2016). As such, education stakeholders may benefit from having a tool to generate feedback about the ways mathematics teachers' instruction promotes the SMPs. The aim of this manuscript is to present evidence connected to validity and reliability for a tool focused on teachers' instruction related to the SMPs. This tool is called the *Revised SMPs Look-for Protocol*.

Literature Review

Prior Validated Tools for Examining Classroom Instruction

There are various tools to examine mathematics instruction. Boston, Bostic, Lesseig, & Sherman (2015) discusses the strengths and limitations of three validated tools used often in educational research (i.e., Reformed Teaching Observation Protocol, Instructional Quality Assessment, and Mathematical Quality of Instruction). Unfortunately, all three were not intended for use in exploring teachers' promotion of the SMPs. On the other hand, the *Mathematics Classroom Observation Protocol for Practices (MCOP^2)* responds to the need for a tool that examines the SMPs (Gleason & Cofer, 2014). "Each of the items on the *MCOP^2* was designed to coordinate with a Standard for Mathematical Practice…for instance, item #9 on the protocol is 'The lesson provided opportunities to examine elements of abstract (symbolic notation, patterns, generalizations, conjectures, etc.),', matching the second Standard for Mathematical Practice that instructors should be aiming to teach their students" (Gleason & Cofer, 2014, p. 96). The *MCOP^2* moves the field forward with a validated tool to examine classroom instruction for the SMPs; however, this observation protocol has been validated for its use with undergraduate mathematics instruction and not for K-12 instruction. Thus, there still exists a need for a validated observation protocol related to K-12 teachers' SMP-focused instruction.

Development of the Standards for Mathematical Practice Look-for Protocol

A year after the large-scale adoption of the CCSSM, Fennell, Kobett, and Wray (2013) created a tool called the Standards for Mathematical Practice Look-for Protocol (*SMP Look-for Protocol*). Their goal was to develop and share a tool to gather evidence related to K-12 mathematics teachers' promotion of the SMPs during classroom instruction. An initial version had only one indicator related to each SMP for teachers' promotion of the SMPs and students' engagement in the SMPs. Later versions included observable mathematical behaviors and habits (as many as eight), related to both teachers' and students' observable mathematical behaviors and habits. A final version of the *SMP Look-for Protocol* was shared at the 2013 Association of Mathematics Teacher Educators' annual meeting. Fennell and his team conducted nearly 300 observations and asked numerous mathematics teacher educators, curriculum coaches, and teachers to examine the protocol for their ideas related to it. Synthesizing across groups' voices, it was clear that the protocol was helpful to examine K-12 teachers' promotion of the SMPs during classroom instruction. Fennell and colleagues further shared openness to additional revisions of the protocol. Moreover, they had not conducted a formal validation study to use the tool in real-time or video-recorded K-12 classroom mathematics observations. The purpose of the present study is to revise and validate this tool for the purpose of analyzing K-12 teachers' promotion of mathematical behaviors and habits framed by the CCSSM SMPs. Our research question is: What evidence supports use of the *Revised SMPs Look-for Protocol* as a tool to examine teachers' mathematics instruction related to promotion of the SMPs?

Method

Context

A validation study for an observation protocol should possess eight stages (Artino, La Rochelle, Dezee, & Gehlbach, 2010; Smith, Jones, Gilbert, & Wieman, 2013). They are (1) conduct a literature review; (2) conduct interviews and focus groups to gather more ideas for items; (3) synthesize data from literature review and focus groups; (4) develop items; (5) conduct expert panel validation; (6) conduct cognitive interviews with potential users of the protocol; (7) conduct pilot testing of protocol; and (8) conduct psychometric analysis using data from the protocol (e.g., reliability analysis). After nearly 60 observations with the protocol (Fennell et al., 2013), our research team felt it was missing some elements related to teachers' promotion of the SMPs. To that end, we conducted stages one though eight, which involved forming focus groups, an expert panel, and working alongside potential users of the tool. These groups, panels, and users included K-12 mathematics teachers, mathematics coaches, curriculum coordinators, mathematics instructors teaching mathematics education courses, and mathematics teacher educators from across the USA who have led professional development focused on the SMPs, including the initial developers of the protocol Fennell, Kobett, and Wray. As a result, we added some observable aspects related to the SMPs and modified some aspects to better capture teachers' instruction that promoted the SMPs. It is this *Revised SMPs Look-for Protocol* that we explore in our current validation study.

Instrumentation

The *Revised SMPs Look-for Protocol* includes two or three observable behaviors related to teachers' promotion of the SMPs as well as specific notes for observers. A selection of the protocol is shared in Figure 1. For instance, an indicator for "SMP 1: Make sense of problems and persevere in solving them" (CCSSI, 2010, p. 6) is "Provide opportunities for students to solve problems that have multiple solutions and/or strategies".

Mathematical Practices	Observable Teacher Moves Related to Practices
SMP 1. Make sense of problems and persevere in solving them	☐ A. Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution ☐ B. Provide opportunities for students to solve problems that have multiple solutions. ☐ C. Encourage students to represent their thinking while problem solving NOTE: Task must be a grade-level/developmentally-appropriate problem. That is, a solution is not readily apparent, the solution pathway is not obvious, and more than one pathway is possible. Comments:

Figure 1. A selection of the Revised SMPs look-for Protocol.

Data Collection

Since an initial protocol was developed previously and our intention was to work towards a revised protocol, we began with stage two of the validation process. For stages two, five, and six of the validation process data were collected from an expert panel consisting individuals from five groups: K-12 mathematics teachers, mathematics coaches, curriculum coordinators, mathematicians, and mathematics teacher educators. For stage two, we communicated with these individuals to make sense of their ideas for a possible tool to gather data about K-12 teachers' promotion of the SMPs during instruction. These data, in addition to a thorough review of relevant literature on the SMPs published since 2010, led to adding and modifying indicators (stage four), and ultimately convening an expert panel of individuals with different backgrounds. At the fifth stage, the panel examined the *Revised SMPs Look-for Protocol* and reflected on the degree to which our revisions and previous statements adequately met the descriptions in the SMPs. For stage six, small-group and one-on-one interviews were made with one member from each group found on the expert panel to further explore their ideas related to its use as a classroom observation tool in research and in teachers' professional development. The goal of these interviews was to learn about the protocol's ease of use and its overall ability to meet the aim of gathering data about K-12 teachers' promotion of the SMPs during classroom mathematics instruction. These data provided evidence for content validity, a measure of the degree to which an item addresses the construct of interest, which is typically examined through the judgment calls of expert panels and cognitive interviews (Gall, Gall, & Borg, 2007).

Data for the quantitative part of this validation study (stages seven and eight) came from two sources. The first source consists of video-recorded data from K-12 teachers located in a

Midwest state that adopted the CCSSM. They participated in one of nine grant-funded mathematics PD programs that lasted a minimum of 100 face-to-face hours during one calendar year. An objective of these PD programs was to foster teachers' sense making of the SMPs so that they might more effectively promote them during classroom mathematics instruction. Teachers consented to providing videos of instruction prior to the PD and again after 80 hours of PD. The second data source consists of observations of live instruction in K-12 classrooms conducted by the authors of this manuscript. In total, 288 observations of teachers' instruction were coded using the *Revised SMPs Look-for Protocol*. Thirty of the 288 observations were made during live instruction while the other 258 were made using videotaped data. Interrater agreement was high across coders (93%), which exceeds the minimum threshold (90%) needed to conduct reliability and factor analysis (James, Demaree, & Wolf, 1984).

Data Analysis

The authors employed inductive analysis (Hatch, 2002) to draw impressions from the interviews and expert panel reviews (stages two, five, and six). Inductive analysis allows users to identify salient themes from data sets (Glaser & Strauss, 1967/2012; Hatch, 2002). Our approach to inductive analysis started with re-reading (or re-listening) to materials (e.g., expert panel written reviews and audio-recorded interviews). Step two was to make memos consisting of initial ideas stemming from this examination of the data. Step three was to reflect on those memos as a way to synthesize them into key impressions, needed as evidence for validity. Step four was to search for evidence within the data sets to support our key impressions. Step five was to search the data for counter evidence. Impressions with a paucity of counter evidence and a large set of evidence were retained. The sixth and final step was crafting clearly written impressions (themes) to share broadly.

Psychometric analysis was conducted during the eighth stage of the validation study was to examine reliability associated with using this tool. Internal consistency (i.e., reliability) was explored in two ways. The first was internal consistency of the protocol using Cronbach's alpha; it indicates the "coefficient of precision from a set of real test scores" (Crocker & Algina, 2006, p. 117). Test-retest reliability using data from pre- and post-PD observations is the second form of reliability evidence investigated. A bivariate correlation was used to determine the relationship between pre-post-PD observations with higher positive relationships indicating a higher level of test-retest reliability.

Results

Impressions from Expert Panel and Interviews

There was a single impression from the inductive analysis. All members involved in the stages consistently agreed that the *Revised SMPs Look-for Protocol* provided a clear vision of gathering meaningful data about K-12 mathematics teachers' promotion of the SMPs. Many on the panel shared how the protocol offered a coherent set of observable aspects related to each SMP. Others who were interviewed supported this. A mathematics teacher commented "This [revised protocol] is helpful for reflecting on what I could be doing in my classroom to promote the SMPs. I feel confident knowing that when I focus on one SMP that my principal, who is a former math teacher, could use this. In fact, I'd prefer that he use this over other observation tools required by our state because we could have a meaningful conversation about ways I might improve my instruction related to the math standards." One mathematics educator shared that the additions found on the revised protocol allowed more teacher moves to be counted as promoting the SMPs, which did not hinder the quality of the observation or overall impressions of the teacher's instruction. He added, "Allowing strategies and solutions to be counted as promoting SMP 1 is more consistent with the literature on problems and problem solving. I'm glad it's there."

Reliability Analysis

Internal consistency was acceptable with a Cronbach's alpha level of .801 for the overall measure. Cronbach's alpha levels between .70 and .90 are considered appropriate for assessments (Tavakol & Dennick, 2011). Measure with internal consistency below .70 could represent an assessment with poorly interrelated items and a measure with internal consistency above .90 could possess too much item redundancy (Tavakol & Dennick, 2011).

Test-retest reliability was acceptable with a correlation coefficient of .721 from pre-PD observation to post-PD observation. This suggests that teacher growth from pre-post-PD is not always consistent across participants. Further investigation of the data clearly demonstrated this phenomenon with teachers performing higher at pre-PD demonstrating less growth by post-PD than teachers who performed lower over time. While conceptually it makes sense that teachers would have the ability to grow more if demonstrating lower levels of performance at pre-PD, it does not allow for production of what are considered good or excellent test-retest reliability coefficients.

Conclusion and Discussion

The purpose of our study was to revise and validate a tool to analyze K-12 teachers' promotion of mathematical behaviors and habits framed by the CCSSM SMPs. We aimed to share validity evidence from cognitive interviews and the expert panel as well as results from internal consistency and reliability analyses. Our content validity evidence was strong hence our conclusion is that the *Revised SMPs Look-for Protocol* appropriately organizes data regarding K-12 teachers' promotion of the SMPs. Internal consistency was strong and relatedly, our test-retest reliability also met the threshold for use in most settings. In sum, a diverse audience may use the *Revised SMPs Look-for Protocol* to gather data about K-12 teachers' promotion of the SMPs during classroom instruction. The protocol may be used with video-recorded data or during live instruction.

This study adds to the growing body of observation protocols validated for use in K-12 mathematics classrooms (see Boston et al., 2015 for a review) and builds upon the Fennel, Kobett, and Wray's (2013) development of a tool to gather observational data about teachers' promotion of the SMPs. Results of our study fill a needed gap as no validated tools currently focus on this area within K-12 instruction. Mathematics teachers, curriculum leaders, and researchers may feel confident using this tool to explore the ways in which teachers foster the SMPs during instruction, and perhaps explore teachers' instructional changes using two or more observations. One caveat with use of this protocol was that everyone who used it had a robust understanding of the SMPs. Observers have either engaged in more than 100 hours of professional development on the SMPs or led professional development on the topic. Thus we do not advocate its use by those unfamiliar with the SMPs or without a coherent understanding of each SMP.

Future Research

While we feel confident with results of this study, we intend to conduct further observations and perform an exploratory factor analysis after more observations. Exploratory factor analysis is appropriate when a researcher has a notion about the nature of the factors measured by an instrument but those factors are not well-defined (Crocker & Algina, 2006). Future researchers might explore the student-version of the protocol developed by Fennell et al. (2013) and explore validity and reliability evidence related to how K-12 students engage in the SMPs during instruction. We also encourage mathematics education researchers to explore connections between the SMPs and the Mathematics Teaching Practices described in *Principles to Action* (National Council of Teachers of Mathematics, 2014). It may be that teachers' promotion of the SMPs might be indicative of one or more Mathematics Teaching Practice, thus relationship to other variables validity evidence should be explored.

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References

Artino Jr., A., La Rochelle, J., Dezee, K., & Gehlbach, H. (2014). Developing questionnaires for educational research: AMEE guide no 87. *Medical Teacher, 36*, 463-474.

Bostic, J. (2015). A blizzard of a value. *Mathematics Teaching in the Middle School, 20,* 350 – 357.

Bostic, J., & Matney, G. (2014). Preparing K-10 teachers through Common Core for Reasoning and Sense Making. *Journal for Mathematics Education Leadership, 15*(2), 3-10.

Bostic, J., Matney, G., & Sondergeld, T. (2016). *A validation study of the revised SMPs look-for protocol.* Manuscript submitted for publication.

Boston, M., Bostic, J., Lesseig, K., Sherman, M. (2015). Classroom Observation tools to support the work of mathematics teacher educators. Invited manuscript for *Mathematics Teacher Educator, 3*, 154-175.

Common Core State Standards Initiative. (2010). *Common core standards for mathematics.* Washington, DC: National Governors Association Center for Best Practices and Council of Chief State School Officers.

Crocker, L., & Algina, J. (2006). *Introduction to classical and modern test theory.* (2nd ed.) Mason, OH: Wadsorth Publishing.

Fennell, F., Kobett, E., & Wray, J. (2013, February). *Using look fors to consider the common core content standards.* Presentation at the Annual Meeting of the Association of Mathematics Teacher Educators: Orlando, FL.

Gall, M., Gall, J., & Borg, W. (2007). *Educational research: An introduction* (8th ed.). Boston: Pearson.

Gleason, J. & Cofer, L.D. (2014). Mathematics classroom observation protocol for practices results in undergraduate mathematics classrooms. In T. Fukawa-Connelly, G. Karakok, K. Keene, and M. Zandieh (Eds.), *Proceedings of the 17th Annual Conference on Research on Undergraduate Mathematics Education,* Denver, CO, 93-103.

Glaser, B. & Strauss, A. (1967/2012). *The discovery of grounded theory: Strategies for qualitative research*. Mill Valley, CA: Sociology Press.

Hatch, A. (2002). *Doing qualitative research in education settings*. Albany, NY: State University of New York Press.

James, L., Demaree, R., & Wolf, G. (1993). R_{wg}: An assessment of within-group interrater agreement. *Journal of Applied Psychology, 78,* 306-309.

Kanold, T., & Larson, M. (2012*). Common core mathematics in a PLC at work: Leader's guide.* Bloomington, IN: Solution Tree Press.

Koestler, C., Felton, M. D., Bieda, K. N., & Otten, S. (2013). *Connecting the NCTM Process Standards and the CCSSM Practices*. Reston, VA: National Council of Teachers of Mathematics.
National Council of Teachers of Mathematics. (2014). *Principles to action.* Reston, VA: Author.
Smith, M., Jones, F., Gilbert, S., & Wieman, C. (2013). The classroom observation protocol for undergraduate STEM (COPUS): A new instrument to characterize university STEM classroom practices. *CBE-Life Sciences Education, 12*, 618-627.
Tavakol, M., & Dennick, R. (2011). Making sense of Cronbach's alpha. *International Journal of Medical Education, 2*, 53-55.

COLLIGATION AND UNIT COORDINATION IN MATHEMATICAL ARGUMENTATIVE WRITING

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Children's development of mathematical argument has long been identified as facilitated by early algebraization. A fundamental feature of argumentation is colligation, or the ability to synthesize one's warrants towards a claim. In this study, we examined the presence of colligation across 168 second and third grade students' written responses to mathematical tasks. Results suggest that students who demonstrate colligation, via operationalizing given information in the task (i.e., detailing), tend to demonstrate higher scores on assessments for two concepts in early algebra: multiplicative reasoning and conception of equivalence.

Research on argumentation and proof in the elementary grades suggests that engaging students in tasks related to early algebra contexts improves the quality of their mathematical arguments (Fosnot & Jacob, 2009; Morris, 2009). Likewise, research on early algebra in elementary grades suggests that more sophisticated engagement in early algebra tasks and processes facilitates engagement in clarifying claims and specifying justifications (Blanton & Kaput, 2011). Although much of the literature base provides evidence for a connection between early algebra tasks and mathematical argument in the elementary grades, there has been little to no focus on the specific features of language that relate to algebraization.

The present study focuses on examining how children's engagement in early algebra relates to a particular linguistic skill necessary for construction of argument referred to as *colligation* (Liszka, 1996). Peirce (1903/1998) described colligation as the first step of argumentation, in which an individual collectively utilizes the warrants for their argument as a singular copulative proposition. For example, in proving *the sum of two consecutive even integers is divisible by two*, an individual may use the equation $2n + (2n + 2) = 4n + 2 = 2(2n + 1)$ as part of their proof. The equation represents the colligation of three expressions into a copulative proposition supporting the general claim. This colligation is facilitated by a linguistic tool Kosko and Zimmerman (2015) refer to as detailing, in which the given information is operationalized through construction and integration of a reference chain through mathematical proposals (i.e., warrants). Examining the detailing in children's mathematical writing, Kosko (in review) noted that students were more likely to attempt detailing when given a multiplicative rather than an arithmetic version of a task. Thus, colligation may be facilitated through use of early algebra tasks. The purpose of the present study is to investigate whether students demonstrating certain understandings of early algebra (i.e., multiplication and equivalence) are more likely to engage in colligation via detailing in their mathematical argumentative writing.

Colligation in Mathematical Argumentative Writing

The present study takes a Peircian semiotic view of mathematical argument. Peirce's (1903/1998) semiotic theory conceived of various signs that, with increasing abstraction, contain more sophisticated and synthesized meanings. *Argument* is defined as a sign that synthesizes various premises and includes inference towards some general claim. *Copulative signs* involve the colligation of various propositions synthesized in a manner to indicate a logical sequence. As such copulative signs do not involve inference towards a general claim, but arguments do. Copulative signs, however, are an essential feature of arguments (Liszka, 1996). In their examination of elementary students' mathematical writing in grades K-3, Kosko and Zimmerman (2015) observed a difference in students' writing that provided procedures of how they completed a task versus how such procedures supported their claim. Specifically, students who used procedures to support a claim operationalized given information from the task to create new propositions that were linked together by a reference chain to the given information. The use of reference chains in this manner is referred to as *detailing*.

Detailing, in the present study, is considered a linguistic tool that facilitates colligation. By facilitate, we infer that there are additional linguistic tools necessary for colligation to occur consistently to construct a copulative sign, or copulative proposition. Not discussed in depth in the present paper, nominalization is one such linguistic tool that, as defined by Halliday and Matthiessen (2004), conveys two or more mathematical linguistic objects metaphorically as one. As such, nominalization has the potential to synthesize multiple propositions into a singular copulative proposition, indicating colligation. Detailing provides an alternative tool that may be used in isolation of, or in combination with other such linguistic tools to colligate mathematical propositions. Returning to the example of the proof for *the sum of two consecutive even integers is divisible by two,* a completed proof may state that:

> Let n be an integer. So, $\boldsymbol{2n}$ and $\boldsymbol{2n+2}$ are consecutive integers. Then $\boldsymbol{2n+(2n+2)=4n+2=2(2n+1)}$, which is divisible by 2. Thus, the sum of two consecutive even integers is divisible by two.

The expressions $2n$ and $2n + 2$ reference the given information that there are two consecutive even integers. These two references are then operationalized initially as $2n + (2n + 2)$ and then further by $4n + 2$ and $2(2n + 1)$. In each expression, the given information is used to construct new variations of mathematical information in a manner that extends beyond reiterating what is explicitly conveyed in the given information. The formation of such a reference chain, concurrent with the operationalization of the given information which is referenced, is the essence of detailing. The observant reader may note that the example of colligation provided above also incorporates the linguistic tool of nominalization. Yet, we focus explicitly on detailing for purposes of simplicity in the present paper.

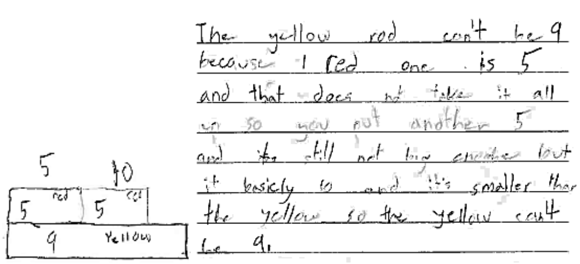
Kosko and Zimmerman's (2015) analysis of K-3 students' mathematical writing suggests that detailing may be much more prevalent in second and third grade students' argumentative writing than that of Kindergarten or first grade students'. Kosko's (in review) subsequent analysis suggests that by varying the nature of the given information in tasks so that it must be accepted at face value, students were statistically more likely to attempt detailing in their mathematical argumentative writing. Kosko's analysis focused on two tasks: one in which it was posited that two red Cuisenaire rods could not be the same length as a yellow Cuisenaire rod, and the other in which it is posited that if a red rod is 5 long, a yellow rod cannot be 9. The first version can be solved arithmetically while the second version requires considering the red rod as a unitized 5 length, which cannot be partitioned into smaller available Cuisenaire rods. Thus, the attempt to reconceptualize the red as a 5, and the yellow rod accordingly in relation to the red rod was related to whether students attempted detailing.

Early Algebra and Colligation in Mathematical Argument

Tall et al. (2011) suggest that as students explore various operations on mathematical objects, they observe patterns which eventually are described as rules of arithmetic. Specifically, describing and examining these rules leads to the development of deductive reasoning and mathematical argument. Indeed, various studies have observed connections between generalizing involved in early algebra and improved precision in claims and conjectures (e.g., Blanton & Kaput, 2011). Specific facets of early algebra lead towards this generalizing, such as looking for and describing patterns and examining the structure involved in arithmetic operations. The latter includes, among other concepts, understanding of equivalence and multiplicative reasoning, of which the present study focuses particularly.

As with other areas of early algebra, equivalence and multiplicative reasoning have been observed to relate to more sophisticated argumentation. Fosnot and Jacob (2009) observed that by encouraging second grade students to examine the meaning and application of equivalence, students began to engage more in deductive reasoning. Similarly, Morris (2009) described the process of using a schematic to model multiplicative relationships as facilitating deductive reasoning. In both studies, the authors describe the importance of students examining the relationships between mathematical objects, or between two or more collections of such objects. In this paper, we argue that such a focus can be extended to explain the linguistic actions of students' mathematical descriptions in tandem with their demonstrating an ability to relate mathematical objects.

Figure 1 provides an illustrative example from the present study's data of a third grade student's use of detailing. The student was provided Cuisenaire rods (color-coded length models) to respond to the statement "If a red rod is 5, a yellow rod can't be 9 because…" All red rods are 2 centimeters long and all yellow rods are 5 centimeters, and neither the red nor yellow rods could be partitioned into the designated lengths with other available rods. As shown in Figure 1, the child identifies red as a unit of 5, which can be operated upon. The child then creates a new unit 10, which is abstracted from their coordination of two red rods, and this is compared to yellow (i.e., 9). We consider each unit, and their abstracted coordinations, as signs. Signs point to meaning, and sometimes different signs can point to the same (or similar enough) meanings. Thus, the linguistic references in the child's writing serve as different signs than the units used by researchers to model the child's mathematical thinking. As such, we might refer to the linguistic references as linguistic units. These linguistic units point towards the same kinds of meanings that our models of the child's unit coordination do. Therefore, the child's ability to engage in detailing is related to the various efforts at unit coordination including those which are more explicitly multiplicative and those which are not.



"The yellow rod can't be 9 because **1 red one is 5** and that does not take it all up so you put **another 5** and it's still not big enough but **it [is] basically 10** and **it's** smaller than the yellow so the yellow can't be 9."

1 red is 5	2 reds are 10	10 < 9

Figure 1. Third grade student #319's response to the prompt "If a red rod is 5, a yellow rod can't be 9 because…" References are in bold in transcribed text with illustrated reference chain below.

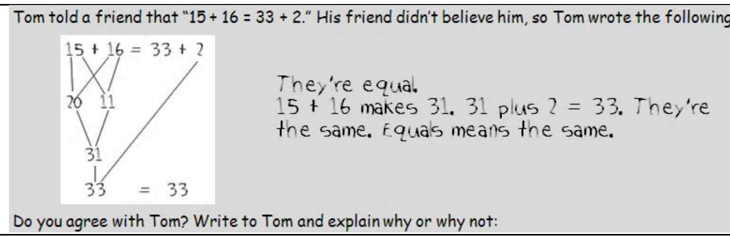
We hypothesize that a more sophisticated ability to coordinate units, often demonstrated via different aspects of early algebra, associate with an increased likelihood to engage in detailing. To test this hypothesis, we compared second and third grade students' scores on two early algebra assessments (multiplicative reasoning and equivalence) with their demonstrated detailing on two argumentative tasks embedded in early algebra (one multiplicative and one related to equivalence).

Methods

Sample and Measures

Data were collected from 168 second and third grade students in two suburban schools in a Midwestern U.S. state in May 2015. Second grade students were enrolled in four different teachers' classrooms (*n*=76) and third grade students were enrolled in three different teachers' classrooms (*n*=92). Participants completed an assessment packet focusing on equivalence and multiplicative reasoning, followed the next week by a packet of six mathematical argumentative writing tasks. The assessment on equivalence included items adapted from Fyfe, DeCaro, and Rittle-Johnson (2014). An overall higher percentage of correct responses is indicative of a relational view: equals means *the same as*. Our use of the assessment showed strong internal reliability (α=.89), supporting prior validation of the assessment by Fyfe et al. (2014). The assessment on multiplicative reasoning developed by Kosko and Singh (in review) is based on the notion that more abstracted unit coordination is representative of more sophisticated multiplicative reasoning. Our use of the assessment showed sufficient internal reliability (α=.79). Higher scores on the assessment are indicative of more sophisticated multiplicative reasoning. Mathematical argumentative writing tasks included six tasks with three centered on length model representations involving multiplicative/proportional comparisons and three centered on arithmetic involving a relational conceptions of equivalence. For sake of space, the present paper discusses only analysis of Tasks 3 and 5 (see Table 1).

Table 1. *Mathematical argumentative writing tasks*.

<table>
<tr><th>Task 3*</th><th>Task 5</th></tr>
<tr><td>If a red rod is 5 long, a yellow rod can’t be 9 because…</td><td>
</td></tr>
</table>

**Students completed task 3, and other length model tasks, with Cuisenaire rods.*

Analysis and Results

Writing samples from Tasks 3 and 5 were coded for evidence of detailing. Each task was initially examined for potential reference chains most likely to be observed. For Task 3, a complete reference chain indicative of detailing should include reference to the red rod being 5 long, operationalization of this given stating that two reds are 10 long, and extension of the latter proposition stating that 10 is less than 9 in the current context and this is not possible. Presence of some variation of these three propositions was coded as detailing. Both authors coded Task 3 independently prior to reconciling coding. Independent coding showed strong interrater reliability (K=.67), suggesting the coding for detailing on Task 3 is reliable (*M*=.22, *SD*=.42). For Task 5, there were two reference chains identified for coding of detailing:

- *15+16=31*; *33+2=35*; *31≠35.*
- *Tom moved the 2 from one side of the equation*; *It is inappropriate to move the 2*.

Presence of some variation of the propositions in each chain was coded as detailing. As with Task 3, independent coding by the authors showed evidence of strong interrater reliability (K=.64). The coding was reconciled by the authors prior to analysis (*M*=.39, *SD*=.49). Although the reference chains described for both tasks were considered the most likely to be used by students, we allowed for unanticipated reference chains evident of our definition of detailing to be coded.

Next, we used two independent t-tests per writing task to examine whether there was a difference in equivalence scores and multiplicative reasoning scores between students who demonstrated detailing and those who did not. Students demonstrating detailing on Task 3 were found to have statistically significant higher scores both on the equivalence assessment (t=3.73, p<.001) and on the multiplicative reasoning assessment (t=4.08, p<.001). Similar results for Task 5 indicate that students demonstrating detailing had statistically significant and higher scores on the equivalence (t=2.42, p=.02) and multiplicative reasoning assessment (t=2.20, p=.03). These results support our hypothesis that higher demonstrated ability to coordinate units mathematically associate with an increased likelihood to engage in detailing.

Discussion

The present study sought to examine whether second and third grade students' colligation, as demonstrated via their use of detailing, was related with their unit coordination involved in early algebra contexts. We found that students who demonstrated detailing in each task had statistically significant higher scores on equivalence and multiplicative reasoning tasks. Our Peircian semiotic view of detailing suggests that linguistic units, as signs, point towards the same kinds of meanings that our models of children's unit coordination do. Although there is no explicit use of equals in Task 3, comparison of *two red rods* (length 10) as being less than a *yellow rod* (length 9) points towards the same sort of coordination that a relational understanding of equivalence does. Similarly, Task 5 includes no explicit application of multiplicative concepts. Yet, construction of either common reference chain defined in our coding requires the child to consider parts of the equation separately while considering the whole equation (i.e., considering *15+16=31* and *33+2=35* separately before considering them together).

The findings presented in this study confirm and extend those of Kosko (in review), which suggested that inclusion of early algebra contexts may encourage detailing. Rather, the present study suggests that students' demonstrated understanding of early algebra concepts may facilitate their use of detailing. The present study also extends the findings of others (Fosnot & Jacob, 2009; Morris, 2009) who suggested relationships between students' deductive reasoning and their application of equivalence and multiplicative relationships. Although prior research has suggested a connection between early algebra and mathematical argumentation, there has been little to no analysis regarding specific linguistic features that evidence this connection. The results presented here provide much needed evidence regarding this connection.

References

Blanton, M. L., & Kaput, J. J. (2011). Functional thinking a s a route into algebra in the elementary grades. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 5-23). Berlin: Springer-Verlag.

Fosnot, C. T., & Dolk, M. L. A. M. (2009). Young mathematicians at work: The role of contexts and models in the emergence of proof. In D. A. Stylianou, M. L. Blanton, & E. Knuth (Eds.), *Teaching and learning proof across the grades*: A K-16 perspective (pp. 102-119). New York: Routledge.

Fyfe, E. R., DeCaro, M. S., & Rittle-Johnson, B. (2014). An alternative time for telling: When conceptual instruction prior to problem solving improves mathematical knowledge. *British Journal of Educational Psychology*, 84(3), 502-519.

Halliday, M. A. K., & Matthiessen, C. M. I. M. (2004). *An introduction to functional grammar* (3rd Ed.). London: Hodder Education.

Kosko, K. W. (in review). Making use of what's given: Children's detailing in mathematical argumentative writing. *Journal of Mathematical Behavior*.

Kosko, K. W., & Singh, R. (in review). Elementary children's multiplicative reasoning: Initial validation of a written assessment. *The Mathematics Educator*.

Kosko, K. W., & Zimmerman, B. (2015). *Emergence of argument in children's mathematical writing*. Paper presented at the annual meeting for the American Educational Research Association (Division C), Chicago, IL.

Liszka, J. J. (1996). *A general introduction to the semeiotic of Charles Sanders Peirce*. Indiana University Press: Bloomington, IN.

Morris, A. K. (2009). Representations that enable children to engage in deductive argument. In D. A. Stylianou, M. L. Blanton, & E. Knuth (Eds.), *Teaching and learning proof across the grades: A K-16 perspective* (pp. 87-101). New York: Routledge.

Peirce (1998). Nomenclature and divisions of triadic relations, as far as they are determined. In N. Houser, A. De Tienne, J. R. Eller, C. L. Clark, A. C. Lewis, & D. B. Davis (Eds.), *The essential Peirce: Selected philosophical writings* (Vol. 2; pp.289-299). Bloomington, IN: Indiana University Press (written originally in 1903).

Tall, D., Yevdokimov, O., Koichu, B., Whiteley, W., Kondratieva, M., & Cheng, Y-H. (2011). Cognitive development of proof. In G. Hanna & M. de Villiers (Eds.), *Proof and proving in mathematics education* (pp. 13-49). New York: Springer.

FACILITATING MATHEMATICAL CONVERSATIONS IN DIVERSE CLASSROOMS: A CASE STUDY

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The researchers explored the use of culturally responsive teaching (CRT) practices to determine the ways that teachers support their students to create viable arguments and critique the reasoning of others (SMP3). The cross-case analysis established that the support that teachers are able to provide to students depends on their teaching experience, teacher content and pedagogical knowledge, and classroom management. Study results provide implications regarding the kinds of support teachers might need as they attempt to motivate culturally diverse students to engage in SMP3.

Introduction

In answer to the less than perfect performance of the United States in international mathematics testing, such as the Trends in International Mathematics and Science Study (TIMSS), and in order to prepare future generations to meet the challenges of a global workplace, the National Governors Association (NGA) along with the Council of Chief State School Officers (CCSSO) developed the Common Core Standards Initiative and the Common Core State Standards for Mathematics (CCSSM). The goal proposed for the CCSSM is an ambitious one: to ensure that students possess the 21st Century skills necessary to successfully compete in a global economy. In order to achieve this goal, it is not enough that teachers know the mathematics described in the content standards. They must also have a deep understanding of the conceptual foundations of mathematics as well as the coherence between and among the content standards. Teachers must be mathematically proficient because the “emphasis on mathematical practices, which require students to be able to think mathematically and apply the techniques they have learned to rich problems in diverse contexts. Achieving this requires changes in the way mathematics is taught and assessed in most schools” (Shell Center for Mathematical Education, 2012, para. 1). To add to the intricacy of this issue, the increase in the numbers of ethnically diverse students in our schools have changed the classroom landscape dramatically and the performance of these students play an important part in the overall performance of US students.

Over time, trends of the achievement gap between minority and white students have been documented using standardized assessments from the National Center for Educations Statistics (NCES). Data from the 2005-2011 National Assessment for Educational Progress (NAEP) reveal that the gap in academic performance between white and minority fourth and eighth grade students in mathematics remains steady (NCES, 2011a; 2011b). These reports suggest that over the past decade, the achievement gap still remains and continues to be of concern as educators and policy makers strive to develop mathematically proficient students.

One prevailing school of thought is that diverse student achievement is adversely affected by "cultural discontinuity" (Tyler et al, 2008, p. 280). The theory of cultural discontinuity refers to the mismatch between students' home and school cultures. As a result, some minority students struggle with going between their home and school culture as no connection exists between them. This system of teaching may contribute to the underachievement of culturally diverse students by ignoring the influence of culture on learning. It incorporates the notion that achievement of ethnically diverse students can be improved by lessening or offsetting the cultural gaps between instructional methods and students' ways of knowing, thereby narrowing the achievement gap. Attention to student engagement may provide insight on narrowing the achievement gap.

Framework

The theory of cultural discontinuity and its relationship to student engagement provided insight in the framing of this study. Culturally responsive teaching (CRT) offers a framework for addressing cultural discontinuity by validating the cultures of students through the use of instructional practices that capitalize on the cultures of students. It is described as "using the cultural knowledge, prior experiences, frames of reference, and performance styles of ethnically diverse students to make learning encounters more relevant and effective for them" (Gay, 2000, p. 29). The use of CRT practices may have an impact on student engagement. Schussler (2009) explains that by making connections between students and the topics of instruction, teachers increase students' interest in learning thereby having a positive effect on engagement.

The focus of Standard for Mathematical Practice 3 (SMP3) is communication, which involves cultural exchange. As a result, considerable emphasis was placed on SMP3, "construct viable arguments and critique the reasoning of others" (NGA & CCSSO, 2010, p. 6). According to this practice, students are expected to build their reasoning in ways that are mathematically sound and logical and be able to assess their peers' logic. In order to create these arguments, students can use any model or representation to support their reasoning, giving validity to their logic (NCTM, 2000; NGA & CCSSO, 2010). Through their own experiences of constructing arguments, students are able to analyze the logic of others. The expectations associated with this standard should be incorporated in the classroom norms, and the classroom environment should be one that encourages discourse among the students.

The need for teachers' deeper understanding of discourse and how to encourage it in the culturally diverse classroom is great, given the body of research that confirms their efficacy in improving students' conceptual knowledge and in closing the engagement gap (Akkus & Hand, 2010; Boaler, 2008; Moses-Snipes, 2005; Powell & Kalina, 2009) and the way that it will facilitate meeting the goals of SMP3 that specifies that students should be able to construct sound arguments for their solutions while being able to assess each other's logic (NCTM, 2000, 2010; Phillips, 2008). By utilizing the 6th grade mathematics class as the setting, this study focused on the following questions:

1. What is the nature of the relationship between teachers' use of CRT practices (teacher characteristics and instructional practices) and students' engagement in SMP3?
2. How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom?
3. How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?

Methodology

The methodology used for this study was qualitative with a collective case study design. The use of a collective case study design allowed the researchers to look at how two teachers who received similar Professional Development (PD) implemented those strategies in their practice.

The following criteria were used to choose the teachers who participated in the study: they must teach mathematics, must demonstrate a willingness to participate in the study, likelihood of demonstrating CRT and SMP3 (as determined by school leaders), and length of teaching experience. The two teachers selected fit the criteria mentioned. One was in her first year teaching mathematics (Ms. Jane) and the other was into his 6th year teaching (Mr. John).

The data for this study were collected in four different stages during a period of two months. Stage one included initial observations and interviews with the teachers. Stage two included classroom observations during PD and teacher reflections. Stage three included classroom observations after PD and teacher reflections. Stage four included classroom observations done four weeks after the PD had concluded. Both researchers performed all observations and interviews; records include video and audio recordings, field notes, and code books.

Results

As students engage in SMP3, the social culture of the classroom and selected mathematical tasks become of great importance especially in environments of diverse students. A finding in this study suggests that a shift in teacher practice will support this reform. Both teachers demonstrated a shift in their practice; they demonstrated an increased ability to use CRT practices and an increased ability to facilitate students' engagement in SMP3. These instructional shifts resulted in a shift in their students' engagement in SMP3. As the teachers displayed more CRT teacher characteristics and more CRT instructional practices, their students demonstrated more student engagement practices consistent with SMP3. Furthermore, it became clear that the teachers' reflections on their practice and the engagement of their students through the lens of CRT impacted the changes in nature of instruction and the level of engagement of students in SMP3.

Another finding was that the participating teachers demonstrated varying levels of proficiency in the use of CRT instructional practices. It was particularly evident that each teacher exhibited a different set of CRT teacher characteristics and employed different CRT instructional practices. Variations in these characteristics and instructional practices related to each teacher's ability to engage students effectively. It became apparent that while the exhibition of specific CRT characteristics and practices draw a parallel to the skillful engagement of students in certain aspects of SMP3, teachers must demonstrate all characteristics and practices to engage students in activities that reflect SMP3. Inversely, the lack of any CRT characteristics or the inability to use each of the instructional practices to some degree undermines CRT as well as the depth and breadth of student engagement.

The results of this study suggest that there is a marked difference in how teachers support their students based on their teaching experience. Ms. Jane struggled with setting a structured classroom that promoted understanding and discourse. In her classroom, there was a lack of routines and students' expectations and students' misbehaviors were common. This affected their ability to be engaged in instructional activities. Mr. John had a classroom that was organized and had set expectations for the students; his classroom environment promoted students' engagement. The students in his class were diligently working and there were no discipline issues.

The teachers' pedagogical and mathematical content knowledge were two factors that were not taken into consideration at the time of the study. However, both proved to be major factors on how teachers were able to support their students' engagement in SMP3. The mathematics content and pedagogical knowledge of the teachers who participated in this study were vastly different. Ms. Jane was unsure of the content she taught, was unable to present context that was relevant to her students, and habitually taught from the teacher's edition textbook. On the other hand, Mr. John demonstrated that he had strong mathematical and pedagogical content knowledge because of the tasks he selected for his students to work with, the learning trajectories he followed, his ability to foresee common roadblocks his students might encounter, and his ability to present mathematical concepts in different contexts.

Classroom discourse entails students creating their own ideas and exchanging them. As they exchange mathematical ideas, the differences within those ideas become apparent. Teachers can use questioning to direct and enable students to communicate their ideas to each other and to examine those differences. The teachers' ability to ask the right questions at the right time was dependent on their content knowledge and their understanding of their students' learning trajectories. Ms. Jane asked closed-ended questions for the most part. Mr. John was able to ask questions that probed his students' knowledge and allowed them to build mathematical reasoning. In order for teachers to support the engagement of their students in SMP3, they must be familiar with questioning schemes that allow for open-ended answers and that enable students to construct and assess mathematical reasoning.

In order to be able to evaluate students' ability to construct and assess mathematical reasoning, teachers must listen to what the students are saying. In the initial interviews, both teachers expressed that if they grouped the students they would talk about the mathematics they were doing. However, they were not listening to the conversations the students were engaged in. Most of the time, these conversations were about subjects other than mathematics with the exception of exchanging answers. Ms. Jane was partially successful in listening to her students' conversations because she was able to determine they were off task in some instances. New teachers have a very difficult time during their first year teaching. The task of teaching can become so overwhelming that sometimes the focus is on controlling students' behavior rather than what is really occurring in classrooms. Mr. John had an easier time listening to his students' conversations. Since he did not have to spend any energy controlling his class, he was able to listen to the students' conversations and was able to direct classroom talk to where it needed to go. This experience allowed Mr. John to shift his views and he was able to see his role as a teacher morph into one of facilitator. Teachers must listen to the conversations students are having as they explain and justify their answers in order to guide them and to enable them to build reasoning based in mathematical concepts (Hadjioannou, 2007; Herbel-Einsenman & Otten, 2011).

Classroom management was not accounted for when planning the PD. The relationship between CRT and students' engagement in SMP3 is greatly impacted by the teacher's classroom management. This unexpected finding provided another lens on current research related to CRT and SMP3. These practices are important as they help to establish the culture of the classroom environment. This finding suggests that when implementing CRT and facilitating SMP3, special attention must be given to classroom management. The effect that poor classroom management had over students' behaviors cannot be ignored. Ms. Jane struggled with classroom management. Her classroom was chaotic at times because her students had no set rules to follow and they were often off task. On the other hand, Mr. John had created a positive environment in his classroom. He had routines in place and clear expectations of behaviors. His students were well behaved and were engaged in all activities he had for them and they knew the expectations he had of them. Creating a positive and nurturing classroom that is supported by CRT practices is vital for students' engagement in SMP3. Students are able to construct arguments and critique others when their classroom environment promotes understanding and where there are set routines and expectations.

Implications and Conclusion

The results of this study have potential implications in the ways teachers support students as they are engaged in SMP3. The position of the researchers is that the shift noticed in the teachers' application of CRT practices did affect the ways their students could explain and justify their answers. The first implication of this study is that the support students receive from their teachers depends on their teaching experience, student expectations, and content knowledge. The second implication is that teachers will need varying degrees of continuous support as they implement the changes necessary to have a classroom that allows students to be engaged in SMP3. The third implication is that teachers must create a classroom environment in which students feel comfortable and are able to exchange ideas, test hypothesis, and conduct experiments. The research findings suggest that teachers that make use of all components of CRT are more likely to effectively facilitate students' engagement in all indicators of constructing viable arguments and critiquing the reasoning of others, especially when working with students from diverse backgrounds.

The disparity noted in the two cases investigated in this study was due to the differences in the participants themselves. The teachers involved in this study had very different teaching styles, teaching experience, mathematical content and pedagogical knowledge, and expectations. These characteristics gave the study rich results and allowed the researchers to conclude that novice teachers need more support than experienced teachers. This diversity of teachers' characteristic should be taken into account in subsequent research as this can inform the support that different types of teachers might need.

References

Akkus, R. & Hand, B. (2011). Examining teachers' struggles as they attempt to implement dialogical interaction as part of promoting mathematical reasoning within their classrooms. *International Journal of Science and Mathematics Education*, *9*(4), 975-998.

Boaler, J. (2008). Promoting 'relational equity' and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167-194.

Gay, G. (2000). *Culturally responsive teaching: Theory, research, & practice*. New York: Teachers College Press.

Hadjioannou, X. (2007). Bringing the background to the foreground: What do classrooms that support authentic discussions look like? *American Educational Research Journal, 44*(2), 370-399.

Herbel-Einsenmann, B. & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education, 42*(5), 451-485.

Moses-Snipes, P. R. (2005). The effect of African culture on African-American students' achievement of selected geometry topics in the elementary mathematics classroom. *The Negro Educational Review, 56*(2/3), 147-166.

National Center for Education Statistics (NCES). (2011a). *Achievement gaps: How blank and white students in public schools perform in mathematics and reading on the national assessment of educational progress*. (NCES 2011-485). Institute of Education Sciences: US Department of Education. Washington, DC: NCES.

National Center for Education Statistics (NCES). (2011b). *The nation's report card. Mathematics 2011 National Assessment of Educational Progress at grades 4 and 8*.

(NCES 2011-458). Institute of education Sciences: US Department of Education. Washington, DC: NCES
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.
National Council of Teachers of Mathematics. (2010). *Making it happen: A guide to interpreting and implementing common core state standards for mathematics*. Reston, VA: NCTM.
National Governors Association (NGA) & Council of Chief State Schools Officers. (2010). *Common core standards for mathematics*. Retrieved from http://www.corestandards.org/
Phillips, R. (2008). Motivating prospective elementary school teaches to learn mathematics by focusing upon children's mathematical thinking. *Issues in Teacher Education, 17*(2), 7-26.
Powell, K. & Kalina, C. (2009). Cognitive and social constructivism: Developing tools for an effective classroom. *Education, 130*(2), 241-250.
Shell Center for Mathematical Education, U. C. at Berkeley, & The University of Nottingham. (2012). *Mathematics assessment resource service*. Available at http://map.mathshell.org/materials/index.php
Schussler, D. L. (2009). Beyond content: How teachers manage classrooms to facilitate intellectual engagement for disengaged students. *Theory into Practice*, *48*(2), 114-121.
Tyler, K. M., Uqdah, A. :., Dillihant, M. L., Beatty-Hazerlback, R., Conner, T., Gadson, N., Henchi, A., Hughes, T., Mulder, S., Owens, E., Roan-Belle, C., Smith, L., & Stevens, R. (2008) Cultural discontinuity: Toward a quantitative investigation of a major hypothesis in education. *Educational Researcher, 37*(5), 280-297.

ACADEMIC RIGOR IN SINGLE-SEX AND COEDUCATIONAL MIDDLE-GRADES MATH CLASSES

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This study examines disparities in academic rigor between single-sex and coeducational public school mathematics middle grades classrooms. Data analyzed includes 122 video recorded instructional sessions from all-boys, all-girls and coeducational classrooms. All sessions were evaluated and rated for academic rigor using the Instructional Quality Assessment rubric, which considers academic rigor, accountable talk, and teacher's clarity of expectations. Findings suggest there are no significant differences in academic rigor between single-sex and coeducational classroom settings. We question the veracity of implementing single-sex educational options in coeducational public schools and posit that it is the teachers, rather than settings, that are greater influencers of academic rigor in classrooms.

Introduction

Gender and gender equity continue to be areas of focus in mathematics education (Hyde & Mertz, 2009; McGraw, Lubienski & Strutchens, 2006). Though educational outcomes for both boys and girls have generally improved over the last 3 decades, incongruities between boys' and girls' performance in mathematics and science continue (Corbett, Hill & St Rose, 2008; Fryer & Levitt, 2009). According to Wilder and Powell (1989) there are a number of theories proposed to rationalize such discrepancies. These include biologically based explanations – particularly in relation to cognitive and spatial abilities; and early sex-typing and socialization processes, which have the potential to promote notions that mathematical activities are more masculine and literal activities are more feminine. Some scholars argue that teacher beliefs and classroom processes fail to identify or frame girls as mathematics and science learners (Pringle, et al., 2012) and that gender differences in performance are closely tied to cultural variations in opportunity structures for female students (Else-Quest, Hyde & Linn, 2010). Frenzel, Pekrun & Goetz (2007) have also hypothesized that it is affective, more so than cognitive variables, that contribute to differences in perceptions about mathematics. Specifically, they posit that variations in performance could be attributed to "girls' low competence beliefs and domain value of mathematics, combined with their high subjective values of achievement in mathematics" (p. 497). Recent findings, however, suggest that gender performance differences are small and correlations to aforementioned factors are complex, and tenuous (Guiso, Monte, Sapienza & Zingales, 2008).

Nonetheless, to address a perceived gender inequity and encourage greater participation in mathematics and science, many schools are increasingly introducing single-sex classroom instruction (SSI) within coeducational (COED) schools (Younger & Warrington, 2006). Unlike other countries such as Australia and the United Kingdom, this is a relatively new phenomenon in U.S. public schools and only became permissible in certain circumstances in October, 2006. Questions have risen about whether or not such settings can enhance the learning experiences of either boys or girls. Currently, no scholarly consensus exists on the efficacy of SSI on students' performance and attitudes (Pahlke, Hyde & Allison, 2014). Given the increasing number of schools and districts experimenting with SSI (Klein, et al., 2014), and the limited scholarly work that explores potential influences of these classroom settings, it is important that we generate a better understanding of mathematics teaching and learning in such environments and examine affordances or limitations, if any, such settings provide to either boys or girls. This study seeks to uncover differences in academic rigor and bring to fore inequities in instruction that may exist across different class structures in public coeducational schools with single-sex classrooms. The study is driven by the following question: To what extent do instructional quality and academic rigor in mathematics classrooms in middle schools with single-sex instruction options vary by class-type?

Theoretical Framework

Our interest in single-sex mathematics classes in public schools stems from our habitual, perpetual concerns about inequity and asymmetries in power relations and access to knowledge. Because we think about the ontology of power and processes through which power constrains or expands one's domain of action—one's agency—and because we seek to analyze, uncover, and transform these hegemonic processes, we are aligned with critical feminist theory (Dillabough & Arnot, 2001). Germane for this study, we are interested in how the quality of the academic environment in single-sex mathematics and science classes compares to coeducational classes in middle grades, and by extension, the impact of these classes on students' academic achievement. We understand the potential for SSI environs to provide nurturing, and supportive environments for students, both girls and boys, to flourish (Morrow & Morrow, 2005). Simultaneously, we are also keenly aware of dangers inherent in single-sex settings as venues for the perpetuation and reification of gendered stereotypes that serve to further marginalize girls. To better evaluate mathematics classroom processes in different class types, we use the Instructional Quality

Assessment rubric, which offers a useful way to assess the degree to which classroom instructional practices support efficacious teaching and learning, and has been found to be effective at teasing out differences in students' opportunities to learn mathematics. As Junker and colleagues (2005) note, assessing instructional quality is central to evaluating the rigor of instruction and the extent to which teachers promote student learning and achievement within given classrooms.

Method

Data for this study includes 122 videos of whole class instructional sessions collected over a three year period from 8 teachers, in 3 middle schools drawn from 3 rural school districts in the south eastern region of the United States. Of the 8 teachers, one taught coeducational classes exclusively, while the remaining seven taught either all three class types, or some combination of boys only, girls only, and coeducational classes. Each video recorded session comprised of video captured by two cameras; one fixed camera, mounted on a tripod at the back of the classroom that captured whole class interactions and board activities, and a second handheld camera operated by one of the research team members that focused on students' tasks, student-student interactions as well as teacher interactions with individuals, groups, and the whole class. Each instructional session was analyzed for rigor of classroom instruction using the Instructional Quality Assessment rubrics [IQA] (Junker, et al., 2005). The IQA has six rubrics organized around three themes, that is: task potential to engage students in rigorous thinking on challenging mathematical content; accountable talk as indicated by the cognitive level of teacher questions and press for evidence supporting reasoning, together with related student responses; and rigor in teacher expectations. Table 1 below gives a summary of the rubric descriptions. Each rubric was rated on a 5-point scale (0= the absence of task engagement, classroom discussion, and teacher expectation; 4 = high level engagement with tasks, classroom discussions undergirded by appropriate mathematical evidence, and teacher expectation for higher order, rigorous thinking). The research team established inter-rater reliability by analyzing videos of SSI and COED classrooms drawn from the existing data set.

Table 1. *Instructional Quality Assessment (IQA)Academic Rigor in Mathematics*

Academic Rigor	
Rubric 1	*Potential of the Task*: Did the task have potential to engage students in rigorous thinking about challenging content?
Rubric 2	*Implementation of the Task*: At what level did the teacher guide students to engage with the task in implementation?
Rubric 3	*Discussion of the Task*: To what extent did students show their work and explain their thinking about the important mathematical content?
Accountability to Knowledge and Rigorous Thinking	
Rubric 4	*Asking (by Teachers)*: Were students pressed to support their contributions with evident and/or reasoning?
Rubric 5	*Providing (by Students)*: Did students support their contributions with evidence and/or reasoning?
Clear Expectations	
Rubric 6	*Academic Rigor in the Teacher's expectations*: At what level did the teacher expect the students to engage in complex thinking and/or explore and understand mathematical concepts, procedures, and/or relationships?

The 122 instructional sessions were scored on the IQA rubrics and mean scores on each rubric calculated for each classroom and teacher. To test for statistical significance in academic rigor scores among the three classroom types, one way analyses of variance (ANOVA) were conducted, with the 'class type' and 'teachers' as independent variables, and scores on the IQA as dependent variables. Data were evaluated for differences by sex for students in SSI classes (boys only compared to girls only); by class-type (three way comparison of boys only, girls only, and coeducational); by comparing combined single-sex against coeducational classes; by teacher, and contrasts done by teacher and class-type. For ANOVA in which a significant difference ($\alpha = .05$) among the means was concluded, Tukey's Pairwise Comparison post hoc test was utilized. All statistical calculations were performed using the software program JMP Pro 11.

Findings

Due to paper length restrictions, abridged results are presented and will be expounded on in the final conference presentation. Preliminary findings indicate that there were no statistical differences in academic rigor among classes for comparisons done by sex for students in SSI classes, by class-type, and by combined single-sex and coeducation class comparisons. Table 2 below shows data from analysis by class-type.

Table 2. *Math IQA Scores by Class-Type*

Class type	*n*	Rubric 1		Rubric 2		Rubric 3		Rubric 4		Rubric 5		Rubric 6	
		M	SD	M	SD	M	SD	M	SD	M	SD	M	SD
Boys	41	2.244	0.5376	2.049	0.3841	1.927	0.5191	1.732	0.7080	1.585	0.6315	2.073	0.4685
Coed	43	2.186	0.5878	2.047	0.4857	1.837	0.6145	1.791	0.9894	1.814	0.9821	2.093	0.4788
Girls	38	2.263	0.5543	2.132	0.4748	1.868	0.5287	1.711	0.7318	1.763	0.7510	2.211	0.4132
p		**0.8101**		**0.6358**		**0.7579**		**0.9009**		**0.4021**		**0.3572**	
F		0.211		0.4546		0.2778		0.1044		0.9182		1.0386	

Table 3 below presents results from further analysis where we compared aggregated teachers' scores on the IQA, irrespective of class-type taught. Results point to a wide disparity in the level of instructional quality among the 8 teachers. It is interesting to note that even though individual teacher scores changed from rubric to rubric, those with better than average scores consistently performed better than average across the rubrics, while those with low level instructional quality scores consistently performed at a low level.

Table 3. *Math IQA aggregated scores by teacher*

Teacher	*n*	Rubric 1		Rubric 2		Rubric 3		Rubric 4		Rubric 5		Rubric 6	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
B	12	2.42^{a}	0.51	2.25^{ab}	0.45	2.00^{ab}	0.43	2.33^{a}	0.65	2.42^{a}	0.67	2.42^{a}	0.51
C	30	1.97^{b}	0.18	1.97^{b}	0.18	1.73^{b}	0.45	1.20^{bc}	0.41	1.23^{bc}	0.43	2.00^{b}	0.00
E	9	2.89^{a}	0.33	2.56^{a}	0.53	2.22^{ab}	0.44	2.89^{a}	0.60	2.89^{a}	0.60	2.56^{a}	0.53
H	30	1.93^{b}	0.25	1.93^{b}	0.25	1.80^{b}	0.41	1.50^{b}	0.51	1.43^{b}	0.50	2.00^{b}	0.00
L	5	2.80^{a}	0.45	2.40^{ab}	0.55	2.20^{ab}	0.45	2.60^{a}	0.55	2.60^{a}	0.55	2.40^{ab}	0.55
M	9	1.44^{c}	0.53	1.44^{c}	0.53	1.00^{c}	0.87	0.67^{c}	0.50	0.67^{c}	0.50	1.33^{c}	0.50
P	13	2.69^{a}	0.48	2.15^{ab}	0.38	2.08^{ab}	0.28	1.69^{b}	0.48	1.69^{b}	0.48	2.15^{ab}	0.38
V	14	2.71^{a}	0.47	2.36^{a}	0.50	2.29^{a}	0.47	2.64^{a}	0.50	2.43^{a}	0.51	2.50^{a}	0.52
p		**<.0001**		**<.0001**		**<.0001**		**<.0001**		**<.0001**		**<.0001**	
F		23.7415		9.0642		8.1114		29.2097		26.4321		13.9909	

Note: *n* = number of video sessions analyzed. Item means under each rubric with a different letter superscript (a, b, c) indicate significant difference among teachers based on F-test with p<.05.

To tease out the existence or lack thereof of significant differences in each teacher's rigor that could be associated with the class-type taught (i.e., single-sex, coeducational), we conducted separate analysis for each teacher and compared each teacher's IQA score for different math classes they taught. Only results from seven of the eight teachers in the study are represented in Table 4 below as one teacher's classes were all coeducational. Additionally, results in Table 4 show the average score per teacher across the 6 rubrics.

Table 4. *Analysis of Individual teacher average scores on IQA based on class type.*

		Average			
		Boys	Girls	Coed	
Teacher	*n*	M (SD)	M (SD)	M (SD)	*p*
B	12	n/a	2.3 (.50)	2.3 (.17)	*0.880*
C	30	1.7 (.19)	1.5 (.13)	1.6 (.16)	*0.163*
E	9	2.8 (.00)	2.7 (.36)	2.4 (.00)	*0.698*
H	30	1.8 (.21)	1.7 (.26)	1.7 (.34)	*0.847*
M	9	1.3 (.30)	n/a	0.9 (.46)	*0.210*
P	13	2.1 (.28)	1.9 (.35)	n/a	*0.224*
V	14	2.4 (.35)	2.6 (.39)	n/a	*0.402*

Note: M = Mean, SD = Standard Deviation. *n/a* indicates that the teacher did not teach that class type.

Discussion and Conclusion

The main goal of the present study was to examine the extent to which instructional quality and academic rigor in middle school mathematics classrooms varied by class type. As the results from our targeted, focused study indicate, there were no significant differences in academic rigor based on class type taught. Rather, as the mean scores by class type (Table 2) show, we had minimal variation in aggregated scores for each class type across the six IQA rubrics. Furthermore, the significant difference in teacher scores (Table 3) suggest teacher based differences rather than the restructured classrooms were the larger influence on instructional quality. This is made more evident when we compare each teacher's level of rigor when single-sex and coeducational classes taught. Individual teacher scores for either single-sex or coeducational classes were stable across the rubrics, with some teachers consistently showing better than average scores across the rubrics, and vice versa, irrespective of class type taught. No consistent partialities were noticed, either in favor of, or against single-sex or coeducational classes. This is in opposition to suggestions that separating students by sex would serve to increase rigor in instruction and hence promote achievement in mathematics (Sax, 2007). Though the sample size is too small to generalize to the broader population of single-sex classrooms across the U.S., findings do raise questions about calls for sex based classrooms as a way to address equity issues in the teaching and learning of mathematics.

Many questions and considerations remain regarding the utility or wisdom of separating students according to their biological sex. It is not clearly evident that students attending single-sex classes are exposed to superior or targeted instruction that might lead to increased student participation and achievement in STEM based courses. We realize that there might be other rationales for instituting single-sex instruction that may be unconnected to the presence or absence of a significant relationship between class type and instructional rigor. As Kaiser and Rogers (2005) would argue, efforts, for example, that seek to promote girls' attitudes and aptitudes towards mathematics would do well to focus on interventions that address inclusive instructional practices. This argument is made more profound when we consider the lackluster instruction observed in participant teachers' mathematics classrooms. Though gaining a fuller picture of the impact of segregation by sex on mathematics instruction will take time to rigorously examine; this study offers measured insight regarding instructional quality and rigor in single-sex and coeducational classroom environments.

References

Corbett, C., Hill, C., & St Rose, A. (2008). *Where the girls are: The facts about gender equity in education*. American Association of University Women Educational Foundation.

Dillabough, J., & Arnot, M. (2001). Feminist sociology of education: dynamics, debates and directions. In Demaine, J. (ed.). *Sociology of education today*, 38-48.

Else-Quest, N. M., Hyde, J. S., & Linn, M. C. (2010). Cross-national patterns of gender differences in mathematics: a meta-analysis. *Psychological Bulletin*, *136*(1), 103.

Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Girls and mathematics—A "hopeless" issue? A control-value approach to gender differences in emotions towards mathematics. *European Journal of Psychology of Education*, *22*(4), 497-514.

Fryer Jr, R. G., & Levitt, S. D. (2009). *An empirical analysis of the gender gap in mathematics* (NBER Working Paper No. 15430). Cambridge, MA: National Bureau of Economic Research.

Guiso, L., Monte, F., Sapienza, P., & Zingales, L. (2008). Culture, gender, and math. *Science*, *320*(5880), 1164-1165.

Hyde, J. S., & Mertz, J. E. (2009). Gender, culture, and mathematics performance. *Proceedings of the National Academy of Sciences*, *106*(22), 8801-8807.

Junker, B. W., Weisberg, Y., Matsumura, L. C., Crosson, A., Wolf, M., Levison, A., & Resnick, L. (2005). *Overview of the instructional quality assessment*. Regents of the University of California.

Kaiser, G., & Rogers, P. (Eds.). (2005). *Equity in mathematics education: Influences of feminism and culture*. Routledge.

Klein, S., Lee, J., McKinsey, P., & Archer, C. (2015, June 4). *Identifying US K-12 public schools with deliberate sex segregation*. Retrieved from http://feminist.org/education/pdfs/IdentifyingSexSegregation12-12-14.pdf

Lubienski, S. T., & Gutiérrez, R. (2008). Bridging the gaps in perspectives on equity in mathematics education. *Journal for Research in Mathematics Education*, *39*(4) 365-371.

McGraw, R., Lubienski, S. T., & Strutchens, M. E. (2006). A closer look at gender in NAEP mathematics achievement and affect data: Intersections with achievement, race/ethnicity, and socioeconomic status. *Journal for Research in Mathematics Education*, *37*(2), 129-150.

Morrow, C. M & Morrow, J. (2005). Connecting women with mathematics. In Kaiser, G., & Rogers, P. (Eds.), *Equity in mathematics education: Influences of feminism and culture*, 13-26. Routledge.

Pahlke, E., Hyde, J. S., & Allison, C. M. (2014). The effects of single-sex compared with coeducational schooling on students' performance and attitudes: A meta-analysis. *Psychological Bulletin*, *140*(4), 1042.

Pringle, R. M., Brkich, K. M., Adams, T. L., West-Olatunii, C., & Archer-Banks, D. A. (2012). Factors influencing elementary teachers' positioning of african american girls as science and mathematics learners. *School Science and Mathematics*, *112*(4), 217-229.

Sax, L. (2007). *Why gender matters: What parents and teachers need to know about the emerging science of sex differences*. Harmony.

Wilder, G. Z., & Powell, K. (1989). *Sex differences in test performance: A survey of the literature*. (College Board Report No. 89-3, ETS RR No. 89-4*)*, New York: College Entrance Examination Board.

Younger, M. R., & Warrington, M. (2006). Would Harry and Hermione have done better in single-sex classes? A review of single-sex teaching in coeducational secondary schools in the United Kingdom. *American Educational Research Journal*, *43*(4), 579-620.

LEARNING ABOUT ELEMENTARY PRESERVICE TEACHERS FROM THEIR OBSERVATIONS OF STRUGGLING LEARNERS

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This study examined elementary prospective teachers' professional noticing of the mathematical thinking among students who have been identified as struggling learners. Prospective teachers observed students and documented their noticings in multiple settings. The analysis of these noticings provides a snapshot into their perceptions and insight into the pedagogical needs of prospective teachers to effectively attend, assess, and respond to the needs of all students. Comparisons between noticings that occurred while prospective teachers were instructing students versus those while observing mathematics lessons indicate the need to support prospective teachers as they move from observers to teachers.

Perspectives and Theoretical Framework

Grounded in the instructional construct of professional noticing (Sherin, Jacobs, & Phillipp, 2011), this study examines elementary preservice teachers' (EPTs') professional noticings about student mathematical thinking and behaviors when teaching and observing in elementary classrooms. Professional noticings are central to the effectiveness of mathematics teachers (Jacobs, Lamb, & Phillipp, 2010; Mason, 2002; Sherin et al., 2011). Professional noticing includes (a) identifying the important elements that occur, (b) utilizing information about the classroom and context to make sense of the situation, and (c) connecting these specific moments in the classroom to larger issues and topics related to teaching and learning (Sherin & van Es, 2009). Being able to identify and respond to student thinking in a strategic manner that advances learning for all is a challenging task for EPTs. When provided specific targeted experiences, it is possible to improve EPTs ability to attend, interpret and decide how to act upon professional observations of student thinking (Schack et al., 2013). By learning what is important to notice and how to respond to these noticings teachers can improve student learning (Sherin et al., 2011).

Teacher education often focuses on the planning and preparing aspect of teaching with little attention to important interactive aspect of teaching that occurs while the lesson is being taught (Grossman et al., 2009). While planning is an essential element, it is also important to be prepared for the in the moment instructional decisions that teachers must make on a daily basis in order to be effective. The decisions that are made through observation and reflection-in-action are often invisible to EPTs, unless they are explicit taught to look for these opportunities, observe teachers making these decisions, and provided opportunities to practice these skills

(Schack et al., 2013). Professional noticing isn't a skill that develops intuitively over time. Instead, it requires intentional focus and practice (Jacobs et al., 2010).

This study provides insights about EPTs perceptions of teaching and learning elementary mathematics based on the notes they take in their mathematics journal while observing, teaching, and interviewing elementary students. This included the noticings made when interacting with the students in the role of the teacher and when they are observing the students while an experienced teacher is instructing. It explored EPTs interpretations and observations about the mathematical thinking among students that are identified as a student struggling in of mathematics using the Tier 2 identification level in the Response to Intervention Framework (Batsche et al., 2005). By analyzing how EPTs interpret student work and what elements of mathematics they emphasize, such as mathematical practices, conceptual understanding, and/ or procedural knowledge, mathematics teacher educators could gain deeper understanding of misconceptions and priorities EPTs are placing in relation to teaching elementary mathematics.

Methodology

Twenty-four EPTs participated in this study that was conducted during a semester long mathematics methods course and 24 hour a week field placement in grades 1-5. Twenty- three EPTs were female and one was male. Each EPT was the only EPT in the classroom and had a cooperating teacher to provide mentorship and support. Also the methods instructor visited each placement weekly to answer questions and provide support beyond the on campus course. Before enrolling in this course, EPTs completed at least two mathematics content courses designed specifically for elementary teacher and completed two semesters as a cohort in the elementary education professional program. The program focused on preparing elementary teachers in grades K-6 to teach in all academic content areas. In addition to the ongoing professional noticing assignment, the EPTs completed multiple experiences in the field that included teaching lessons, observing students, working in small groups, and working one on one.

During the tri-weekly field placement in a rural setting, EPTs observed 2 students who were identified as having difficulty in the area of mathematics using the Tier 2 identification level in the Response to Intervention Framework. EPTs documented their noticings in a variety of ways as described below (see Data Sources). Each record was coded and analyzed using content analysis. Content analysis is a careful examination of materials that will lead to an understanding of meanings (Berg & Lune, 2012; Bryman, 2004; Guba & Lincoln, 1981). The codes developed inductively from the data. All journal notes, work samples, lesson plans, and reflections of video-taped lessons from one EPT were analyzed collectively before moving to another EPT's data for analysis. Summaries were developed on the first reading, but were revised as needed upon future readings and analysis. These summaries led to explication occurring to clarify and make annotations about context as needed. For example, if EPTs were observing students in a classroom that was rich with dialogue and exploration as opposed to a fairly traditional one, this might not be noted by on in the journal and reflection, but would impact what the EPT could observe from each student. Analysis of the situation and context surrounding the data was an important aspect of analysis. The multiple data in each set was analyzed using cross case summative content analysis. This allowed for meaningful patterns across the data sources to emerge as well as to identify the role context played in the interpretations made by EPTs. This cross case summative analysis served to delineate a combination of possible factors that may have contributed to the data. It also supported triangulation of data within the data from moments as observer and data from moments as teacher (Yin, 2014).

After each data set was analyzed, data sources were cross-analyzed by type. For example, all journals were read, analyzed, and coded together without attention to previously determined codes from the individual EPT analysis. Cross analysis allowed the researcher to compare across settings and EPTs to identify patterns within each data source that may be unique to that data source. Identifying commonalities among multiple cases contribute to generalizations (Miles & Huberman, 1994). Data from noticings when the EPT was observer were separated from noticings when the EPT was teacher. Once this analysis was conducted, generalizations from the categories and codes were revisited focusing on EPTs observations, mathematical noticings, generalizations, beliefs and beliefs about mathematics learning and instruction.

Data Sources

There were multiple data sources. Observations, student work samples chosen by the EPTs, and their analysis of these artifacts were used for analysis of the EPTs professional noticings. The EPTs compiled a final summary and analysis of their observations and understandings of the students as mathematical learners based on the data they collected during the field placement. These sources provided triangulation of data from moments when the EPTs were directly interacting with the students as well as when the EPTs were observing.

EPTs kept a journal of their observations of students throughout their time in their field placement. This journal was contained open-ended notes that the students took while observing and interacting with students. Before the placement began, EPTs watched a 15-minute mathematics lesson on video and practiced taking observation notes as part of the methods course. They discussed the importance of only noting observable behaviors, avoiding jumping to conclusions and making judgments, and focusing on content knowledge and strategies. The field placement observations began by the EPT observing the students while the teacher taught mathematics and while students were working in groups and independently. Then the EPTs conducted an interview with each student to assess their perceptions and attitudes about mathematics as well to discuss mathematical problems with the student and determine areas of strengths and weaknesses regarding numbers and operations. As the semester progressed EPTs began to teach mathematics lessons and recorded observations of students during this time as well. They worked with students in small groups and taught at least two mathematics lessons to the entire class that were videotaped for reflection after the teaching experience. EPTs made in the moment notes on student work as it was collected, to allow for later interpretation.

Before each mathematics interaction, the EPT noted in the journal if they were teaching or observing the student during instruction by the classroom teacher. After each mathematics interaction they led, EPTs added to their "in the moment notes." When they led the instruction, they reflected upon specific observable things they noticed about the students' mathematical thinking, their interpretations of this thinking, and how they responded. After each mathematics interaction that the cooperating teacher led and the EPT observed, the EPTs made notes of what they noticed about the students' mathematical thinking, the interpretations of the students' thinking, and how they would respond if they were leading the instruction.

Each of these elements was used for analysis of the EPTs professional noticings. In addition, the EPTs watched their videotaped lesson and added entries to their journals of what they noticed about the identified students during the lesson that they did not notice when they were actually teaching the lesson. Finally the EPTs compiled a final summary of their observations and understandings of the students as mathematical learners based on the data they collected during the field placement. These sources provided triangulation of data from when the EPTs were directly interacting with the students as well as when the EPTs were observing.

Findings

This study found that EPTs struggled to focus on mathematical thinking during both teaching and observing. Behavior was the initial focus of notes. EPTs noted things such as if the student was off task, didn't complete the assignment in time, or talked to their neighbor, etc… more often than they noted things such as how a student solved a double-digit addition problem. For example, one EPT wrote, "Student X is playing in desk during instruction." While these observations can be informative, attention to attempts to solving mathematics, discussions on the mathematics, and use of manipulatives are also important in order to understand student mathematical thinking. Initially if mathematics content was included in the observational notes, the notes stated if answers were accurate, rather than examining strategies, models, or questions related to the content. For example, "student completed 20 single digit addition facts correctly." Often the errors were attributed to lack of effort. For example, one EPT wrote, "student missed last 3 word problems, because he was hurrying and didn't care." These observations missed the fact that often the final problems involved more contextualization or higher order thinking, which could account for the errors.

As the semester progressed, data began to shift towards student thinking. They began to write more about the mathematics and less about the behaviors. For example, one EPT who initially only wrote about a student's lack of effort and misbehaviors wrote, "Student Y seems to grasp double digit multiplication using the partial product method, but skips steps when applying the traditional algorithm. This needs to be revisited one on one in order to identify where the break down seems to happen." During observations, EPTs were able to take more detailed notes about student thinking, but their inability to discuss the thinking with the student impacted their interpretations and understanding of the student thinking. For example, one EPT saw a computational error that appeared to be related to regrouping. However, as an observer, she noted that she didn't feel comfortable to ask the student follow up questions to fully grasp the misunderstandings that caused the error or provide instructional support to help the student make sense of the problem. Also, EPTs often focused on what the teacher did to cause misbehaviors, rather than focusing on the student thinking. While observing and critiquing teachers can be helpful, this assignment was to focus on student thinking and affect.

When in the role of teacher, whether as one-on-one interviewer, small group teacher, or whole class teacher, EPTs were easily distracted by things such as management or preparing for the next instructional step. For example, one student took a page of notes during a math lesson observed, but wrote one note about what was observed when teaching. Although they did not notice as much from their students, the insight they had into the observed moments was greater. When they took the time to notice the students, they would probe further, push discussions among peers, and try to explore content from different angles all to support the understanding of the students. For example, the EPT who took one note about a lesson they taught noted that a student struggled to translate the group activity of regrouping with base ten blocks to the abstract concept of solving equations. In a follow up activity and in the reflection, the EPT was able to provide more scaffolding and questioning to aide in this transition. Another EPT noted that a student wasn't leaving a zero as a placeholder when appropriate in regrouping. The EPT designed questions based on this observation to allow students to discuss the reason that zero needed to be used in these problems. This trajectory of learning to professionally notice is illustrated in Figure 1.

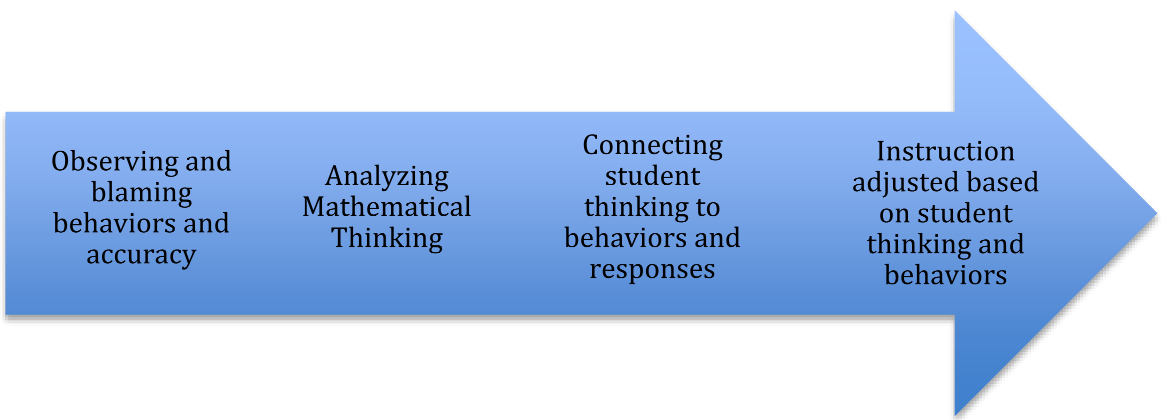

Figure 1. The developmental trajectory of professional noticing for EPTs

It was interesting to note that when students struggled with content when the EPT was observer, they often noted ways they would change the lesson to ensure the student understood. Initially this came in the form of a confident critique of the teacher without as much attention to student thinking and needs. For example, one student wrote, "The teacher told students what to do and didn't enough time to complete the worksheet or to ask questions about what they didn't understand." However, as time progressed, they began to identify elements the student did understand and would share ways to build off of these strengths. This same EPT later wrote about an observed lesson, “Student X showed difficulty with using regrouping when needed in double digit addition, but was able to solve double digit addition problems where regrouping isn’t necessary.” There was less confidence when the EPT was in the role of teacher. They would often blame the student for “not paying attention,” “rushing,” or “not trying.” Often the EPT would suggest more repeated practice, or express that “the content had been explored every way possible.” EPTs expressed more of a deficit way of thinking about students and fixed mindset when they were in the role of teacher.

Professional noticings are central to the effectiveness of mathematics teachers (Jacobs, et al., 2010; Mason, 2002; Sherin et al., 2011). The ability to identify mathematical thinking, interpret this thinking and respond based on the needs of the individual during instruction is an expertise that is extremely complex. For an EPT this can be challenging when working with students who are identified as having difficulty in mathematics. While professional noticing is usually described as the “ in the moment decision making” that is required of teachers during instruction (Jacobs , Lamb, & Phillipp, 2010), for this study an additional element was added. EPTs also shared their professional noticings while observing their cooperating teacher during instruction. This allowed the EPTs to focus on the students without the demands of the other critical elements of instruction. This could be related to other studies on professional noticing that used videotaped lessons (Schack et al., 2013, Sherin & Van Es, 2009).

Scholarly Significance

This study contributes to the growing body of research on professional noticing by exploring the impact of observing versus teaching in professional noticings among EPTs. While it does raise the same concerns found in other studies on the ability of EPTs to notice and utilize student thinking in-the-moment, it also provides insight into EPTs ability to make the most of these moments when they are observed. It extends beyond video professional noticings, because these observed incidences occurred in settings where the EPTs were able to observe the same students over extended periods of time. They seemed to move beyond behaviors and explore the thinking based on a well-rounded view of the student. However, when teaching it still was difficult for EPTs to utilize this newfound skill in action. Professional noticing is extremely complex. If EPTs are going to be effective in their instruction, they need to learn to attend and respond to these teachable moments. By identifying the areas of struggle for EPTSs when transitioning to the role of teacher, teacher educators can focus and support these areas. Teacher educators need to continue to provide opportunities for EPTs to focus on the strengths of all learners and to provide strategies for EPTs to build on these strengths to increase the mathematical competence of their students.

References

Batsche, G., Elliott, J., Graden, J. L., Grimes, J., Kovaleski, J. F., Prasse, D., Schrag, J., & Tilly, W.D. (2005). *Response to intervention policy considerations and implementation.* Reston, VA: National Association of State Directors of Special Education.

Berg, B. L., & Lune, H. (2012). *Qualitative research methods for the social sciences* (8th ed.). Boston: Pearson.

Bryman, A. (2004). *Social research methods* (2nd ed.). New York: Oxford University Press.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, 111, 2055–2100. http://www.tcrecord.org.

Guba, E. G., & Lincoln, Y. S. (1981). *Effective evaluation: Improving the usefulness of evaluation results through responsive and naturalistic approaches.* San Francisco, CA: Jossey-Bass.

Jacobs, V. R. Lamb, L. L. & Philipp, R. A. (2010). Professional noticing of children’s mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169-202.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: Routledge-Falmer.

Miles, M.B. & Huberman, A. M. (1994). *Qualitative data analysis*. Thousand Oaks, CA: Sage Publications.

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers’ professional noticing of children’s early numeracy. *Journal of Mathematics Teacher Education*, 16(5), 379-397.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (eds) (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York: Routledge.

Sherin, M. G., & Van Es, E. A. (2009). Effects of video club participation on teachers’ professional vision. *Journal of Teacher Education*, 60(1), 20–37.

Yin, R. K. (2014). *Case study research: Design and methods* (5th ed., p. 312). Thousand Oaks, CA: SAGE Publications.

DRAW YOURSELF DOING MATHEMATICS: ASSESSING A MATHEMATICS AND DANCE CLASS

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RCML-developed drawings prompts were adapted for use in a general education mathematics classroom to understand the affective factors influencing the learning of mathematics, trends in stereotypical views of the subject, and changes in views toward mathematics over a semester. This prompt was used as part of a larger study to assess the overall effectiveness of a course pairing the study mathematics and dance. Efforts to develop standardized coding instruments for this drawing prompt are reported. Also, trends in the drawings, changes in pre to post drawings, and differences in drawings from the traditional and experimental classrooms are shared.

Related Literature

Chambers (1983) designed an instrument called the Draw-a-Scientist Test to investigate the development of stereotypical views of scientists among children. This test was an adaptation of the previously developed Draw-a-Man and Draw-a-Person prompts (Chambers, 1983). In the Draw-a-Scientist Test, Chambers asked students to "draw a picture of a scientist." Chambers then went on to analyze the drawings for evidence of stereotypical scientist items such as lab coat, eyeglasses, facial hair, scientific instruments and lab equipment, books and filing cabinets, and formula. Many researchers have since developed additional tools for analyzing the Draw-a-Scientist Test (Farland-Smith, 2012; Finson, Beaver, Cramond, 1995). In particular, Farland-Smith (2012) delineated the categories of appearance, location, and activity and designated particular rubric scores for each category. The instrument was modified by Thomas, Pedersen, and Finson (2001) to ask participants to draw a science teacher.

More recently the method was again modified to create prompts regarding mathematics. Burton (2012) asked preservice teachers to draw their initial impressions of math. Mcdermott and Tchoshanov (2014) also worked with preservice teachers and asked them to "draw yourself learning mathematics" and "draw yourself teaching mathematics." Utley, Reeder, and Redmond-Sanago (2015) presented their work on the Draw-A-Mathematics-Teacher test and accompanying rubric at the 2015 RCML Annual Conference. Like previous drawing prompts, this test was used with preservice teachers.

While extensive work surrounds the use of drawing prompts to understand views of science, science teaching, and mathematics teaching, further study was needed to investigate the use of such prompts to understand the views toward mathematics of mainstream students. This paper reports on an attempt to fill this gap in the research.

Methodology

The prompt "draw yourself doing mathematics" was used as one of several tools to assess the effectiveness of an innovative general education course pairing mathematics and dance. The course used short movement studies to get students exploring and experiencing the recognition, creation, and analysis of patterns. These activities were based on extensive research on arts integration methods that draw on students' abilities in other modes of learning to study mathematics (Rinne, Gregory, Yarmolinskaya, & Hardiman, 2011; Schaffer, Stern, & Kim, 2001). Not only did this class attempt to introduce mathematics through the use of real activities requiring active participation, the instructors built a classroom environment that emphasized the collaborative nature of mathematics, deemphasized lecture and memorization, celebrated multiple problem solving strategies, and dealt with counterproductive affective factors such as math anxiety. This drawing prompt was chosen as one of the assessments used to evaluate this course because of the potential for the prompt to uncover incoming student images toward mathematics and the influence an innovative mathematics class could have on these images in one semester. The phrase "draw yourself doing mathematics" was chosen over the prompt used by Mcdermott and Tchoshanov (2014) to "draw yourself learning mathematics" because the researchers wanted to avoid biasing responses to classroom learning.

Students from the experimental course and students from a traditional mathematics course were asked to respond to this prompt on the first and last day of their respective courses. The course instructors were not in the classroom when the prompt was administered. The prompt was typed at the top of an 8.5"x 11" page, and students had the remainder of the page to draw responses. The students were asked to provide their names on the prompt so that pre and post drawings could be compared. Prior to distributing the prompt, the researcher explained that the quality of the drawings did not matter to the researchers and instructed the students to draw what came to mind when thinking of themselves doing mathematics. Also, the full prompt read "Draw yourself doing mathematics. Don't worry about the quality of the drawing. Just sketch what comes to mind." The students were given about 10 minutes to complete their drawings. The students received no monetary or grade incentive for participating in this research.

To analyze the responses to the prompt the research team completed several iterations of coding. All of the coding of the pretest and posttest drawings were completed blindly so the researchers were unable to distinguish between treatment and control drawings. First, the team open-coded the pretest drawings to explore themes emerging in the drawings. Secondly, the team explored related literature for more systematic ways of coding the drawings and adapted the three categories of appearance, location, and activity delineated by Farland-Smith (2012). The team developed a spreadsheet to group the features of the drawings into these three categories: location or setting of the picture (e.g., desk, chalkboard, and daydream), appearance of the student (e.g., portion of body drawn, and type of mouth drawn), activity shown in the picture (e.g., writing, studying, and panicking).

The third stage of coding assigned a numerical value to each picture ranging from 1 (severely negative) to 7 (extremely positive). Each member of the team assigned a value to each picture and debated any pictures for which there was disagreement. After all the pictures had been coded, the pictures were grouped by number to make sure all the drawings in a category were alike. The team then wrote descriptions of each group (see Figure 1 for examples).

Drawings receiving a “1” represented students with a severely negative view of mathematics. These drawings included at least one of the following: expletives, statements of hate (e.g., “I hate math”), intense crying, vomiting, suicide attempts, display of failing grade, or another communication of anger or intense sadness. Drawings receiving a “2” represented students with a negative view of mathematics. These drawings had at least two negative components but lacked any severely negative actions mentioned in the first category. These drawings tended to communicate confusion, frustration, and feelings of being overwhelmed. Often these drawings featured several question marks and frowns. Drawings receiving a “3” portrayed math as unpleasant but the students in the pictures were not pushed to the point of confusion or frustration. The drawings typically only included one negative element (e.g., frown) and no positive elements. These students were in semi-productive states in very traditional math environments (e.g., alone studying at a desk). To receive a score of “4,” the drawing was seen as a neutral depiction of math. Most of the students drew a flat line mouth and communicated no clear positive or negative emotion.

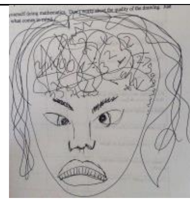
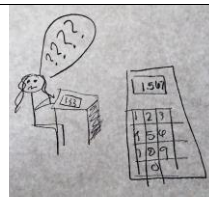
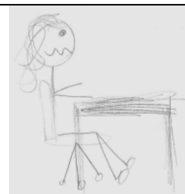
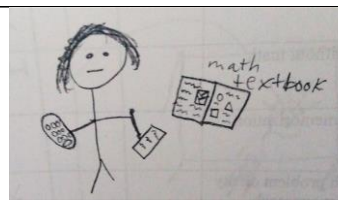
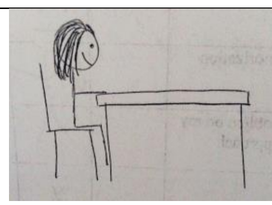
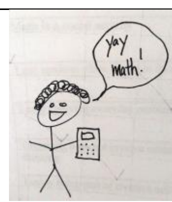
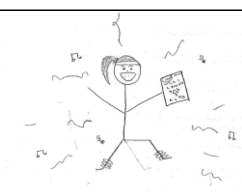
Numerical Code	Example
1 – Extremely Negative	
2 - Negative	
3 – Math is Unpleasant	
4 - Neutral	
5 – Math is Pleasant	
6 - Positive	
7 – Extremely Positive	

Figure 1. Examples of different categories of coded drawing

If positive and negative elements were present, the two aspects are shown in equal proportion. Drawings receiving a "5" portrayed math as pleasant but not overly enjoyable. These drawings typically had one positive element (e.g., a smile) and no negative elements. The drawings tended to represent a very traditional setting (e.g., at a desk writing or studying). To receive a score of "6," the drawing had to include more than one positive element (e.g., a smile and positive thought bubble). Also, the drawing needed to show some non-traditional aspect (e.g., movement, other people). Many of these drawings captured part of the thinking process of mathematics (e.g., working through confusion to find a "light bulb" moment). To reach the highest score of "7," the picture needed to show an element of being extremely excited and show a complete lack of worry, confusion, and unproductive struggle. This score also shows an element of a nontraditional setting or activity. The student displays their full body and portrays competency with mathematics.

An interesting subset of pictures emerged where the student drew himself in a daydreaming state escaping the mathematics at hand (see Figure 2 for examples). The researchers decided that pictures involving daydreaming, where the student was disengaged from the mathematics, could score no higher than 3. The leftmost drawing featured in Figure 2 shows a chain link fence topped with barbed wire separating the student's "happy place" and mathematics. This drawing scored a 2 for the negative components of escaping mathematics and needing to do so over barbed wire. The drawing shown in the middle shows a student fantasizing about burning his mathematics textbook. This drawing received a score of 1 because of the anger communicated in the picture. The drawing on the right received a score of 3 because the only negative component of the picture was the need to escape mathematics and think about pleasant things.

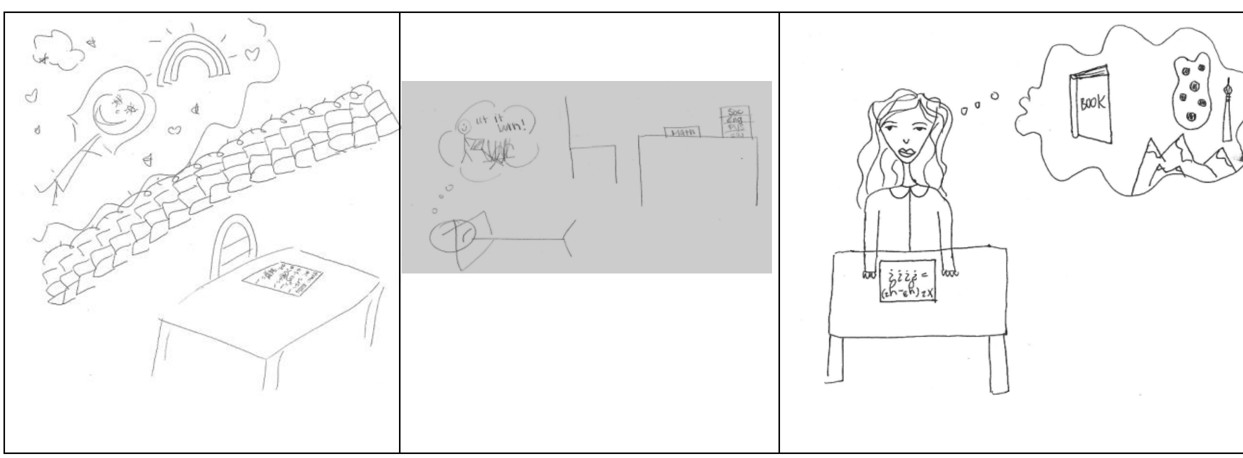


Figure 2. Examples of drawings featuring daydreaming

Findings

On the pretest, the math dance class scored an average of 2.5 out of 7 points and the traditional math class scored 2.94 points. On the posttest, the treatment group increased to an average of 2.75 points and the control group decreased to an average of 2 points. Only one (6%) student in the treatment group received a lower score on the posttest than the pretest while five (28%) students from the control group scored lower on the posttest. Overall, 75% of the treatment group drew more positive pictures about themselves doing mathematics on the posttest than the pretest compared to only 39% of students improving in the control group. Furthermore, none of the posttest treatment drawings received the most negative score of “1” while five students (28%) from the control group received this score on the posttest. Also, only one student (6%) from the control group scored above a 5 on the posttest while five students (42%) received a 6 or better from the treatment group.

On the pretest, the most typical drawings displayed in both groups featured a student sitting alone at a desk, frowning, and confused (see middle drawing in Figure 3). In many of the pictures, students drew themselves unproductively struggling with the mathematics (see Figure 3 for examples). None of the pretest drawings featured nontraditional productive work; all of these students drew themselves working alone at a desk. On the posttest, the treatment group was more likely to draw a nontraditional setting or activity than the control group. The treatment group was also less likely to focus the activity of the picture on confusion or panic. Five students (28%) in the control group focused on confusion while only one student from the treatment group (6%) focused on confusion on the posttest.

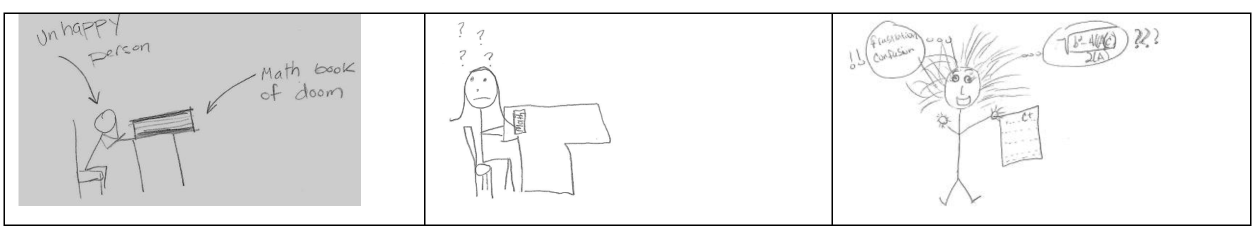


Figure 3. Unproductive struggle commonly seen in the drawings.

Only one student drew anyone else in the picture on the pretest, and that student drew a teacher saying “blah, blah, blah” (see Figure 4). Many of the drawings continued to follow this trend by the posttest. However, students in the treatment group were more likely than the control group to draw another person in their picture by the posttest. Three students (25%) in the treatment group drew other people (e.g., teachers and classmates) in the posttest picture while no one from the control group drew another person on the posttest.

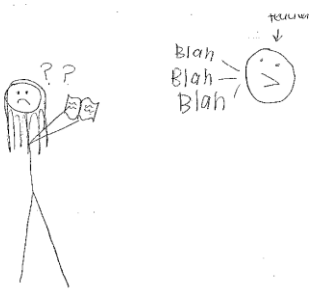


Figure 4. The only pretest picture including someone else in the drawing

Another striking feature of the drawings was 82% of the pretest drawings included at least one affective factor in the drawing. Of these students, 81% of them drew a negative affective component such as tears, a frown, or vomit. Analysis revealed powerful images of the negative emotions students encountered when doing mathematics including anxiety, confusion, and anger.

Further study and Implications

While significant work has already been done to delineate numerical scores for the overall view of mathematics communicated in a drawing, further work continues to refine the rubric descriptions for these scores. As part of this continued work, plans include designing rubric criteria for scoring the three categories of drawing components: appearance, location, and activity. Work has begun to design a scoring system for these three areas similar to that developed by Farland-Smith (2012). For example, for “Location,” a drawing will receive a location score of “0” when the location “cannot be categorized” (p. 111), “1” for a “sensationalized” (p. 111) or fantasy environment, “2” for a “traditional” (p. 112) classroom or desk setting, and “3” for showing a location “broader than traditional” (p. 112). Similar categories will be established for analyzing the student appearance and activity shown in the picture.

In addition to developing evaluation criteria for this drawing prompt, much work is needed to determine the degree to which the trends perceived in this study capture widespread views of mathematics as an unpleasant endeavor, navigated alone, studied at desks, and plagued by unproductive struggle. While many of these drawings unveiled potent negative images associated with the teaching and learning of mathematics, this prompt also showed shifts away from extremely negative and traditional attitudes toward math for students in a nontraditional classroom where students studied mathematics through physical motion activities and were equipped with tools to conquer mathematics anxiety and moments of unproductive struggle.

References

Burton, M. (2012). What is math? Exploring the perception of elementary pre-service teachers. Issues in the Undergraduate Mathematics Preparation of School Teachers: *The Journal, 5*.

Chambers, D. W. (1983). Stereotypical images of the scientist: The draw-a-scientist test. Science Education, *67*(2), 255-265.

Farland-Smith, D. (2012). Development and field test of the modified draw-a-scientist test and the draw-a-scientist rubric. *School Science and Mathematics, 112*(2), 109-116.

Finson, K. D., Beaver, J. B., Cramond, B. L. (1995). Development and field test of a checklist for the draw-a-scientist test. *School Science and Mathematics, 95*(4), 195.

Mcdermott, B., & Tchoshanov, M. (2014). Draw yourself learning and teaching mathematics: A collaborative analysis. *Proceedings of the 41st annual meeting of the research council on mathematics learning*, pp. 34-40.

Rinne, L, Gregory, E., Yarmolinskaya, J., & Hardiman, M. (2011). Why Arts Integration Improves Long-Term Retention of Content. Mind, Brain and Education, *5*(2), 89-96

Schaffer, K., Stern, E., & Kim, S. (2001). Math dance with Dr. Schaffer and Mr. Stern. Santa Cruz, CA: MoveSpeakSpin.

Thomas, J. A., Pedersen, J. E., & Finson, K. (2001). Validating the draw-a-science-teacher-test checklist: Exploring mental models and teacher beliefs. *Journal of Science Teacher Education*, *12*(3), 295-310.

Utley, J., Reeder, S., & Redmond-Sanago. (2015). *Development of the draw a mathematics teacher test and rubric*. Presentation at the 42nd annual meeting of the Research Council on Mathematics Learning.

PRE-SERVICE TEACHERS' ACCEPTANCE OF NUMBER CONCEPTS INSTRUCTION IN BASE 8

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Pre-service elementary school teachers maintain beliefs and conceptual knowledge not aligned to current standards that focus on conceptual understanding. Research indicates that university courses taught in an alternate base may have a positive impact on the conceptual understanding of content in pre-service teachers. This qualitative study investigated why pre-service teachers accept instruction in an alternate base. Researchers found that students who were exposed to this type of instruction experienced cognitive dissonance, which helped them more deeply understand the content. Consequently, prospective teachers felt they were more likely to be understanding of their students' frustrations, and more capable of teaching the material.

Related literature

Research shows that prospective teachers enter universities with beliefs and mindsets incongruent with conceptual instruction techniques required for making sense of problems (Stohlmann, Cramer, Moore, & Maiorca, 2014). Additionally, many pre-service elementary school teachers lack conceptual knowledge in the mathematics area of whole number and operations When asked to solve a subtraction problem for example, pre-service teachers reason through their solutions poorly, and those solutions are tied to standard algorithms (Thanheiser, 2012). Consequently, when prospective teachers enter the classroom, they are not prepared to teach to the depth of required standards. This could cause them to employ instructional techniques which center around lecture, rules, and procedures rather than techniques that foster student discourse and exploration of concepts. This could be a result of prospective teachers, in their own schooling, never being taught the conceptual foundations of mathematics. One cannot teach what one does not know. Upon entering university programs with this knowledge and certain beliefs, it can be difficult to convince pre-service teachers to instruct mathematics conceptually rather than procedurally (Naylor, & Keogh, 1999) Therefore we must change our approach to teacher education. The significance in this study lies in the small body of research available in regards to instructing pre-service elementary teachers in a different base.

One study focused on a methods course that was designed to prepare pre-service elementary teachers to implement mathematics curriculum in a conceptual manner. Students were taught place value and number operation in base 8. Emphasis was placed on solution processes and participants' explanations and justifications of solutions. The vehicle for such practices was an instructional sequence called Candy Factory, which involved learning place value by thinking of a candy shop. In this particular candy shop, single pieces of candy were packaged in rolls of 8, and there were 8 rolls in a box. This was similar to a third grade activity in which candies were packaged in tens, however eights were used for pre-service teachers to "problematize the mathematics" (McClain, 2003, p. 286). Results indicated that teaching in this fashion resulted in an increase in pre-service teachers' ability to reason mathematically, and that it forced them to assume a dual role, one of student, and one of prospective teacher. They came to realize the importance of teaching mathematics in a conceptual way.

In the second study, researchers observed a mathematics content course in which number operations and place value were taught using base 8. For that particular class, special wording was adopted for base 8, i.e. 8 was referred to as one-e. A university goal in this study was to develop a belief in pre-service teachers that "mathematical knowledge is not a pre-given, external body of knowledge to be acquired, but rather is built up by cognizing individuals as they engage in mathematical activity, including discussions of their own and others' mathematical actions" (Yackel, Underwood, & Elias, 2007, p. 363). Another goal of the university was for pre-service teachers to develop conceptual basic number facts, number relationships, grouping strategies, and thinking strategies. Data were collected in the form of observations and performance in subsequent courses. Researchers concluded that both goals were achieved. Students reported that learning in base 8 gave them a new view of arithmetic and how it can be taught.

Three significant research projects regarding instruction in base 8 took place in a similar environment to aforementioned studies. Andreasen (2006), Roy (2008), and Safi (2009) conducted research that examined prospective teachers understanding of place value and number operation in different iterations of an elementary mathematics course in base 8. Like McClain (2003), Andreasen used the theoretical framework of how children learn place value and number operations. She designed a hypothetical learning trajectory and mathematical tasks for the course. Andreasen focused on the instructional sequence and social context of the classroom and ways in which both contributed to the development of pre-service teachers' understanding of place value and number operations. Unlike McClain (2003), the majority of the place value and number operations unit in Andreasen's study was in base 8. Prospective teachers were expected to think and reason through whole number problems in base 8, and only changed to base 10 at the end of the unit. Andreasen chose to switch in order to encourage a conceptual bridge between reasoning in base 8 and base 10 (Andreasen, 2006). The following social norms were established in the classroom: "1) explaining and justifying one's solution and solution processes and 2) making sense of other students' solutions by asking questions of classmates or the instructor" (p. 160). Sociomathematical norms that were partially established during the teaching experiment were determining: "1) what constituted a different mathematical solution and 2) what made a good explanation" (p. 160). Mathematical practices including unitizing, flexibly representing numbers, and reasoning about operations were also established. A hypothetical learning trajectory was recognized through prospective teachers' learning, which was supported by tasks integrated as part of the instructional sequence (Andreasen, 2006).

Utilizing the same setting and the same learners, Roy (2008) built on Andreasen's work, concentrating on mathematical practices established by prospective teachers. Roy focused on individual pre-service teacher's understanding of whole number concepts and operation as it occurred within an established classroom environment. Roy refined the hypothetical learning trajectory described by Andreasen (2006) and concluded that the revised instructional sequence supported the following normative mathematical practices "(a) developing small number relationships using Double 10-Frames, (b) developing two-digit thinking strategies using the open number line, (c) flexibly representing equivalent quantities using pictures or Inventory Forms, and (d) developing addition and subtraction strategies using pictures or an Inventory Form" (p. 136).

Additionally, Roy (2008) utilized an instrument developed by Hill and colleagues (2004) to measure mathematical content knowledge necessary to teach elementary mathematics. The Mathematical Knowledge for Teaching (MKT) Measures was administered before and after the base 8 unit of the elementary mathematics course. Roy reported a statistically significant difference in prospective teachers' mean scores indicating the possibility that pre-service teachers' content knowledge for teaching increased as a result of classroom instruction.

Safi (2009) extended the research of Roy (2008) and Andreasen (2006) by examining how prospective teachers progress in their learning of number concepts and operations. Safi outlined and analyzed the mathematical conceptual understanding of two pre-service teachers via the emergent prospective. Two individuals were selected based upon their scores on the MKT

(Hill et al., 2004), one scoring lower and one higher. The individual who scored lower progressed in her understanding of how to solve problems. She was able to explain how to solve the problems, however struggled to justify her solution strategies, displaying a lack of conceptual understanding. The higher scoring individual's conceptual understanding increased, and progressed her capability to reason through others' solutions. Safi's findings provided a psychological perspective as it relates to the social perspective's mathematical practices.

Pre-service teachers enter universities with certain beliefs and attitudes not aligned to current standards suggested approach to teaching, which includes conceptual based instruction (Stohlmann et al., 2014). This could be a consequence of pre-service teachers, in their own schooling experience, being taught mathematics via rules and procedures. It is often difficult to convince them of the benefits of conceptual instruction. The purpose of this grounded theory study was to add to the small pool of research regarding instruction in an alternate base. While there seems to be a consensus that instruction in base 8 can aide in the development of prospective teachers' ability to reason mathematically, this study aims to discover the reason pre-service teachers accept instruction on mathematics in base 8. For this particular study, acceptance is defined as students' reactions to certain activities and students responses to questions asked regarding learning in base 8.

Methodology

This study utilizes a grounded theory approach. Before collecting data, the researcher obtained approval from an institutional review board (IRB), and requested permission from the Elementary Mathematics Content course instructor to use her students. Next, the researchers spoke with students about aspects of the study and explained confidentiality. Students were also given the opportunity to opt out of the study.

Participants and Context

Convenience sampling was employed as the researcher chose participants that were readily available (Gall, Gall, & Borg, 2007). Participants included students enrolled in an elementary mathematics content course at a university in the southeastern United States. Upon obtaining permission for inclusion in the study, one table with six students was chosen as the observation site. The entire class was observed, however, the researchers focused on data that were obtained from four students who agreed to be interviewed.

The elementary mathematics content course included a whole number and operations unit taught entirely in base 8. The course utilized a hypothetical learning trajectory and instructional sequence designed to influence pre-service teachers' mathematical knowledge in regards to number concepts and operations. Students began with counting in base 8 and practiced using tasks such as skip counting and open number line problems with tools including double 10-frames and an open number line. The second phase included unitizing and flexible representations, and students were introduced to a Candy Factory scenario. Their tasks included estimation and problems involving packaging and un-packaging candy in rolls and boxes of eight. In phase three, students developed invented computational strategies in base 8. They continued to use the Candy Factory scenario and inventory forms to perform transactions. Finally, students were introduced to multiplication and division word problems (Roy, 2008).

Data Collection

The researchers attended all seven classes in the base 8 unit of the course. The researchers sat at a table with the six students and observed them while they participated in class and had group discussions. The researchers did not participate in discussions. The researchers took detailed field notes as well as audio-recorded classroom discourse. Observations were employed to get a feel of the classroom and how students felt about learning in a different base system. Many things were observed including conversations, comments, facial expressions, and overall demeanor of the students. All audio was transcribed immediately proceeding each class to look for rich conversations during group discussions.

Little interaction occurred between participants and the researcher other than greetings and during interviews. Four students were interviewed at three different times during the unit. Students were chosen to interview based upon availability before or after class. The initial interview occurred after the first class; the second after they turned in their first homework assignment, and the last was after the unit exam. During the first interview students were asked questions to get an idea of how they felt upon being introduced to base 8. Example questions included: What are your initial thoughts about base 8 after the first lesson? Why do you think the professor has you learning this unit in base 8? Can you tell me more about what happened when you were solving the problems? Questions after they turned in the homework were framed to get an idea of how their feelings regarding learning in base 8 might be changing. Example questions included: How did class prepare you for the homework? Why do you think your instructor has you learning this unit in base 8? How beneficial is it for you to see multiple students' strategies presented in front of the class? The final interview questions were framed to attempt to see how students' feelings changed or stayed the same during the unit. Questions included: What did you learn about base 8 in this unit? Do you think learning about place value and whole number operations in base 8 is worthwhile? Why or why not? Do you think this is how the unit should be taught, or should pre-service teachers learn about place value and whole number operations in base 10? Why? Research questions were framed to attempt to paint a picture of students' feelings and attitudes regarding learning in base 8, which could help answer the research question: Why do students accept instruction on mathematics in another base system?

Data Analysis

Data were analyzed upon reviewing field notes, transcribed audio recordings, and interview responses. The researchers looked for common themes and utilized an open coding method. The constant comparative method was employed in attempt to saturate the themes (Creswell, 2007). Repetitive instances that represented the theme were noted and the researchers continued to look for these instances until no further themes were produced from the data. Sub-themes were created from the main themes that represented multiple perspectives in regards to the themes. The researchers reviewed the themes and selected one that was represented by the most by participants. This was considered the main phenomenon of interest. Data were reviewed again in a way that helped the researchers understand how the other themes were connected to the main phenomenon. Theories were generated from this data. Researchers then formed a hypothesis that related the themes (Creswell, 2007).

Limitations to this study lie in the data collection and the sampling method. In regard to data collection, having researchers try to absorb everything that was going on in the class at every moment was difficult. A student could be talking, and then another student would talk over him, so that the researchers could not complete accurate notes. It was also difficult to write reflective notes in the moment if a student was already moving on to another topic that needed to be recorded. To address this, the researchers immediately transcribed the rough notes into field notes and utilized audio recordings of the class to glean information missed. As for the population, researchers only had access to one class, and even then, not every student volunteered to be part of the study. In addition, participants selected for interviews were based solely on who had time or was willing to stay after to do the interview. This hampered the validity of the study because perhaps the students who were willing to stay were more motivated and eager to learn. The researchers attempted to overcome this issue by selecting two seemingly different students who volunteered to interview. This was based upon the students' actions during the first class. One student seemed eager to learn, was taking notes, and participating in group discussions, while the other student did not take many notes, did not speak much during group discussions, and did not volunteer his answers when having class discussions.

Findings

A common theme was that students felt problems took more time and more reasoning because they did not have access to standard algorithms, however, students felt that reasoning through multiple solution strategies in class was valuable. Students believed that learning in base 8 was worthwhile. One student explained, "I see the validity of relearning how to learn math. Given that we're going into schools where they have changed how kids learn math. And also it kind of bumps up our frustration levels so we understand how the kids feel so we're more likely to be patient with them." Another student saw the value of learning in base 8 because it reminded him of the struggles kids will have when learning a number system. It was concluded that students believed that learning in base 8 was beneficial because they had a better understanding of why students are taught math a certain way and they feel they will be more understanding when their future students became frustrated.

Students also indicated that they would feel more comfortable teaching in base 10 because of learning in base 8. They indicated that they could translate the instructional techniques that they were exposed to when learning in base 8 to teaching in base 10. Additionally, students felt that the course should continue to be taught in base 8 because it would make them better teachers in the future. One student expressed: "That's how it should be taught…the whole point of the class is to learn and I guess to remember how we felt, and if we do the base ten system, it's just regular. It's just easy work and we don't remember the struggles." Another student explained that if she had learned in base 10 instead, she would just be bored because she already knew the methods for solving the problems. She said if she learned in base 10 they wouldn't get the benefit of learning about place value and number operations all over again. She was adamant that learning in base 8 really got her thinking from the perspective of an elementary student and that she felt it should continue to be taught in base 8.

Findings in this study are consistent with previous research in regards to students' coming to realize the benefit of a conceptual understanding of mathematics (McClain, 2003; Yackel et al., 2007). The main take away from this study is that students felt it was beneficial to their learning and understanding of mathematics content, and they felt that they would be more competent as elementary mathematics teachers as a result of learning in base 8. This has a significant implication for the field of mathematics education, as it may influence mathematics educators to consider instruction in an alternate base as a method for teacher preparation.

Although prospective teachers may enter universities with certain beliefs and attitudes not aligned with conceptual based instruction (Stohlmann et al., 2014), participants in this study saw the benefits of instruction in base 8, which encouraged their conceptual understanding. Instruction in an alternate base intends to minimize prospective teachers' formal understanding of mathematics so that they experience cognitive dissonance and have to re-learn number concepts and operations conceptually. Additionally, the hope is that this experience will allow them to see things from an elementary student's perspective, that they will be better able to understand challenges and foresee error patterns that may arise for their future students, and have a deeper, more conceptual understanding of the mathematics that they will be teaching. Furthermore, students experience what it feels like to be an elementary student learning place value and number operations for the first time so they are a) likely to be more patient when they have students of their own and b) they will be more competent in teaching these concepts.

References

Andreasen, J. B. (2006). Classroom mathematical practices in a preservice elementary mathematics education course using an instructional sequence related to place value and operations. Unpublished Doctoral Dissertation, University of Central Florida.

Creswell, J.W. (2007). *Qualitative inquiry & research design: Choosing among five approaches* (3rd Ed.). Sage.

Gall, M. D., Gall., J. P., & Borg, W. R. (2006). *Educational research: An introduction* (8th ed.). Boston: MA: Allyn & Bacon.

Hill, H. C., Schilling, S. G., & Ball, D. L. (2004). Developing Measures of Teachers' Mathematics Knowledge for Teaching. *Elementary School Journal*, *105*(1), 11 – 30.

McClain, K. (2003). Supporting pre-service teachers' understanding of place value and multi-digit arithmetic. *Mathematical Thinking And Learning*, *5*(4), 281-306.

Naylor, S., & Keogh, B. (1999). Constructivism in classroom: Theory into practice. *Journal of Science Teacher Education*, *10*(2), 93-106.

Roy, G. J., (2008). *Prospective teachers' development of whole number concepts and operations during a classroom teaching experiment*. Unpublished Doctoral Dissertation, University of Central Florida.

Safi, F. (2009). Exploring the understanding of whole number concepts and operations: A case study analysis of prospective elementary school teachers. Unpublished Doctoral Dissertation, University of Central Florida.

Stohlmann, M., Cramer, K., Moore, T., & Maiorca, C. (2014). Changing pre-service elementary teachers' beliefs about mathematical knowledge. *Mathematics Teacher Education & Development*, *16*(2), 4 – 24.

Thanheiser, E. (2012). Understanding multidigit whole numbers: The role of knowledge components, connections, and context in understanding regrouping 3+-digit numbers.*Journal of Mathematical Behavior*, *31*(2), 220 – 234.

Yackel, E., Underwood, D., & Elias, N. (2007). Mathematical tasks designed to foster a reconceptualized view of early arithmetic. *Journal of Mathematics Teacher Education*, *10*(4-6), 351-367.

BELIEFS ABOUT SOCIAL JUSTICE AMONG ELEMENTARY MATHEMATICS TEACHERS

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The purpose of this study was to measure teacher beliefs about social justice over the course of an elementary mathematics teaching methods course. The participants in the study came from three unique groups of in-service and preservice teachers in a master's degrees program at a medium-size university in New York. Findings revealed that while there were no differences in beliefs over the course of the semester, one group had more positive beliefs about social justice than did another group. Teachers felt positively about several important variable related to social justice in their classrooms.

Social Justice in Mathematics

Stinson and Wager (2012) said that teaching for social justice in mathematics is "rooted, in part, in the belief that all children should have access to rich, rigorous mathematics that offers opportunities and self-empowerment for them to understand and use mathematics in their world" (p. 10). To facilitate this, teachers for social justice in mathematics need to be introspective toward their own identities as agents, as both individuals and teachers, for social change (Gonzales, 2009). Gau Bartell (2012) said teachers are generally unprepared "to teach mathematics to an increasingly racially, ethnically, linguistically, and socioeconomically diverse student population with which they often have had limited previous interactions" (p. 113). Mathematics teachers must be given the opportunities to directly reflect upon their own conceptions of social justice and learn how to teach from a social justice perspective, which can be conducted through professional development and work with after school programs (Gonzales, 2009; Leonard & Evans, 2008). Leonard and Evans (2012) said, "An explicit focus on teachers' beliefs and expectations [about social justice] should be a component of (mathematics) teacher education" (p. 101).

Theoretical Framework

This study is grounded in critical race theory (CRT) in education (e.g., Ladson-Billings & Tate, 1995), which examines race and racism as it applies to education. CRT acknowledges that racism is pervasive throughout society, which means in an educational context racism affects not only children's learning, but also all aspects of the social and academic realities of schooling. Cultural responsiveness, a social justice orientation, and fostering of trust and care in the classroom are important components for educating students who have been traditionally underrepresented in mathematics related fields (Ladson-Billings, 1994). While strong content knowledge is important for effective teaching, these variables are equally important in their impact on learning for underrepresented students (Martin, 2007).

This study is also grounded in Freire's (1970/2000) concept of *Conscientização,* or critical consciousness, which allows the individual to critically perceive injustice and provide an intellectual means for opposing injustice. Teachers need to be given the opportunity to critically approach injustice in society in general, and in the schools in particular, as an important process in assisting them to critically reflect upon institutional and personal teaching practices.

Purpose of the Study and Background on Participants

The purpose of this study was to measure teacher beliefs about social justice over the course of an elementary mathematics teaching methods course at a university that emphasizes social justice for teachers in its conceptual framework. The course was standards- and reform-based, and addressed Common Core Standards, mathematics content and pedagogy, and social justice issues through the works of Gutstein, Leonard, Martin, and the National Council of Teachers of Mathematics. The participants in the study came from three unique groups of in-service and preservice teachers in master's degrees programs at a medium-size university in New York: New York City Teaching Fellows (NYCTF), Teacher Education Assessment and Management (TEAM) program, and traditional preservice teacher preparation program. The two-year graduate program for all three groups was designed to prepare teachers to teach in urban schools in New York with certification in elementary and special education.

The NYCTF program is an alternative certification program developed in 2000 in conjunction with the New Teacher Project and the New York City Department of Education (Boyd, Lankford, Loeb, Rockoff, & Wyckoff, 2007). NYCTF had the goal of bringing professionals from other careers to fill large teacher shortages in New York public schools. NYCTF teachers begin their program in June when they are immersed in coursework at their partnering universities in New York. In September they become the teachers of record in their classrooms while continuing their graduate work in education in a master's degree program. NYCTF teachers receive a Transitional B teaching license in New York State, which is valid for three years provided they remain with the program and complete program requirements. After this successful completion of this commitment they are eligible to apply for initial certification.

The TEAM program is a collaboration between the TEAM organization and the partnering university. The university partnership began in 2009 and had its first cohort begin the program in 2010. Cohorts consist of 12 to 20 Orthodox Jewish teachers separated by sex in the classroom due for religious purposes but in this study all TEAM participants were women. TEAM participants enrolled in the program to prepare for certification to teach in Yeshiva and Hebrew Academies. Several of the participants were already teaching in Yeshiva and Hebrew Academies during this study, while most of the participants were preparing to become teachers, but not currently teaching.

Traditional preservice teachers were enrolled in the university's graduate program, which required extensive fieldwork. Participants in the program were required to have 10 hours of fieldwork for each three credit class in which they were enrolled. Much of the work done in the classes was related to the fieldwork experience including lesson and unit planning, as well as reflection on the teaching experience. Participants in the program were encouraged to incorporate the theory and teaching techniques they were learning in their graduate program into classroom practice.

Research Questions

1. Were there differences in beliefs about social justice over the course of a semester in a reformed-based mathematics methods course?
2. Were there differences in beliefs about social justice between the NYCTF, TEAM, and traditionally prepared teachers?
3. What were teacher beliefs about social justice in the classroom?

Methodology

This study used a quantitative methodology and the sample consisted of 115 preservice and new in-service teachers. All NYCTF teachers were in-service teachers, and TEAM and traditional teachers were preservice teachers, with several TEAM participants teaching in Yeshiva and Hebrew Academies. There were 84 NYCTF teachers, 16 TEAM teachers, and 15 traditional teachers. Participants were enrolled in an inquiry- and reformed-based elementary mathematics methods course that involved both pedagogical and content instruction and was aligned with the NCTM *Principles and Standards for School Mathematics* (2000).

Teachers were given the Learning to Teach for Social Justice Scale (LTSJ) at the beginning and end of the semester, which was developed by Enterline, Cochran-Smith, Ludlow, and

Mitescu (2008) and Ludlow, Enterline, and Cochran-Smith (2008), and measured participants' beliefs about teaching from a social justice perspective. The LTSJ is a 12-item 5-point Likert scale instrument that solicits participant beliefs about social justice in the classroom based upon diversity issues such as race, culture, language, gender, disability, and sexual orientation.

Results

Paired-samples *t*-test was conducted to answer research question one in order to determine differences in the LTSJ scores over the course of the semester. No statistically significant differences were found.

One-way ANOVA was conducted to answer research question two in order to determine differences in LTSJ scores between NYCTF, TEAM, and traditional teachers. A statistically significant difference was found at the 0.05 level for pre- and post- test LTSJ scores with $F(2, 112) = 3.592$, $p = 0.031$, $\eta^2 = 0.06$ and $F(2, 112) = 5.247$, $p = 0.007$, $\eta^2 = 0.09$, respectively. A post hoc Tukey HSD test was conducted to determine exactly where the means differed among the programs (see Table 1). On the pretest it was found NYCTF teachers ($M = 3.94$, $SD = 0.459$) had more positive dispositions toward social justice than did TEAM teachers ($M = 3.61$, $SD = 0.433$) with $p = 0.023$. On the posttest it was also found NYCTF teachers ($M = 3.94$, $SD = 0.529$) had more positive dispositions toward social justice than did TEAM teachers ($M = 3.51$, $SD = 0.382$) with $p = 0.006$. The effect sizes for both pretest and posttest were in the small to medium range. There were no other statistically significant differences.

Table 1. *Mean Scores for NYCTF, TEAM, and Traditional Teachers*

Learning to Teach for Social Justice Scale (LTSJ) Scores	Pretest Mean (SD)	Posttest Mean (SD)
NYCTF	3.94* (0.459)	3.94** (0.529)
TEAM	3.61* (0.433)	3.51** (0.382)
Traditional	3.87 (0.443)	3.76 (0.399)

Note. $N = 115$. * $p < 0.05$. ** $p < 0.01$.

Descriptive statistics were used to answer research question three (see Table 2). Results indicated teachers felt most positively about incorporating diverse cultures and experiences into classroom lessons and discussions; self-examination of attitudes and beliefs about race, class, gender, disabilities, and sexual orientation; and teaching students to think critically about government positions and actions. Teachers felt most positively about negative attitudes such as preparing students for lives they are most likely to lead; student success in school being dependent on how hard they work; and the teachers' jobs as not being agents of societal change. These final three items reflect negative attitudes toward social justice.

Table 2. *Survey Results for Beliefs about Social Justice*

Learning to Teach for Social Justice Scale (LTSJ)	Pretest Mean	Posttest Mean
1. An important part of learning to be a teacher is examining one's own attitude and beliefs about race, class, gender, disabilities, and sexual orientation.	4.27	4.28
	4.08	3.93
2. Issues related to racism and inequity should be openly discussed in the classroom.	4.00	3.99
3. For the most part, covering multicultural topics is only relevant to certain subject areas, such as social studies and literature.	4.60	4.50
4. Good teaching incorporates diverse cultures and experiences into classroom lessons and discussions.	3.75	3.74
5. The most important goal in working with immigrant children and English language learners is that they assimilate into American society.	4.12	4.10
6. It's reasonable for teachers to have lower classroom expectations for students who don't speak English as their first language.	3.90	4.05
7. Part of the responsibilities of the teacher is to challenge school arrangements that maintain societal inequities.	4.17	4.18
	3.99	3.93
8. Teachers should teach students to think critically about government positions and actions.	3.70	3.51
	3.15	3.11

9. Economically disadvantaged students have more to gain in schools because they bring less to the classroom.	2.90	2.96
10. Although teachers have to appreciate diversity, it’s not their job to change society.		
11. Whether students succeed in school depends primarily on how hard they work.		
12. Realistically, the job of a teacher is to prepare students for the lives they are likely to lead.		

Note. $N = 115$.

Items are from Enterline et al. (2008) and Ludlow et al. (2008).

Negative items were reversed scored so that high scores still represented positive attitudes (items 3, 5, 6, 9, 10, 11, and 12).

Discussion

It was found that while there were no differences in beliefs about teaching for social justice over the course of the semester, teachers from the NYCTF program had more positive beliefs about social justice than did teachers from the TEAM program. This result was not surprising given that teachers from the NYCTF program generally teach in high-need urban schools throughout New York. The mission of NYCTF is “to recruit and prepare high-quality, dedicated individuals to become teachers who raise student achievement in the New York City classrooms that need them most” (NYCTF, 2012). Thus, it is not surprising that NYCTF teachers would hold a positive social justice disposition. Furthermore, TEAM participants were religious Orthodox Jewish teachers who held traditional Orthodox values. As earlier stated, an example of their traditional beliefs is TEAM teachers required their classes to be separated by sex for religious purposes. It is possible that religious beliefs and culture contributed to the differences in beliefs about social justice in the classroom. These results should be interpreted with caution given the imbalance in sample size due to teacher availability.

It was found that teachers felt positively about incorporating diverse cultures and experiences into classroom lessons and discussions; self-examination of attitudes and beliefs about race, class, gender, disabilities, and sexual orientation; and teaching students to think critically about government positions and actions. While it is important that teacher educators continue to encourage teachers in these areas, it is more important teacher educators work with teachers in areas in which they felt less positively. Teachers felt least strongly that they should prepare students for life outside of the lives they will most likely lead; student success is contingent upon external variables outside of student hard work; and teachers did not feel strongly that their jobs were to be agents of societal change.

Teachers who focus on the lives they *expect* for their students reduce the possibilities for the students, which is a major issue in teaching for social justice because teachers may possess lower expectations for students of lower socioeconomic status and underrepresented groups. Teacher educators need to work with teachers to help expand possibilities for these students. One method is to help the teacher focus on the individual's aptitude; rather than assume that because a student is from a less affluent family, it is not worthwhile exploring future options.

References

Boyd, D., Lankford, S., Loeb, S., Rockoff, J., & Wyckoff, J. (2007). *The narrowing gap in New York City qualifications and its implications for student achievement in high poverty schools* (CALDER Working Paper No. 10). Washington, DC: National Center for Analysis of Longitudinal Data in Education Research. Retrieved from http://www.caldercenter.org/PDF/1001103_Narrowing_Gap.pdf

Enterline, S., Cochran-Smith, M., Ludlow, L., & Mitescu, E. (2008). Learning to teach for social justice: Measuring changes in the beliefs of teacher candidates. *The New Educator, 4(4),* 267–290.

Freire, P. (2000). *Pedagogy of the oppressed* (M. Bergman Ramos, Trans.). New York, NY: Continuum. (Original work published 1970)

Gau Bartell, T. (2012). Is this teaching mathematics for social justice? Teachers' conceptions of mathematics classrooms for social justice. In A. A. Wager & D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 113-126). Reston, VA: National Council of Teachers of Mathematics.

Gonzales, L. (2009). Teaching mathematics for social justice: Reflections on a community of practice for urban high school mathematics teachers. *Journal of Urban Mathematics Education*, *2*(1), 22-51.

Ladson-Billings, G. (1994). *The dreamkeepers: Successful teachers of African American children.* San Francisco, CA: Jossey-Bass.

Ladson-Billings, G., & Tate, W. F. (1995). Toward a critical race theory of education. *Teachers College Record, 97*(1), 47-68.

Leonard, J., & Evans, B. R. (2008). Math links: Building learning communities in urban settings. *Journal of Urban Mathematics Education, 1*(1), 60-83.

Leonard, J., & Evans, B. R. (2012). Learning to teach mathematics for social justice: Teachers'beliefs and dispositions. In A. A. Wager & D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 99-112). Reston, VA: National Council of Teachers of Mathematics.

Ludlow, L., Enterline, S., & Cochran-Smith, M. (2008). Learning to teach for social justice-beliefs scale: An application of Rasch measurement principles. *Measurement and Evaluation in Counseling and Development, 40*(4), 194–214.

Martin, D. B. (2007). Beyond missionaries or cannibals: Who should teach mathematics to African American children. *High School Journal*, *91*(1), 6-28.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

New York City Teaching Fellows. (2012). *Our mission and values.* Retrieved from https://www.nycteachingfellows.org/purpose/mission.asp

Stinson, D. W., & Wager, A. (2012). A sojourn into the empowering uncertainties of teaching and learning mathematics for social change. In A. A. Wager & D. W. Stinson (Eds.), *Teaching mathematics for social justice: Conversations with educators* (pp. 3-18). Reston, VA: National Council of Teachers of Mathematics.

THE NATURE OF MATHEMATICAL CONVERSATIONS AMONG PROSPECTIVE MIDDLE SCHOOL TEACHERS IN A MATHEMATICS CONTENT COURSE

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This qualitative research study examined the nature of mathematical conversations among prospective middle school teachers' (PMSTs) in an undergraduate mathematics content course. In particular, this study considered the ways that PMSTs might respond to ideas shared by peers as they work together to make sense of mathematical relationships. Results indicated that PMSTs (a) often addressed peers' ideas in their responses but (b) were not always explicit about problematic aspects of their peers' mathematical thinking.

Introduction

Standards for K-12 mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010; National Council of Teachers of Mathematics 2000) and for the preparation of mathematics teachers (Conference Board of the Mathematical Sciences, 2012), call for teachers to foster individual and collective mathematics learning through meaningful mathematical conversations among students. Research suggests that mathematical conversations among peers can be a powerful learning tool as students articulate their thinking to others and consider others' ideas relative to their own (e.g., Cobb, Boufi, McClain, & Whitenack, 1997; Ellis, 2011; Pierson, 2008). However, little is known about what these interactions look like among prospective middle school teachers as they co-construct their understanding of mathematics. The study reported herein contributes to this body of research, and is part of a larger project that closely examines mathematical conversations that occurred among prospective middle school teachers (PMSTs) in an undergraduate mathematics content course as they co-constructed an understanding of relationships between numeric, algebraic, and geometric representations and operations involving even and odd numbers. The specific research question that guided the present study was: *In what ways do prospective middle school teachers listen and respond to their peers during a whole class discussion about generalizations of even and odd numbers?*

Conceptual Framework

Research suggests that listening and responding to others' mathematical ideas is a worthwhile and educative endeavor for teachers. Specifically, considering others' thinking supports teachers' content knowledge and pedagogical content knowledge by changing the way they think about the content as they determine if students' ideas are mathematically valid and have connections to other mathematical ideas; expanding their conception of what students are able to do mathematically; increasing their understanding of common misconceptions; and expanding their understanding of how different representations of mathematical ideas can be used to make sense of particular mathematics concepts (e.g., Driscoll & Moyer, 2001; Fennema & Franke, 1992; Herbel-Eisenmann & Phillips, 2005; Kazemi & Franke, 2004).

According to Empson and Jacobs (2008), listening responsively can be acquired over time as teachers engage in authentic learning experiences where they practice listening carefully to make sense of and respond to others' ideas in a variety of ways including considering students' thinking based on their written work, examining their own and colleagues' teaching of lessons with attention to students' thinking and mathematical principles, and practicing responsive listening by working with individual or small groups of students. Examining and making sense of students' written work, giving attention to different ways of thinking relative to mathematical principles, and listening and responding to others' mathematical ideas are all experiences within the scope of undergraduate mathematics content courses for PMSTs; and provide opportunities for PMSTs' to engage in meaningful conversations as they co-construct their understanding of mathematics.

Methods

This study was conducted during spring 2011 at a four-year, public university in the Southeastern United States. The 22 participants (14 female; 8 male) were prospective middle school teachers enrolled in a required mathematics content course that met for two hours and thirty minutes once every week for 16 weeks. The course focused on three content strands central to middle school mathematics: 1) number and operations; 2) algebra and functions; and 3) geometry and measurement. The classroom was organized into six groups of three to five persons per group, and PMSTs often worked on tasks in cycles of think-pair-shares – individual think time, followed by pair discussions, and then coming together for whole-class sharing. The author and teacher-researcher (hereafter referred to as TR) regularly elicited PMSTs' thinking and encouraged dialogue among peers about the mathematics, one another's ideas, and connections between different mathematical representations. The overarching mathematical goal of the unit was for PMSTs to make explicit and meaningful connections between algebraic generalizations for even and odd numbers ($2n$ and $2n+1$ [or $2n-1$]), respectively, and generalizations of a geometric representation for even and odd numbers in Tilo's Model (from the *Connected Mathematics Project* curriculum – see Figure 1).

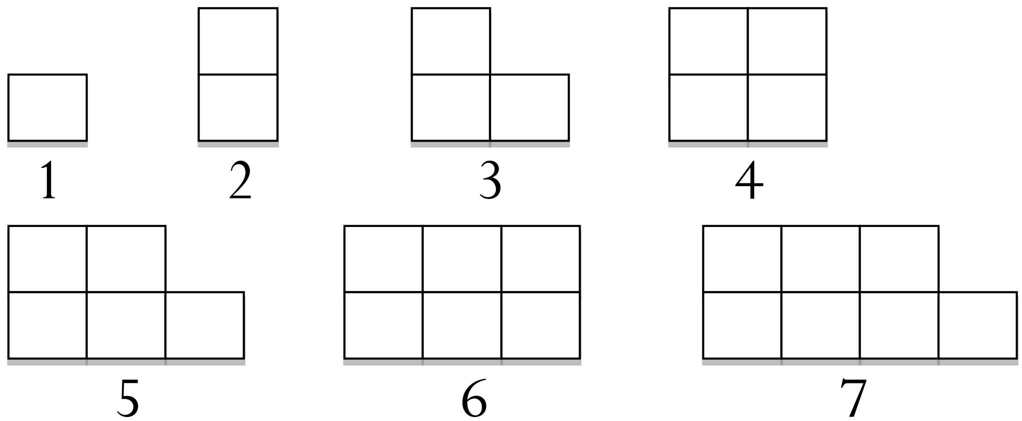

Figure 1. Tilo's Model (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006, p. 25)

Data collection occurred in the middle of the semester during a five-week unit that included considering numeric, algebraic, and geometric representations of even and odd numbers; exploring how to add and multiply even and odd numbers using these different representations; and identifying patterns and making conjectures about whole numbers. Each class meeting was videotaped by a second researcher, and whole class discussions were transcribed. The third class meeting of the unit (the seventh class meetings of the semester) was selected for this study because there was a lot of mathematical conversations among PMSTs as they grappled with the meaning of variables, how to represent geometric generalizations of even and odd numbers, and made explicit connections between algebraic and geometric generalizations to show sums of even and odd numbers. Data analysis involved a two-cycle grounded-theory technique (Saldana, 2009) that began with open-coding each occurrence of a PMST responding to a peer's idea for emergent themes to characterize those responses. The data revealed that PMSTs were responding in ways that align with Pierson (2008) characterizations of High I and High II Responsiveness. Thus, the second-cycle of pattern coding further parsed the data into one of the following two categories:

High I Responsiveness. Follow-up that is responsive to ideas, questions, or perceived misconceptions (one's own thinking is on display, but in response to the others' ideas)

High II Responsiveness. Follow-up that explores others' thinking and allows their reasoning to be the focal point; responding to and building on others' ideas; probing their thinking; expanding, clarifying, or giving an example based on others' ideas.

Results

Analysis of this data revealed that as PMSTs co-constructed their understanding connections between algebraic and geometric representations of even and odd numbers, they (a) often addressed peers' ideas in their responses but (b) were not always explicit about problematic aspects of their peers' mathematical thinking. Data illustrating these findings follow.

As PMSTs worked to identify how the components of the algebraic generalizations of even and odd numbers are related to Tilo's model and what geometric generalizations of Tilo's model to represent even and odd numbers would look like, they grappled with the meaning of the '2' and the meaning of the '*n*'. They also struggled to come to agreement on how to represent the sum of any two even numbers, any two odd numbers, or any even and any odd number. When they presented their work, Group 4, expressed the sum of any two even numbers as "$2n+2n=4n$." Some PMSTs recognized this use of '*n*' as limiting the sum to only represent adding an even number to itself. However, some PMSTs did not see an issue with using the same variable for both of the addends. These ideas were considered and reconsidered through the discussion as PMSTs worked together to come to some collective agreement about how to describe even and odd numbers algebraically in a way that align with generalizations of Tilo's model.

The except that follows provides an example of how PMSTs were making sense of their peers' ideas and then referring to those ideas in their responses. This excerpt begins with a question from a member of Group 4, Terri (all names are pseudonyms), where she was trying to make sense of how the '2' and the '*n*' in the algebraic expressions are related to the height of Tilo's model. Uberto and Edward describe how changing the height of Tilo's model to be three tiles high would change the '2' to '3' making the algebraic expression '$3n$' (as opposed to $2n$) to describe the altered Tilo's model. Ellis also responded to Terri's question about the height of Tilo's model by indicating that altering Tilo's model to have a height greater than two would not work for making a clear distinction between even and odd numbers.

Terri:	If we were not looking at Tilo's model and it was drawn three tiles in each column, would that change what you're saying?	
Uberto:	Then it would just be $3n$.	High II

Edward:	Yeah. If we had three tiles to every column. Right.	High II
Terri:	Ok. So, even numbers are any multiple of two, so 4*n* is still an even number. But in our case because we're adding 2*n* + 2*n* and we got 4*n*, I would say that 4 times n would give me an even number.	High I
	…	
Ellis:	If there's more than two tiles in a column, it doesn't make sense anymore that full columns mean even. … Three full columns would be nine, that's odd, and it doesn't make sense anymore. You have to have two [tiles per column] 'cause we're talking about, it can only be even or odd. There can only be two states.	High I

One of the challenges of engaging PMSTs in mathematical conversations with their peers is that the conversation is not linear, and at times can be difficult for both PMSTs and teacher educators to follow. In addition, in their responses to peers, PMSTs do not always clearly identify what is problematic about the mathematical thinking. For example, in the above excerpt, none of the three PMSTs who responded to Terri were explicit about the way that her group defined '*n*' (as one tile) as being problematic in trying to determine an algebraic generalization for adding any two even numbers. Neither did they clearly state that by using the same variable in the expressions of both of the two even numbers, Terri's group was suggesting that they were always adding the same number to itself (rather than being able to add two different even numbers). Some PMSTs made this connection while others did not, and as a result these two issues kept resurfacing throughout the discussion.

In the following excerpt, Kaila explains why she did not see an issue with using the same variable for both of the addends when summing two even or two odd numbers. Other students listen carefully to her thinking and give examples to convince Kaila that the subscripts are important in the algebraic generalizations for the sums of even and odd numbers.

Kaila:	It's irrelevant to have the subscript. You don't need it.	
TR:	Why don't you need the subscript, Kaila?	
Kaila:	Because *n* can be whatever, *n* is a variable, it changes.	
Tina:	But it would look like they're the same thing.	High I
	…	

Calvin:	[If you let them both equal *n*] they're not different variables, they're the same variable happening at two different occurrences.	High I
	…	
Kaila:	But if you're just trying to find how to add two even numbers, then you can still call them both n.	
	…	
Tina:	If you were to add two and four, then number of columns, [for the number two] is one [and for the number four] is two. And one and two are not the same thing, so they can't both be represented by *n*. That's why one has to be n_1 and one has to be n_2.	High I

In the above excerpt, Tina and Calvin identify something about Kaila's understanding of the how the variable, '*n*,' is being used that is problematic. Although Tina argues that "it would look like they're the same thing" and Calvin states that if *n* is used for both numbers, "they're the same variable happening at two different occurrences" neither of them were specific about why this is problematic. When Kaila's response remains focused on the algebraic expression alone when thinking about what is being added together, Tina points to the geometric model and gives a more specific response to try draw Kaila's attention to the connection between the meaning of '*n*' in the algebraic expression and the total number of complete columns in the geometric representation that '*n*' is being used to represent. Tina asserts that the two numbers being added might not have the same number of columns, so the variable that represents the number of columns for the first number being summed must be different than the variable that represents the number of columns for the second number being summed.

Discussion

This study was concerned with examining the ways in which PMSTs' listened to and responded to their peers' mathematical ideas during an undergraduate mathematics content course. Results indicated that these PMSTs were engaging in productive mathematical conversations where they were co-constructing their understanding of algebraic and geometric generalizations of the set of even and the set of odd numbers. Much of PMSTs' responses were at the *High I Responsiveness* level which involved follow-up that addressed others' ideas, questions, or perceived misconceptions – even though their own thinking was on display. Nevertheless, there was evidence that some PMSTs were responding at the *High II Responsiveness* level; at times they responded to and built on others' ideas in ways that allowed those ideas to be the focal point of the discussion.

This work has important implications because it suggests that undergraduate mathematics content courses can provide opportunities for PMSTs to gain experience listening and responding to others' mathematical thinking in meaning ways. By creating opportunities for PMSTs to participate in peer-to-peer dialogue that encourages them to (a) listen carefully to and consider others' ways of thinking about mathematics and (b) respond to others' mathematical thinking in meaningful ways that support the development of individual and collective understanding of mathematical ideas, mathematics teacher educators are modeling one way PMSTs are expected to foster their future students' learning. Additional research is needed to determine the extent to which such learning experiences in undergraduate mathematics content courses shape the value that PMSTs place on the co-construction of mathematics knowledge among peers.

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References

Conference Board of the Mathematical Sciences. (2012). *The mathematical education of teachers II.* Providence, RI and Washington, D.C.: American Mathematical Society and Mathematical Association of America.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J., (1997). Reflective discourse and collective reflection. *Journal of Research in Mathematics Education, 28*(3). 258−277.

Ellis, A. B. (2011). Generalizing-promoting actions: How classroom collaborations can support students' mathematical generalizations. *Journal for Research in Mathematics Education, 42*(4), 308−345.

Driscoll, M., & Moyer, J. (2001) Using students' work as a lens on algebraic thinking. *Mathematics Teaching in the Middle School, 6*(5), 282-287.

Empson, S. B. & Jacobs, V. R. (2008). Learning to listen to children's mathematical thinking. In D. Tirosh & T. Wood (Eds.)*, The international handbook of mathematics teacher education, volume 2: Tools and processes in mathematics teacher education* (pp. 257-281. Rotterdam, The Netherlands: Sense Publishers.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: MacMillan.

Herbel-Eisenmann, B. A., & Phillips, E. D. (2005). Using student work to develop teachers' knowledge of algebra. *Mathematics Teaching in the Middle, 11*(2), 62-66.
Kazemi, E. & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*, 203-235.
Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2006). *Connected Mathematics 2: Prime Time*. Needham, MA: Prentice Hall.
National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, Virginia: Author.
National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, D.C.: http://corestandards.org
Pierson, J. L. (2008). *The relationship between patterns of classroom discourse and mathematical learning*. Unpublished Dissertation. University of Texas, Austin.
Saldana, J. (2009). *An introduction to codes and coding: The coding manual for qualitative researchers.* Thousand Oaks, CA: Sage.

EXPLORING MENTAL MODELS OF "DOING MATH" THROUGH DRAWINGS

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This study explores the mental models pre-service teachers (PSTs) hold of "doing" math. To explore the models, PSTs drew images of mathematicians doing math. Using comparative judgements, they then selected the image they believed best represented a mathematician doing math. In follow-up prompts, they reflected on the choice of this image. This image was of a smiling, stereotypically attired Caucasian male standing in front of a blackboard, in a teaching position. Although some in the mathematics community view doing math to be creating new mathematics knowledge, the PSTs' model suggests a strong belief that mathematicians also do math by teaching math.

In a recent article of the *MAA FOCUS* magazine, Francis Su, newly installed president of the MAA was asked the following question, "What is your earliest memory of doing mathematics?" Dr. Su spoke of solving arithmetic problems on worksheets, prior to being of kindergarten age, given to him by his father. He further clarified that, at that time, this was what he believed mathematics to be (Peterson, 2015). What does it mean to do math? Due to their early interactions with students and mathematics, a better understanding of teachers' perceptions regarding this question is important. This current study aims to explore mental models held by pre-service elementary teachers.

Mental Model Theory

Mental model theory is a theory of how people reason about the world. A mental model is a cognitive structure constructed by an individual as a representation of a possibly real, imaginary, or hypothetical external reality (Gentner, 2002; Jacob & Shaw, 1999; Johnson-Laird, Girotto, & Legrenzi, 1998; Jones, Ross, Lynam, Perez, & Leitch, 2011). Due to cognitive limitations of an individual, models cannot contain every detail of the reality and thus are not complete or technically accurate representations (Gentner, 2002; Jones et al., 2011; Norman, 1983/2014). However, structural relations present in the reality will have analogous representations in the individual's mental model (Johnson-Laird, 1998). Thus, a model will have structural features in common with the represented domain and be as iconic as possible (Johnson-Laird, 2004).

An individual constructs a mental model through experience, by perceiving or imagining the reality, or by understanding discourse and gaining formal knowledge (Jacob & Shaw, 1999; Johnson-Laird et al., 1998; Jones et al., 2011). An individual uses mental models as conceptual frameworks through which to interpret, understand, and reason about the world (Gentner 2002; Jacob & Shaw, 1999). New information filters through the model (Jones et al., 2011), and the individual reasons about situations, leading to predictions and decisions through mental manipulations of the models (Johnson-Laird, 2005). Because of how models are constructed, a mental model is contextually bound, constrained by an individual's experiences with the represented domain (Norman, 1983/2014). In addition to experience, an individual's goals and motives for construction of the model also influence the structural aspects of the reality that end up being represented in the model (Jones et al., 2011).

In addition to representing physical aspects of a particular domain, mental models also incorporate an individual's beliefs related to the domain; thus, mental models are reflective of belief systems (Libarkin, Beilfuss, & Kurdziel, 2003; Norman, 1983/2014). This connection allows an exploration of belief systems through an individual's mental model. Yet, being internal constructs, mental models are difficult to explore. While one method of exploration is the direct questioning of an individual's beliefs, people generally have difficulty clearly articulating their beliefs (Gentner, 2002). As a result, novel methods can be useful in constructing external representations of internal mental models (Jones et al., 2011).

Efforts continue in order to improve methods for constructing such representations. Mental models are more general instances of a mental image. Hence, underlying any mental image is a mental model, with the image being the projection of the mental model's visualizable aspects (Johnson-Laird, 1998; Johnson-Laird, Girotto, & Legrenzi, 1998). Some recent studies have explored mental models via participant-made drawings, which would be physical manifestations of mental images. For example, drawings were analyzed to explored elementary and middle school students' mental models of circuits (Jabot & Henry, 2007), pre-service teachers' mental models of themselves as teachers of science (Thomas, Pederson, & Finson, 2001), pre-service agriculture teachers' mental models of effective teaching (Robinson, Kelsey, & Terry, 2013), and pre-service teachers' mental models of the environment (Moseley, Desjean-Perrotta, & Utley, 2010). While not explicitly using mental model theory, other studies have used a drawing methodology to explore pre-service elementary teachers' visual images of themselves as mathematics teachers (Utley & Showalter, 2007) and middle and secondary students images of mathematicians at work (Aguilar, Rosas, Zavaleta, & Robo-Vazquez, 2014; Picker & Berry, 2000; Rock & Shaw, 2000).

In their work, Picker and Berry (2000) theorized how a stereotypical cultural image of mathematicians and their work is formed. A young learner, someone unfamiliar with the stereotypical cultural view of mathematics, begins school. Through exposure to cultural stereotypes via media, adults, and peers, through interactions with teachers lacking rich images of mathematics, through a pedagogy that reinforces stereotypes, and through the lack of clear intervention by the mathematics community, the student begins forming a deficient image of mathematics. Stereotypes fill the void left vacant by desirable alternatives, and the student's forming mental model is validated through experience. Teachers play a key early role in inculcating students into the stereotypes of mathematics. However, the teachers would need to hold a healthy model of mathematics themselves to have any positive effect, as a teacher's beliefs influence the mathematical experiences they have with their students and so can influence the model that the students form (Mewborn & Cross, 2007). If students do not have healthy images of mathematics, they may choose to pursue other vocations, potentially robbing society of valuable mathematical innovation. Thus, exploring pre-service teachers' mental models related to mathematics is of importance.

Doing **Math**

From a survey of twenty-five post-secondary mathematics professors, Latterell and Wilson (2012) formulated a working definition of doing math, stating that in order to be considered doing math, mathematicians must be *creating* new mathematics. Schoenfeld (1994) stated, "research – what most mathematicians would call *doing* mathematics – consists of making contributions to the mathematical community's knowledge store" (p. 66). As a result of their definition, Latterell and Wilson excluded teachers of mathematics from being considered as mathematicians and only included mathematics professors if they were engaged in research mathematics. However, the general populace does not necessarily hold to this same understanding.

Through a survey of children in grades K-8, Rock and Shaw (2000) determined that they believed mathematicians did the same kind of math the students did in the classroom, only with larger numbers and harder problems. Many images drawn by the participants showed a mathematician in a classroom setting. Picker and Berry (2000) found similar results when they explored the images that 12-13 year olds made of mathematicians at work. About one-fifth of the drawings were of a teacher. In a follow-up prompt, the plurality of students mentioned that mathematicians were hired to teach math, suggesting that students have a limited idea of what mathematicians did. As a result, Picker and Berry suggested that mathematicians and their work were basically invisible to the students.

From a study of images of mathematicians at work created by high-achieving high school students attending a mathematics and science school, Aguilar, Rosas, Zavaleta, and Romo-Vázquez (2014) discovered that while the images were mostly male figures and contained many images of teachers, the students had a richer conception of what mathematicians did. They suggested this richer view developed from more exposure to advanced mathematics. Also, since many of the images contained items found in school settings, the students' limited interactions with math, mainly in the schools, heavily influenced their image of what it means to do math.

Due to the important role that teachers and the school setting play in the formation of a student's mental model of mathematics, this study explored the following question: What shared mental model of *doing mathematics* is held by pre-service elementary teachers (PSTs) in a mathematics content course? To address this question, this study used mental model theory to explore drawings of "doing math" generated by the participants. The drawings created by the participants were assumed to be external representations of their own mental images, which were in turn the projections of the visualizable aspects of their corresponding internal mental model.

Methodology

The study was conducted at a regional university in the southeastern United States. Participants (PSTs) in the study were undergraduate students in a teacher preparation program. The PSTs were enrolled in one of three sections of a mathematics content course for pre-service teachers. The course was the third in a sequence of four mathematics content courses required by the program. Forty-six PSTs were enrolled in the sections. The PSTs were divided between two disciplines, early childhood education (31, 67.4%) and special education (15, 32.6%). Of these students, 4 (8.7%) were male and 42 (91.3%) were female.
Additionally, 2 were Hispanic (4.3%), 10 were African-American (21.7%), and 34 (73.9%) were Caucasian.

During the sixth week of classes, PSTs responded in an at-home activity consisting of several drawing activities. Germane to this current study was the prompt: Draw a picture of a mathematician doing math. PSTs had approximately one week to create the drawings. The drawings were subsequently collected and scanned to create electronic files. The *Mathematician* *doing Math* drawings were uploaded to the No More Marking website (nomoremarking.com), a website that facilitates and calculates comparative judgements to explore preferences. Comparative judgement (CJ) is a method to measure subjectiveness such as individuals beliefs and is based on the idea that a person assigns a value to a phenomenon; when asked to choose between two phenomena, the person will base the decision on a comparison of the phenomena's values; the values are based upon a shared consensus of those making the judgements (Pollitt, 2012). In other words, with many judges participating, the preference of a phenomena is based upon the shared cultural preferences of the judges.

Figure 1. Judges view while judging at No More Marking website.

During the ninth week, for an at-home activity, PSTs were invited to perform comparative judgments on the two sets of drawings with the following question: Which best represents a mathematician doing math? Furthermore, PSTs were instructed to compare each drawing and choose the one they believed best answered the questions, to give honest responses, and to not judge the pictures on artistic merit. Each PST made 40 comparisons per data set. Figure 1 shows what the PST would see on his or her screen while judging. The image receiving the highest overall score, estimated by using the Bradley-Terry model and calculated internally on the website, was taken to be the image that best represented a mathematician doing math in the opinion of the participants. That is, the image was taken to represent the ideal image based upon shared standards of the students. Finally, during the twelfth week, for an at-home activity, PSTs were shown the image selected through comparative judgment as the best representative of a *Mathematician doing Math* and answered the following prompts: 1.) Why do you believe this picture was selected as the best representation of a mathematician doing math? 2.) To what extent does this picture align with your beliefs of what it means for a mathematician to do math? 3.) To you, what does it mean to be a mathematician?

Participant drawings and responses were explored for common themes using an open coding procedure. The drawing of a mathematician doing math selected through comparative judgement was analyzed in order to develop an initial model of the shared mental model the class had of a mathematician doing math. Student responses were coded separately and used to triangulate and refine the initial model resulting from the image analysis. In this manner, the shared mental model held by pre-service teachers of what it means for mathematicians to do math was constructed.

Results and Discussion

In this section, I discuss the analysis of the selected image and the PSTs' responses to the

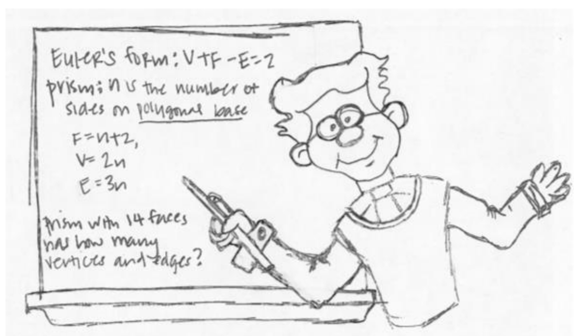

Figure 2. Image selected as best representing a mathematician doing math.

prompts. The conclusions were drawn through a comparison of the image to the PST responses to the prompts. Figure 2 shows the image PSTs selected through comparative judgement as their preferred representation of a mathematician doing math. The image shows a stereotypical Caucasian male wearing glasses, a long-sleeve shirt, a vest, and a tie. Of the top 11 drawings selected through CJ, 8 of the drawings showed the mathematician wearing glasses. 6 of the drawings were identifiable as male mathematicians, 2 were female mathematicians, and 3 were unidentifiable. On the week-12 bonus assignment, PSTs commented that the individual looked like a mathematician due to his dress. Students also commented that the image showed the mathematician as being well-organized, a quality needed in mathematics. Additionally, the writing on the board is neatly organized. As a possible interpretation and synthesis of these results, glasses could be taken to be a symbol of intelligence, indicating the deep knowledge of mathematics that must be possessed by a mathematician. The appearance of the mathematician reflects the order nature of mathematics.

In the image, the mathematician is smiling. 8 out of the 11 top drawings display the mathematician's face; 4 of these images show the mathematician smiling. The phrase "I love math" is written on a chalkboard in 2 images. PSTs indicated that mathematicians needed to actually like math to do math. Thus, the smile of the mathematician showed the joy he felt in doing math. According to the student comments, mathematicians have a deep passion for mathematics and desire to share their understanding of math with others. In the image, he is holding a pointer, the pointer being a symbol of a teacher, and using it to point to information on the board, indicating that he is teaching mathematics. Facing outward with his arms opened, the mathematician welcomes others to learn from him. In their responses to the week-12 bonus, many PSTs mentioned this need to share knowledge, as indicated in the following comment: "I believe that Mathematicians are continuously working through math. problems and explaining math to others. The mathematicians do this because they enjoy math. In the illustration, he is smiling, which shows confidence and enjoyment."

That the participants chose a teacher teaching as their representation of a mathematician doing math could have various explanations. The image could indicate the limited experiences students have had with mathematics. Most students have only experienced math in a classroom setting. The content on the board, Euler's formula, was being discussed in class at the time when the drawings were made, lending credence to this explanation. Alternatively, the participants were all PSTs; the image chosen could be biased toward their intended profession. That is, teaching is the lens through which they view the world.

A richer explanation was mentioned by several participants and best encapsulated by the following comment: "I believe that this picture was chosen because, not only do mathematicians sit there and solve math problems all day in an office, but they also share their solutions and findings with the world." Thus, according to the PSTs, a mathematician is a teacher; he or she solves difficult problems and then must effectively communicate this knowledge to others. Hence, part of doing math is teaching math, passing along knowledge. That PSTs emphasized the teaching aspect recalls the communication process standard of the NCTM. That is, mathematics is not merely solving problems, reasoning, proving, connecting, and representing. Communication is important, and communicating appears to be a necessary facet of the PSTs' mental model of doing math.

While more data would need to be collected from different populations, there does appear to be some misalignment between the mathematics community and the general population regarding *doing* math. Perhaps discussion within the mathematics and education communities would be warranted in order to help PSTs develop a mental model of math that would encourage robust models within students. If students view *doing* math as just teaching math, then they may become discouraged from entering the mathematics field.

References

Aguilar, M. S., Rosas, A., Zavaleta, J. G. M., & Romo-Vazquez, A. (2014). Exploring high-achieving students' images of mathematicians. *International Journal of Science and Mathematics Education, 1*(22). doi:10.1007/s10763-014-9586-1

Gentner, D. (2002). Psychology of mental models. In J. J. Smelser & P. B. Bates (Eds.), *International encyclopedia of the social and behavioral sciences* (pp. 9683-9687). Amsterdam: Elsevier.

Jabot, M., & Henry, D. (2007) Mental models of elementary and middle school students in analyzing simple battery and bulb circuits. *School Science and Mathematics, 107*(1) 371-381.

Jacob, E. K., & Shaw, D. (1999). Sociocognitive perspectives on representation. In M. E. Williams (Ed.), *Annual review of information science and technology: Volume 33, 1998* (pp. 131-185). Medford, NJ: Information Today.

Johnson-Laird, P. N. (1998). Imagery, visualization, and thinking. In J. Hochberg (Ed.), *Perception and cognition at the century's end* (pp. 441-467). San Diego, CA: Academic Press.

Johnson-Laird, P. N. (2004). The history of mental models. In K. Manktelow & M. C. Chung (Eds.), *Psychology of reasoning: theoretical and historical perspectives* (pp. 179-212). Hove, England: Psychology Press.

Johnson-Laird, P. N. (2005). Mental models and thought. In K. J. Holyoak & R. G. Morrison (Eds.), *The Cambridge handbook of thinking and reasoning* (pp. 185-208). Cambridge, England: Cambridge University Press.

Johnson-Laird, P. N., Girotto, V. & Legrenzi, P. (1998). Mental models: A gentle guide for outsiders. Retrieved from http://musicweb.ucsd.edu/~sdubnov/Mu206/MentalModels.pdf

Jones, N. A., Ross, H., Lynam, T., Perez, P., & Leitch, A. (2011). Mental models: An interdisciplinary synthesis of theory and methods. *Ecology and Society, 16*(1), 46. Retrieved from http://www.ecologyandsociety.org/vol16/iss1/art46/

Latterell, C. M., & Wilson, J. L. (2012). Students' perceptions of what mathematicians do. *The Mathematics Educator, 13*(2), 73-84.

Libarkin, J., Beilfuss, M., & Kurdziel, J. (2003). Research methodologies in science education: Mental models and cognition in education. *Journal of Geoscience Education 51*(1), 121-126.

Mewborn, D. S., & Cross, D. I. (2007). Mathematics teachers' beliefs about mathematics and links to students' learning. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 259-269). Reston, VA: National Council of Teachers of Mathematics.

Moseley, C., Desjean-Perrotta, B., & Utley, J. (2010). The Draw-An-Environment Test

Rubric (DAET-R): exploring pre-service teachers' mental models of the environment. *Environmental Education Research, 16*(2), 189-208.

Norman, D. A. (2014). Some observations on mental models. In D. Gentner & A. L. Stevens (Eds.), *Mental models* (pp. 7-14). New York, NY: Psychology Press. (Original work published 1983)

Peterson, I. (2015). The humanity and community of mathematics: Interview with Francis Edward Su, new MAA president. *MAA FOCUS, 35*(1), 4-6.

Picker, S. H., & Berry, J. S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics, 43*(1), 65-94.

Pollitt, A. (2012). Comparative judgement for assessment. *International Journal of Technology and Design Education*, 22(2), 157-170.

Robinson, J. S., Kelsey, K. D., & Terry, R., Jr. (2013). What images show that words do not: Analysis of pre-service teachers' depictions of effective agricultural education teachers in the 21st century. *Journal of Agricultural Education, 54*(3), 126-139.

Rock, D. & Shaw, J. M. (2000). Exploring children's thinking about mathematicians and their work. *Teaching Children Mathematics, 6*(9), 550-555.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53-70). Hillsdale, NJ: Erlbaum.

Thomas, J. A., Pedersen, J. E., & Finson, K. (2001). Validating the Draw-A-Science-Teacher-Test Checklist (DASTT-C): Exploring mental models and teacher beliefs. *Journal of Science Teacher Education, 12*(3), 295-310.

Utley, J., & Showalter, B. (2007). Preservice elementary teachers' visual images of themselves as mathematics teachers. *Focus on Learning Problems in Mathematics, 29*(3), 1-14.

FACTORS THAT INFLUENCE TEACHERS' GEOMETRY LEARNING FOR TEACHING

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This paper reports the quantitative portion of a study that examined cognitive and affective factors that influence teachers' geometry learning. Although teachers of all aged students are expected to possess content knowledge and pedagogical content knowledge for teaching, research reveals that few experience sufficient instruction or gain adequate understanding of the geometry concepts they must model and discuss with their students. A multi-measure approach was used to investigate relations between teachers' spatial ability, geometry attitudes, and geometry content learning. Findings revealed geometry attitudes predicting content pretest scores and spatial ability predicting growth of geometry understanding during the study period.

Rationale

Raising mathematics achievement at all levels of student learning is of critical importance. Both the National Council of Teachers of Mathematics and the National Governors Association reframed guidelines for K-12 mathematics instruction given compelling need for a mathematically literate workforce within the global economy (NCTM, n.d.; National Governors Association Center for Best Practices, 2014). The geometry domain received new attention within these frameworks, reflecting research on the influence of geometry understanding on mathematics learning overall (Clements & Sarama, 2011).

Important studies underscore the notion that geometry knowledge together with spatial ability enhances mathematics comprehension beyond what seems inherently geometric (de Hevia, Vallar, & Girelli, 2008; Stavridou & Kakana, 2008). For example, this may be the case when imagining the difference between fractional values on a number line, determining the number of people who might suitably occupy a particular space, or visualizing a representative line of scattered data points. To raise student achievement, it stands to reason that all areas of possible improvement deserve attention, especially if there is added value in joining multiple influences. This could be the case if we expose students to rich geometry ideas in tandem with opportunities to spatially represent mathematics. Problematic in this combined process of improving geometry teaching and invigorating mathematics with relevant spatial tasks is the fact that too many K-12 teachers have little experience with geometry concepts and spatially representing mathematics for teaching (Murphy, 2011; van der Sandt & Nieuwoudt, 2003).

Related Literature

A number of studies report that elementary and middle school teachers' geometry content knowledge is reliably insufficient, leaving them underprepared to teach in ways that help students internalize salient concepts (Ball & Bass, 2000; Murphy, 2011). Teachers who are unable to make sense of the mathematics they must teach are also less likely to ask important questions of their students during instruction. Furthermore, they may not uncover students' mathematical misconceptions that proliferate throughout the elementary and middle school years (Lin, Luo, Lo, & Yang, 2011; Ma, 2010). In many cases, elementary and middle school teachers rely on no more than their own secondary school coursework for geometry content knowledge in their teaching practice (Swafford, Jones, & Thornton, 1997).

Teacher Attitudes Toward Geometry

It stands to reason that teachers who express enthusiasm for learning and teaching geometry will communicate to their students a more positive outlook on this important area of mathematics. Ly and Malone (2010) studied geometry learning in high schools and found a positive relationship between views of geometry and meeting classroom learning goals. Campbell et al. (2014) purport that student achievement is positively affected by the interaction of teacher attitudes and mathematics content knowledge. Other researchers examining relationships between teacher content knowledge, pedagogical content knowledge, and student achievement report that improved content knowledge enhances beliefs about mathematics and influences the level of classroom discourse and use of problem-solving strategies (Lin, Luo, Lo, & Yang, 2011; Wilkins, 2008). Notably, preservice teachers report that geometry is a difficult subject, to learn and to teach (Barrantes & Blanco, 2006; Clements & Sarama, 2011).

Spatial Ability and Learning Geometry

Mental reasoning in mathematics requires a wide range of spatial tasks. Clements and Sarama (2011) argue that the asset of strong spatial ability reaches beyond geometry learning to all areas of mathematics. For this reason, teachers who must engage students in rich mathematics learning need to model and discuss content using spatial strategies and examples.

Spatial ability in this study comprises two of three complex cognitive elements suggested in the literature: 1) spatial visualization–the ability to form mental images of ideas or inherently visual objects (Halpern, 2000), and 2) mental rotation–rotation of a mentally imaged figure (Voyer, Voyer, & Bryden, 1995). Unal, Jakubowski, and Corey (2009) conclude in their study of teachers' learning that those with elevated spatial visualization ability made greater strides learning geometry than those with lower ability. Lord and Rupert (1995) found that undergraduate elementary education students scored poorly on measures of visual-spatial aptitude. The only high scorers were those pursuing a mathematics or science concentration, suggesting that people who aspire to such study already possess robust spatial abilities. Importantly, Moore-Russo et al. (2013) found that teachers struggle with an important teaching competence, communicating spatial reasoning during instruction. Effective communication is vital to geometry teaching and translates to instructional efficacy.

Methodology

The purpose of this study was to examine cognitive and affective factors that influence the geometry learning of teachers charged with supporting student mathematics learning. A multi-measure approach was used to examine relationships between teachers' spatial ability, geometry attitudes, and geometry content learning.

Participants

Forty-one students (29 females, 12 males) in two graduate sections of a mathematics education geometry course volunteered for this study. Participants were also K-8 teachers in underperforming schools where improving student mathematics achievement is essential.

Table 1. *Participant Descriptors*

Subject Age Range	Mean Age	Grades Taught *(range)*	Grades Taught *(median)*	Females/Males
24-62 yrs.	37 yrs.	K - 8	5-6	29/12

Alignment of test results was facilitated by participant use of anonymous alphanumeric codes created during enrollment in the mathematics education program.

Procedures

Tests of geometry content knowledge. Participants completed a geometry content knowledge inventory to begin the course and again on the final day of instruction. Coursework presented by the same instructor included concepts investigation before skills practice using various strategies. The inventory measure addressed a majority of course concepts such as triangle congruency and similarity, trigonometry, geometry proofs, application of the Pythagorean theorem, quadrilaterals, and polygon attributes. Two independent geometry instructors twice reviewed inventory scores where a total of 40 points was possible. Analyses incorporated scores as percentages.

Geometry attitudes inventory. This study employed the 32-item Utley Geometry Attitudes Scale (Utley, 2007) to examine the affective factors of usefulness, confidence and enjoyment as they relate to geometry learning. Utley reports validity and internal reliability for the inventory. Measure completion occurred on the third day of instruction in order to 1) inhibit potential effects of content exposure and grades awareness on measure results, and 2) provide participants a context for response to questions about such things as confidence in solving problems, given little or no prior experience with geometry beyond a standard high school geometry year.

Tests of spatial ability. The 24-item Vandenberg and Kuse Mental Rotations Test (MRT) (Vandenberg & Kuse, 1978) and the 30-item Revised Purdue Spatial Visualization Test (PSVT:R) (Yoon, 2011), assessed mental rotation and spatial visualization abilities in this study. The tests measure closely related cognitive abilities and together served to strengthen analyses given the participant number. Test administration occurred early on the second instructional day with a course activity conducted between the two sessions. The literature supports high reliability and validity for these spatial measures (Voyer, Voyer, & Bryden, 1995; Maeda & Yoon, 2013).

Results

Research questions for this study were as follows: Do *spatial abilities* predict teachers' initial geometry knowledge scores? Do *spatial abilities* predict teachers' geometry knowledge growth? Do teachers' *attitudes* about geometry predict their initial geometry knowledge scores? Do teachers' *attitudes* about geometry predict their geometry knowledge growth?

The spatial abililty variables of mental rotation and spatial visualization in this study were highly correlated ($p < 0.001$). Due to strong correlation and because the participant number was likely too small to ascertain distinctive contributory factors for each spatial test, the two spatial measures were combined to form a composite spatial ability score representing the mean of each participant's spatial test results (Tabachnick & Fidell, 2012).

Pretest

Geometry pretest scores examined as a function of geometry attitudes and composite spatial scores show a model summary in Table 2 indicating that these two predictor variables account for approximately 34% of variance in geometry pretest scores ($R^2 = 33.7\%$).

Table 2. *Regression Model Summary*

R	R Square	Adjusted R Square	Std. Error of the Estimate
.580[a]	.337	.302	18.05373

a. Predictors: (Constant), Spatial Composite, Geometry Attitudes

Table 3 presents an analysis of variance, a comparison of the means of the three variables geometry attitudes, composite spatial, and geometry pretest.

Table 3. *ANOVA of Dependent Variable Geometry Pretest*

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	6284.876	2	3142.438	9.641	.000[a]
Residual	12385.611	38	325.937		
Total	18670.488	40			

a. Predictors: (Constant), Spatial Composite, Attitude

The analyses shown below in Table 4 reveal no significance with regard to composite spatial scores predicting geometry pretest scores. However, geometry attitudes were significantly predictive of geometry pretest scores ($p < .01$).

Table 4. *Regression Coefficients for Geometry Attitudes and Composite Spatial with Dependent Variable Pretest*

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	β	*t*	Sig.
(Constant)	-30.749	20.235		-1.520	-.137
Geometry Attitudes	19.625	4.975	.523	3.944	.000**
Composite Spatial	2.606	1.640	.211	1.589	.120

** Correlation is significant at the 0.01 level (2-tailed).

Growth

Paired samples t-tests examined statistical significance for pre-to posttest geometry content knowledge growth before addressing the main research questions related to growth predictors. Significance was determined ($p < .01$) with a large effect size (1.04).

Table 5 presents the regression model summary. The demonstrated correlation of geometry attitudes and geometry growth was small and negative (small $R^2 = .01$ to .29), implying that a small increase in geometry attitudes correlates with a small decrease in geometry growth. Predictor variables account for approximately 18% of growth variance.

Table 5. *Regression Model Summary*

R	R Square	Adjusted R Square	Std. Error of the Estimate
.422[a]	.178	.128	12.52238

a. Predictors: (Constant), Spatial Composite, Geometry Attitudes

Table 6 presents an analysis of variance, a comparison of the means of the three variables geometry attitudes, composite spatial, and geometry growth.

Table 6. *ANOVA of Dependent Variable Geometry Growth*

Model	Sum of Squares	df	Mean Square	F	Sig.
Regression	1122.246	2	561.123	3.578	.039[a]
Residual	5174.726	33	156.810		
Total	6296.972	328			

a. Predictors: (Constant), Spatial Composite, Attitude

Next, regression analyses examined dependent variable geometry growth (Table 7) as functions of predictor variables geometry attitudes and composite spatial ability. Results show spatial ability moderately predicting the geometry growth between pre- and posttest.

Table 7. *Regression Coefficients of Geometry Attitudes and Composite Spatial with Dependent Variable* ***Growth***

	Unstandardized Coefficients		Standardized Coefficients		
	B	Std. Error	β	*t*	Sig.
(Constant)	40.314	15.242		2.645	.012
Geometry Attitudes	-3.774	3.838	-.155	-.983	.333
Composite Spatial	2.932	1.187	.390	2.471	.019*

* Correlation is significant at the 0.05 level (2-tailed).

Discussion

The significant predictive nature of geometry attitudes on pretest scores in this small study suggests that the affective quality predisposes participants to better scores than those they might realize simply by chance. Positive attitudes toward geometry may reflect facility with this element of mathematics at an earlier time, contributing to openness toward the content in the context of one's life as well as a more confident approach during test taking. A negative disposition toward geometry might interact with test taking in opposite fashion.

Attitudes toward geometry influenced pretest scores although did not predict the growth of geometry understanding in this study. For one thing, it is possible that participants with better geometry attitudes had less room to grow in terms of content knowledge. Furthermore, the specific geometry attitudes measured in the Utley scale may not have provided adequate variation for interaction with learning gains. However, the moderately predictive nature of spatial ability on knowledge growth is supported in this study and warrants further investigation.

The combined effect of mental rotation and spatial visualization skills on growth in this study is reasonable given that course content necessarily included mental manipulation of both 2D shapes and 3D figures. Such exploration calls upon cognitive resources that support spatial processing. Interestingly, Huk (2006) purports that high spatial ability students profit from 3D objects use during learning and that low spatial ability students may suffer some level of cognitive disengagement. If this is so, it may help to explain variability in knowledge growth within the participant group. Understanding the predictive quality of spatial ability in this study would benefit from repeating the study with more participants. A larger sample size might also allow inspection of differences between participants' mental rotation and spatial visualization abilities, which in turn could influence instructional strategies for all aged students.

Implications

Elementary and middle school teachers must develop good attitudes toward geometry as well as sufficient geometry knowledge for teaching if they expect to assist young learners in gaining adequate geometry understanding throughout schooling. Correcting student misconceptions is difficult if the basis for misunderstandings is not discernible by teachers due to their own inadequate comprehension (Ma, 2010) or interest in the content. Furthermore, teachers and teacher educators may need to appraise the use of spatial tasks in their teaching, especially if such opportunities contribute to improved student spatial ability over time. Understanding the malleability of spatial ability was not part of the current study. However, there is growing evidence within the literature that spatial enhancement is possible (Uttal et al., 2013). Importantly, study results suggest addressing both cognitive and affective factors from the earliest stages of geometry learning.

References

Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 83-104). Westport, CT: Ablex.

Barrantes, M., & Blanco, L. J. (2006). A study of prospective primary teachers' conceptions of teaching and learning school geometry. *Journal of Mathematics Teacher Education, 9*(5), 411-436.

Campbell, P. F., Nishio, M., Smith, T. M., Clark, L. M., Conant, D. L., Rust, A. H., …Choi, Y. (2014). The relationship between teachers' mathematical content and pedagogical knowledge, teachers' perceptions, and student achievement. *Journal for Research in Mathematics Education, 45*(4), 419-459.

Clements, D., & Sarama, J. (2011). Early childhood teacher education: the case of geometry. *Journal of Mathematics Teacher Education, 14*(2), 133-148.

de Hevia, M. D., Vallar, G., & Girelli, L. (2008). Visualizing numbers in the mind's eye: The role of visuo-spatial processes in numerical abilities. *Neuroscience & Biobehavioral Reviews, 32*(8), 1361-1372.

Halpern, D. (2000). *Sex differences in cognitive abilities* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum.

Huk, T. (2006). Who benefits from learning with 3D models? The case of spatial ability. *Journal of Computer Assisted Learning, 22*(6), 392-404.

Lin, C.-Y., Luo, F., Lo, J.-J., & Yang, D.-C. (2011). U.S. and Taiwanese pre-service teachers' geometry knowledge and thinking. *International Journal for Mathematics Teaching and Learning*. www.cimt.plymouth.ac.uk/journal/lin.pdf

Lord, T. R., & Rupert, J. L. (1995). Visual-spatial aptitude in elementary education majors in science and math tracks. *Journal of Elementary Science Education, 7*(2), 47-58.

Ly, R. K., & Malone, J. A. (2010). *Teachers' perceptions of geometry instruction and the learning environment in years 9-10 ESL classrooms*. http://files.eric.ed.gov/fulltext/ED520927.pdf

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States* (2nd ed.). NY: Routledge.

Maeda, Y., & Yoon, S. (2013). A meta-analysis on gender differences in mental rotation ability measured by the Purdue Spatial Visualization Tests: Visualization of rotations (PSVT:R). *Educational Psychology Review, 25*(1), 69-94.

Moore-Russo, D., Viglietti, J. M., Chiu, M. M., & Bateman, S. M. (2013). Teachers' spatial literacy as visualization, reasoning, and communication. *Teaching and Teacher Education, 29*, 97-109.

Murphy, C. (2011). The role of subject knowledge in primary prospective teachers' approaches to teaching the topic of area. *Journal of Mathematics Teacher Education*, 1-20.

NCTM. (n.d.) Math standards and expectations. Retrieved from http://www.nctm.org/standards/content.aspx?id=4294967312

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2014). *Common core state standards for mathematics.* Washington, D.C.

Stavridou, F., & Kakana, D. (2008). Graphic abilities in relation to mathematical and scientific ability in adolescents. *Educational Research, 50*(1), 75-93.

Swafford, J. O., Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education, 28*(4), 467-483.

Tabachnick, B. G., & Fidell, K. S. (2012). *Using multivariate statistics* (6th ed.). Upper Saddle River, NJ: Pearson.)

Unal, H., Jakubowski, E., & Corey, D. (2009). Differences in learning geometry among high and low spatial ability pre-service mathematics teachers. *International Journal of Mathematical Education in Science & Technology*, *40*(8), 997-1012.

Utley, J. (2007). Construction and validity of geometry attitude scales. *School Science & Mathematics, 107*(3), 89-93.

Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin, 139*(2), 352-402.

Vandenberg, S. G., & Kuse, A. R. (1978). Mental rotations, a group test of three-dimensional spatial visualization. *Perceptual and Motor Skills, 47*(2), 599-604.

van der Sandt, S., & Nieuwoudt, H. D. (2003). Grade 7 teachers' and prospective teachers' content knowledge of geometry. *South African Journal of Education, 23*(3), 199-205.

Voyer, D., Voyer, S., & Bryden, M. P. (1995). Magnitude of sex differences in spatial abilities: A meta-analysis and consideration of critical variables. *Psychological Bulletin, 117*(2), 250-270.

Wilkins, J. L. M. (2008). The relationship among elementary teachers' content knowledge, attitudes, beliefs, and practices. *Journal of Mathematics Teacher Education, 11*(2), 139-164.

Yoon, S. Y. (2011). Revised Purdue Spatial Visualization Test: Visualization of Rotations (Revised PSVT:R) [Psychometric Instrument].

TEACHERS' SELF-EFFICACY AND KNOWLEDGE FOR THE INTEGRATION OF TECHNOLOGY IN MATHEMATICS INSTRUCTION AT URBAN SCHOOLS

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This study examined teachers' technology integration (TI) self-efficacy and technological pedagogical content knowledge (TPACK). We surveyed 80 K-12 mathematics teachers from urban school districts before and after a three-week professional development (PD) program. Results indicated that: a) beliefs about mathematics and mathematics instruction were associated with TI self-efficacy, TPACK dimensions, and instructional technology use, b) TI self-efficacy and TPACK dimensions improved upon PD completion, and c) teachers' perception of technology instruction through PD predicted two dimensions of TPACK. This study has implications for instruction in the use of technology for mathematics teaching provided by teacher preparation and PD programs.

Theoretical Framework

It is unequivocal that emerging instructional technologies have the potential to bolster mathematics learning and instruction at urban schools (e.g., International Society for Technology in Education, 2007; National Council of Teachers of Mathematics, 2008). With appropriate use, instructional technology can help teachers enact their teaching-related tasks more effectively, and in turn, facilitate students' learning of mathematics. Given the importance of effective technology integration in mathematics education, we utilized the technological pedagogical content knowledge (TPACK) model—a theoretical framework that addresses teachers' knowledge of effective technology integration in instruction (Mishra & Koehler, 2006)—to investigate the effect that beliefs about mathematics, mathematics teaching, and professional development have on technology integration self-efficacy and knowledge.

Teachers' Self-efficacy for Technology Integration

Not only do teachers need knowledge of how to incorporate technology into instruction, but they also need to believe that they have the ability to use technology effectively. This belief is known as technology integration (TI) self-efficacy and is associated with technology use in the classroom (e.g., Albion, 1999, c.f. Wang, Ertmer, & Newby, 2004). TI self-efficacy is also closely related to TPACK, a concept that will be further discussed in the next section (Wang et al., 2004).

Teachers' Technological Pedagogical Content Knowledge

Technological pedagogical content knowledge (TPACK) is a theoretical framework that addresses teachers' knowledge of effective technology integration in instruction. Developers of TPACK contend that in addition to the importance of teachers' content-specific knowledge and pedagogical knowledge for effective teaching, teachers should also gain knowledge of how to integrate technology in their instruction (Thompson & Mishra, 2007). Through the lens of the TPACK framework, we identified three technology-specific knowledge dimensions as proposed by Mishra and Koehler (2006): technological content knowledge (TCK; the knowledge of how technology can provide new representations of specific content), technological pedagogical knowledge (TPK; the knowledge of how different technologies can be utilized for teaching), and technological pedagogical content knowledge (TPCK; the knowledge necessary for teachers to integrate technology into their teaching of a specific content area). It is important to understand the factors that influence TPACK dimensions because this knowledge can inform PD and teacher education programs about how to best approach curriculum and instruction related to technology use in the classroom (Mishra & Koehler, 2006; Schmidt et al., 2009).

Effect of Teacher Beliefs on Technology Integration Self-efficacy and Knowledge

Investigating teachers' educational beliefs about mathematics teaching and learning is important given that mathematics teachers' self-efficacy and epistemic beliefs predict their mathematical knowledge for teaching and their instructional practices (Corkin, Ekmekci, & Papakonstantinou, 2015; Pajares, 1992). Moreover, when considering the influence professional development may have on teachers' integration of technology, it is also essential to examine the effect that teachers' fundamental educational beliefs about teaching and learning have on technology integration, given that teacher beliefs have been identified as barriers to technology use (see Kim, Kim, Lee, Spector, & DeMeester, 2013). For example, a recent study found that less sophisticated epistemic beliefs were associated with lower levels of technology integration in the classroom (Kim et al., 2013). Little research, however, has examined the extent to which fundamental beliefs about teaching and learning relate to the differential utilization of technology in instruction. Moreover, the studies that have examined this relationship have not emphasized domain-specific beliefs (e.g., Kim et al., 2013). Therefore, this study extends extant work by investigating whether key teacher beliefs about mathematics and mathematics instruction, namely, mathematics self-concept, epistemic beliefs about mathematics, and mathematics teaching self-efficacy, play a role in teachers' TI self-efficacy and TPACK dimensions.

Effect of Professional Development on Technology Integration Self-efficacy and Knowledge

Similar to other social-cognitive types of self-efficacy, research indicates that vicarious learning (modeling) influences TI self-efficacy (Wang et al., 2004). Furthermore, researchers identify modeling as a frequently used strategy to enhance in-service teachers' TPACK (Voogt, Fisser, Roblin, Tondeur, & van Braak, 2012). Thus, this study focuses on whether the quality of a PD program that emphasized modeling of technology use and promoted collaboration would positively influence TI self-efficacy and TPACK.

Research Questions

The following research questions guided this study:

1. To what extent do teachers' beliefs about mathematics and mathematics instruction relate to their technology integration (TI) self-efficacy, technological pedagogical content knowledge (TPACK), and their self-reported frequency of technology use in mathematics instruction?
2. To what extent do teachers' TI self-efficacy beliefs and TPACK change upon participation in professional development (PD)?
3. What effect does teachers' perceptions of the quality of PD instruction in the use of technology have on teachers' TI self-efficacy and TPACK?

Method

Program Description

We surveyed 80 K-12 in-service mathematics teachers from urban school districts in Texas who participated in a three-week rigorous PD program focusing on pedagogical content knowledge and effective technology integration. The teachers volunteered or were selected by school administration to participate in the program. The mathematical content focus was: (a) numbers, operations, and quantitative reasoning; and (b) patterns, relationships, and algebraic reasoning. Integration of technology for effective mathematics instruction focused on three main objectives. Teachers would need to learn to effectively use technology to (1) collaborate and plan for instruction, (2) enhance student learning in numbers and operations, patterns, functions, and algebraic reasoning, and (3) monitor student progress and provide immediate help to students (formative assessment), as well as evaluate student learning (summative assessment).

Technology activities included demonstrations by master teachers (modeling), technology-shares where each participating teacher shared their opinions about an app/software that they found useful, and technology integrated lesson plan assignments where teacher participants received critical and constructive feedback from master teachers. Since the summer PD included teachers from all grade bands at primary and secondary levels, the type of technological devices that teachers modeled and practiced varied. For example, graphing calculators with network ability were used in the high school class but not in the elementary class. The software and apps also varied due to the nature and rigor of mathematics topics covered across classes (from fractions and place value apps in the elementary class to function transformation software in the high school class). However, computers, iPads, interactive white boards, smart phones, GeoGebra, online collaboration and course management tools, presentation tools, and polling apps were modeled and practiced across all grade bands.

Procedure

We surveyed teachers several weeks before and after the three-week summer PD program. Survey items were adapted from valid and reliable instruments: 6 items for mathematics self-concept (Marsh, 1990); 8 items for epistemic beliefs in mathematics (Hofer, 2000); 13 items for self-efficacy in teaching mathematics (Enochs, Smith, & Huinker, 2000); 16 items for TI self-efficacy (Wang et al., 2004); and 11 items for technology knowledge (1 TCK, 5 TPK, and 5 TPCK items; Schmidt et al., 2009). In addition to these items, the pre-survey also included items measuring teachers' frequency of use of several technologies for planning, instruction, and assessment (e.g., virtual manipulatives, document cameras). The post survey also included items to assess teacher perceptions about the PD instruction in specific technologies. These perceptions served as a proxy for the program's quality of instruction in the use of technology for planning, instruction, and assessment. Items for frequency of technology use were on a 4-point Likert-scale (0-never to 3-almost always). All other survey items were on a 5-point Likert-scale: 1 (strongly disagree) to 5 (strongly agree) for mathematics epistemic beliefs, mathematics teaching self-efficacy, TI self-efficacy, and TPACK items; 1 (not like me) to 5 (very much like me) for mathematics self-concept items; 0 (not provided) to 4 (excellent) for perceptions of PD instruction in the use of technology. Cronbach's alphas for the scales were: mathematics self-concept (.84), mathematics epistemic beliefs (.67), math-teaching self-efficacy (.83), TI self-efficacy (.94), TPK (.79), and TPCK (.89)

Participants

In this study, 80 K-12 mathematics teachers representing several urban school districts took the pre- and post- surveys. Ethnic composition of the teachers was 25% White, 40% African American, 21% Hispanic, 13% Asian, and 1% other. There were 63 female teachers (79%) and 17 male teachers (21%). Of all the teachers, 20 attended the elementary class (grades K-3); 19 attended the intermediate class (grades 4-6); 21 attended the middle school class (grades 7-8); and 20 attended the high school class (grades 9-12).

Findings

Correlation results (Table 1) indicated that teacher beliefs about mathematics and mathematics teaching were associated with TI self-efficacy, TPACK dimensions, and frequency of technology use. Specifically, teachers' personal beliefs about their mathematics ability (mathematics self-concept) was positively associated with their TI self-efficacy ($p < .01$), TPK ($p < .01$), TPCK ($p < .01$), and with their frequency of technology use for planning ($p < .05$) and instruction ($p < .01$). Teachers' beliefs about the certainty of mathematics knowledge—an epistemic belief dimension where the stability of mathematics knowledge is viewed as either certain or evolving—was negatively associated with their frequency of technology use for instruction ($p < .05$) and assessment ($p < .01$). In other words, less sophisticated epistemic beliefs (certainty) were associated with lower frequencies of technology use. Teachers' self-efficacy for mathematics instruction emerged as having statistically significant associations with all technology-related variables, and most of these correlations were stronger compared to the correlations between teacher beliefs about mathematics and technology. Specifically, teachers' self-efficacy for mathematics instruction was positively correlated with TI self-efficacy ($p < .01$), TCK ($p < .01$), TPK ($p < .001$), TPCK ($p < .05$), and with the frequency of technology use for planning ($p < .05$), instruction ($p < .001$), and assessment ($p < .05$). Finally, frequency of technology use was positively associated with both TI self-efficacy and all TPACK dimensions. These correlations ranged from small to moderate ($r = .31$ to $r = 53$).

We conducted paired-samples t-tests to investigate whether changes occurred in teachers' self-efficacy and knowledge about the integration of technology (see Table 2). Overall, the changes were statistically significant ($p < .001$) with practically significant effect sizes (ranging from Cohen's $d = 0.58$ to Cohen's $d = 0.75$; see Ferguson, 2009). Specifically, teachers' TI self-efficacy, TCK, TPK, and TPCK increased (0.22, 0.32, 0.53, 0.35, and 0.51 points, respectively).

Table 1. *Means, Standard Deviations, and Pearson Correlations among the Main Variables*

Variable	*M*	*SD*	1	2	3	4	5	6	7	8	9
1. Math Self-Concept	3.71	0.68	---								
2. Math Epistemic Belief: Certain Knowledge	2.74	0.52	-.07	---							
3. Self-efficacy for Math Teaching	4.04	0.46	$.32^{**}$	-.02	---						
4. Technology Integration (TI) Self-efficacy	3.80	0.59	$.37^{**}$	-.07	$.38^{*}$	---					
5. Technology Content Knowledge (TCK)	3.73	0.84	.20	-.00	$.30^{**}$	$.61^{***}$	---				
6. Technology Pedagogical Knowledge (TPK)	3.81	0.63	$.34^{**}$	-.03	$.41^{***}$	$.81^{***}$	$.69^{***}$	---			
7. Technology Pedagogical Content Knowledge (TPCK)	3.63	0.75	$.31^{**}$	-.04	$.28^{*}$	$.88^{***}$	$.62^{***}$	$.82^{***}$	---		
8. Technology Use: Planning	1.16	0.39	$.29^{*}$	-.12	$.25^{*}$	$.46^{***}$	$.39^{***}$	$.49^{***}$	$.48^{***}$	---	
9. Technology Use: Instruction	1.14	0.39	$.37^{**}$	$-.28^{*}$	$.43^{***}$	$.45^{***}$	$.43^{***}$	$.53^{***}$	$.46^{***}$	$.78^{***}$	---
10. Technology Use: Assessment	0.71	0.39	.19	$-.37^{**}$	$.23^{*}$	$.45^{***}$	$.31^{**}$	$.39^{***}$	$.44^{***}$	$.59^{***}$	$.62^{***}$

$N = 80$; $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$.

Table 2. *Paired-Samples t-test Results for Change in Teachers' TI Self-efficacy and TPACK*

	Paired differences (post – pre)				
Variable	N	Mean gain	SD	*t*-value	Cohen's *d*
TI Self-Efficacy	80	0.32	0.52	5.50^{***}	0.61
TCK	80	0.53	0.86	5.48^{***}	0.61
TPK	80	0.35	0.60	5.20^{***}	0.58
TPCK	80	0.51	0.68	6.72^{***}	0.75

Notes. $^{***}p < .001$.

Table 3 displays the results of four multiple linear regression analyses predicting TI self-efficacy, TCK, TPK, and TPCK at the end of the three-week PD. Teachers' TI self-efficacy and TPACK dimensions at the onset of PD were entered as control variables in order to understand the extent to which perceptions about the quality of PD technology instruction predicted TI self-efficacy and TPACK dimensions at the end of PD beyond teachers' initial levels of TI self-efficacy and TPACK. The models with TPK and TPCK as the outcome variables were statistically significant ($F(2, 77) = 17.92$, $p < .001$, $R^2 = 32\%$; $F(2, 77) = 17.53$, $p < .001$, $R^2 = 31\%$, respectively. Results indicated that more positive perceptions of PD quality were statistically significantly associated with TPK ($\beta = .25$, $p < .05$) and TPCK ($\beta = .19$, $p < .05$) after controlling for initial TPK and TPCK levels.

Table 3.

Summary of Multiple Linear Regression Analyses Predicting Technology Beliefs and Knowledge

Variable	Post-TI S.E.	Post-TCK	Post-TPK	Post-TPCK
	β	β	β	β
Control Variables				
Pre-TI Self-Efficacy	.59***	---	---	---
Pre-TCK	---	.40***	---	---
Pre-TPK	---	---	.48***	---
Pre-TPCK	---	---	---	.51***
Main Predictor				
Perceptions of PD Tech Instruction	.18	.17	.25*	.19*
R^2	.40***	.20***	.32***	.31***

Notes. $N = 80$; $^{*}p < .05$. $^{**}p < .01$. $^{***}p < .001$. "---": variable not included in analysis.

Discussion and Conclusions

The study expands our knowledge about the types of teacher educational beliefs that may impede the integration of technology in mathematics instruction. One significant and perhaps novel finding is that mathematics teaching self-efficacy is moderately associated with numerous indicators of technology use in the classroom (TI self-efficacy, TPACK domains, and self-reported use of technology). Current findings suggest that for teachers to feel confident in integrating technology, they must also feel self-efficacious about mathematics teaching in general. A second significant finding is that teachers' perceptions of PD instruction in technology integration are positively associated with TPK and TPCK at the end of the PD program. This finding is consistent with research indicating that modeling technology integration and enacting technology-based lessons through collaboration in teacher education programs are effective means of developing TPACK (see Voogt et al., 2012). In closing, these findings are important because they may inform teacher preparation and PD programs about the importance of incorporating technological instruction to promote teachers' use of technology in mathematics teaching. These findings also suggest that PD instructors keenly assess teachers' fundamental beliefs about mathematics and mathematics instruction, as these beliefs may inhibit teachers from incorporating technology in their classrooms.

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References

Corkin, D, Ekmekci, A., & Papakonstantinou, A. (2015). Antecedents of teachers' educational beliefs about mathematics and mathematical knowledge for teaching among in-service teachers in high poverty urban schools. *The Australian Journal of Teacher Education, 40*(9), 31–62.

Enochs, L. G., Smith, P. L., & Huinker, D. (2000). Establishing factorial validity of the mathematics teaching efficacy beliefs instrument. *School Science and Mathematics, 100*(4), 194–202.

Ferguson, C.J. (2009). An effect size primer: A guide for clinicians and researchers. *Professional Psychology: Research and Practice*, 40(5), 532–538.

Hofer, B. K. (2000). Personal epistemology research: Implications for learning and teaching. *Journal of Educational Psychology Review*, *13*(4), 353–382.

International Society for Technology in Education (2007). *National educational technology standards for students (2nd ed.)*. Washington, DC: Author.

Kim, C., Kim, M.K., Lee, C., Spector, M., DeMeester, K. (2013). Teacher beliefs and technology integration. *Teaching and Teacher Education, 29*, 76–85.

Marsh, H. W. (1990). The structure of academic self-concept: The Marsh/Shavelson model. *Journal of Educational Psychology, 82*(4), 623–636.

Mishra, P., & Koehler, M. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *The Teachers College Record, 108*(6), 1017–1054.

National Council of Teachers of Mathematics. (2008). *Position statement: The role of technology in the teaching and learning of mathematics*. Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Technology-in-Teaching-and-Learning-Mathematics/

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–332.

Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J., Shin, T. S. (2009). Technological pedagogical content knowledge (TPACK)-The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Technology in Education, 42*, 123–149.

Thompson A. & Mishra P. (2007) Breaking news: TPCK becomes TPACK! *Journal of Computing in Teacher Education*, 24, 38–39.

Voogt, J., Fisser, P., Roblin, N.P., Tondeur, J., & van Braak, J. (2012). Technological pedagogical content knowledge—a review of the literature. *Journal of Computer Assisted Learning, 29*(2), 109–121.

Wang, L., Ertmer, P.A., & Newby, T.J. (2004). Increasing preservice teachers' self-efficacy beliefs for technology integration. *Journal of Research on Technology in Education, 36*(3), 231–250.

DEEPENING STATISTICAL CONTENT KNOWLEDGE FOR THE COMMON CORE

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With the adoption of the Common Core State Standards by many states, many in-service middle and secondary teachers find themselves with a shortage of statistical content knowledge to teach the statistics standards. This paper discusses the development and implementation of a one-week intensive professional development workshop designed to increase statistical and pedagogical content knowledge, increase self-efficacy related to statistics standards, and positively impact attitudes towards statistics.

Introduction

Kentucky Senate Bill 1 (2009) addressed many areas related to K-12 education, including the revision of the academic standards for what Kentucky students should know and be able to do at each grade level. The Kentucky Department of Education, in conjunction with the Council on Postsecondary Education, was tasked with planning and implementing new content standards across all grade levels. In February 2010, Kentucky was the first of 48 states to adopt the Common Core State Standards (CCSS, 2010). The mathematics (and English/language arts) standards were first implemented in Kentucky schools in the 2011 – 2012 academic year.

Prior to the implementation of the mathematics standards in the K-12 programs, P-16 educators had voiced some concerns over the rigor of the standards, as well as the in-service teachers' content knowledge on the standards they would be teaching (Jenkins & Agamba, 2013). One particular area of concern with the content knowledge was in the realm of Statistics and Probability. Although many college programs in teacher education include some statistics coursework in their preparation of teachers (specifically for middle and high school programs), the amount and approach of traditional statistics coursework did not align with the new CCSS. George Cobb (2007) commented about traditional statistical coursework with the statement, "Our curriculum is needlessly complicated because we put the normal distribution, as an approximate sampling distribution for the mean, at the center of our curriculum, instead of putting the core logic of inference at the center," (p. 4). The CCSS attempts to introduce the logic of inference without some of the complications. However, current in-service (as well as many pre-service) teachers lacked any statistical coursework using randomization-based procedures since the idea of teaching this way was quite new (Rossman & Chance, 2008).

Review of Literature

The statistical preparation of in-service (and pre-service) teachers has been an area of concern for quite some time. In particular, the Mathematical Education of Teachers (MET) report discussed middle-grades mathematics teachers' lack of preparation to teach the topics of statistics and probability (Conference Board of the Mathematical Sciences (CBSM), 2001). Although the recommended statistical coursework has increased and evolved since then, the MET II Report found the importance of a firm statistical education for teachers has not evolved (CBMS, 2012). This report also states that "Many teachers prepared before the era of the CCSS will need opportunities to study content that they have not previously taught, particularly in the areas of statistics and probability" (CBMS, 2012, p. 68). Scheaffer and Jacobbe (2014) recently tracked the history of statistics education, which include the two MET reports and the CCSS. They specifically address in their conclusion that a group of educators are working on making a list of recommended but necessary changes in statistics education.

There has been some PD opportunities offered for in-service teachers, but up until the implementation of the CCSS, this was done primarily in the area of probability (Baterno, Godino, & Roa, 2004) with minimal statistical training targeted at high school and college teachers (Garfield & Everson, 2009). In addition, there is some literature on the development of new programs aimed at in-service middle school teachers in both mathematics (Heaton, Lewis, & Smith, 2013; Heaton, Lewis, Homp, Dunbar, & Smith, 2013) and statistics (Schmid, Blankenship, Kerby, Green, & Smith, 2014) that have adapted to include elements of the CCSS.

After surveying the literature, one thing is clear – there is a need to increase the statistical training and content knowledge of both in-service and pre-service secondary teachers.

Methodology

Given this vital need for statistical content training related to the CCSS, the authors submitted a proposal to host a one-week intensive workshop for middle and high school in-service and pre-service teachers in June 2015 that focused on statistical content. This workshop was supported and funded by the Kentucky Center for Mathematics (KCM) and information about the workshop was distributed across northern Kentucky (near Northern Kentucky University where the workshop was held).

The workshop was designed to span five days (Monday – Friday) with a different statistical content area emphasized each day. The areas covered included: descriptive statistics, sampling distributions, simulation-based hypothesis testing, bootstrap confidence intervals, and regression. The topics were chosen because of their ability to span across the secondary standards, as well as their connection to recommended content in the Statistical Education of Teachers (SET) report (ASA, 2015). For example, regression topics aimed at the middle school level included an informal lesson on fitting a line to a dataset and assessing the strength of the linear relationship using the quadrant count ratio. More advanced topics included determining the line of best fit and assessing the strength of the linear relationship using Pearson's correlation coefficient; these two topics appear in the secondary standards.

The goals of the workshop were to increase the participating teachers' statistical content knowledge, positively impact the teachers' attitudes towards statistics, and to increase the teacher's self-efficacy on teaching statistical content. In order to assess these goals, validated instruments, discussed below, were selected from literature.

To assess statistical content knowledge, the Basic Literacy in Statistics (BLIS) was used. This instrument was developed by Ziegler in 2014 to measure students' ability to read, understand, and communicate statistical information after completing an introductory statistics course which included simulation-based inferential techniques. The assessment was designed for students completing an introductory statistics course including the following topics: data production, graphs, descriptive statistics, sampling distributions, confidence intervals, randomization distributions, hypothesis tests, regression, and correlation. While those involved in this workshop were not traditional college students, the topics covered in this assessment fit nicely with topics covered during the workshop, and this instrument seemed well-suited to assess the growth in statistical content knowledge.

To assess self-efficacy for teaching statistics, the Self-Efficacy to Teach Statistics (SETS) instrument was administered. This instrument was developed by Harrell-Williams, et. al. (2009), as a tool to measure a pre-service teacher's efficacy to teach statistical concepts discussed in the Common Core State Standards (CCSS). While this instrument has been validated for use in the pre-service population, it has not been used on in-service teachers. Unfortunately, there are few self-efficacy instruments related to statistics, and these tend to focus on student attitudes, not teacher attitudes. While validation of the instrument was based on pre-service teachers, its central focus relates to topics in the CCSS; the instrument lent itself to use in this environment.

To assess attitudes towards statistics, the Survey of Attitudes Toward Statistics (SATS) instrument was administered. This survey was developed by Schau to assess six components: affect, cognitive competence, value, difficulty, interest, and effort. The SATS instrument has been validated for use with students in introductory statistics courses. Because the topics covered during the workshop would often be considered introductory topics, this instrument was deemed appropriate for use in this setting.

Results

The SATS and SETS were given to the participants online, and they were asked to complete these instruments before the first day of the workshop. Not wanting survey fatigue, and wanting the participants to take content seriously, the BLIS instrument was given at the very beginning of the first day of the workshop in paper form. On the first day of the workshop, we discovered that the number of anticipated participants (7) was not the same as the number that attended the workshop (3). Although the sample size was very small, post-tests of the same instruments were given at the conclusion of the workshop in the same form used for the pre-test. Due to the low turnout, the results from these instruments will not allow for any conclusions to be drawn, but could suggest if the content and delivery of the workshop was inclined to meet the goals set.

The results on the attitudes from the SATS were given as the standard pre-test and post-test SATS-36 (Schau, 2003). The first comparison to be made for the SATS results would be to the historical SATS data (Schau & Emmioglu, 2012) shown in Table 1 below. Although the sample size was small, the average changes of the attitude components (Table 2) were not much different from what has been found from historical SATS data. More specifically, the areas of Effort, Cognitive Competence, and Interest all had negative changes while Affect and Difficulty had positive changes. No real change was found in the Value component. The fact that the Value component did not decrease much is interesting to note as historical data suggests that this component tends to decrease as well.

Although the changes in attitudes of the workshop participants was not much different from the historical data, if one were to just look at the pre-test means for each component, these starting places are quite different. For all components except Difficulty, the mean pre-test score of the workshop participants were significantly higher than that of the historical data from introductory statistics students. As all workshop participants were either in-service or pre-service teachers as well as voluntary participants, this substantially better starting attitude is not that surprising. However, participants came in to the workshop with lower attitudes concerning Difficulty; that is, the workshop participants perceived statistics as more difficult. Due to the depth at which they have learned mathematics and statistics content compared to those in a traditional introductory statistics course, this belief of statistics being challenging is expected.

Table 1. *Student-based means and standard deviations for pretest, posttest, and change scores by attitude component (Historical)*

Component	n	Pretest		Posttest		Change	
		M	SD	M	SD	M	SD
Affect	2209	4.16	1.12	4.30	1.32	0.13	1.23
Cognitive Competence	2192	4.94	1.04	5.03	1.16	0.10	1.06
Value	2186	5.04	0.99	4.72	1.12	-0.32	0.96
Difficulty	2204	3.75	0.81	3.90	0.96	0.15	0.84
Interest	2219	4.51	1.27	4.00	1.44	-0.50	1.25
Effort	2246	6.32	0.90	5.84	1.09	-0.48	1.14

Table 2. Student-based means for pretest, posttest, and change scores by attitude component

Component	Pretest	Posttest	Change
	M	M	M
Affect	5.67	5.83	0.16
Cognitive Competence	6.17	5.83	-0.24
Value	6.06	6	-0.06
Difficulty	3.36	3.79	0.43
Interest	6.5	6	-0.5
Effort	7	6.25	-0.75

The BLIS test had 37questions and was administered face-to-face as both a pre- and post-test. To determine if the statistical content knowledge of the participants had increases, the change in the number of questions answered correctly was recorded. Although not all questions directly related to the randomization-based curriculum, it could be argued that a stronger foundational knowledge of statistics from this perspective could improve overall statistical content knowledge. Thus, all questions were included in the analysis. On the pre-test, the participants got 21, 28, and 32 correct (an average of 27 correct, or 72.98% of questions). On the post-test, the participants got 28, 27, and 34 (respectively) correct (an average of 29.67 correct, or 80.18% of questions).

The SETS instrument posed some issues as we delved into looking at the results. We found participants came in to the experience with quite a bit of confidence in their ability to teach the topics on the SETS instrument, even though some of the topics they had never seen in their coursework or in their curriculum. The post-SETS instrument showed that their confidence had a tendency to decrease as perhaps they discovered that they weren't as familiar with the content as they had initially believed.

Conclusion

Although the low participation in the workshop prevents the generalization of results, the data does suggest that the participants' statistical content knowledge increased after the workshop and that their attitudes towards statistics improved in the Affect and Difficulty categories, with no real change in the Value component. Future work would suggest offering the same workshop to a larger number of participants, pending support and demand for the workshop. One hurdle we believe we need to overcome to increase the demand is a realization that the statistical content of the CCSS will not only be taught in a "statistics" course by one teacher at the school, but instead that the standards are taught across the mathematics curriculum. This concern stresses the importance of all mathematics middle and high school teachers deepening their understanding of statistics and probability standards' content.

References

American Statistical Association (2015). The Statistical Education of Teachers (SET) report. Washington, DC: ASA. Retrieved from http://www.amstat.org/education/SET/SET.pdf

Batenero, C., Godino, J.D., and Roa, R. (2004), "Training teachers to teach probability," *Journal of Statistics Education* [online], 12, 1. Available at http://www.amstat.org/publications/jse/v12n1/batanero.html

Cobb, G. (2007), "The Introductory Statistics Course: A Ptolemaic Curriculum?," *Technology Innovations in Statistics Education* [online], 1, 1. Available at http://escholarship.org/uc/item/6hb3k0nz.

Common Core State Standards Initiative (2010), *Common Core State Standards for Mathematics*, Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

Conference Board of the Mathematical Sciences (2001), *The Mathematical Education of Teachers*, Providence, R.I. and Washington, DC: American Mathematical Society andMathematical Association of America.

Conference Board of the Mathematical Sciences (2012), *The Mathematical Education of Teachers II,* Providence, R.I. and Washington, DC: American Mathematical Society and Mathematical Association of America.
Garfield, J., and Everson, M. (2009), "Preparing teachers of statistics: A graduate course for future teachers," *Journal of Statistics Education* [online], 17, 2. Available at http://www.amstat.org/publications/jse/v17n2/garfieldpdf.html
Heaton, R. M., Lewis, W. J., and Smith, W. M. (2013), "The Math in the Middle Institute: Strengthening middle level teachers' mathematical and pedagogical capacities," in *Resources for Preparing Middle School Mathematics Teachers,* eds. C. Beaver, L. Burton, M. Fung, and K. Kruczek, Mathematical Association of America Note Series, pp. 47-58.
Harrell-Williams, L.; Sorto, M. A.; Pierce, R. L.; Lesser, L. M.; Murphy, T. J. (2014). Validation of scores from a new measure of pre-service teachers' self-efficacy to teach statistics in the middle grades. Journal of Psychoeducational Assessment, 32(1), 40-50.
Heaton, R. M., Lewis, W. J., Homp, M. R., Dunbar, S. R., and Smith, W. M. (2013), "Challenging and rigorous yet accessible and relevant mathematics courses for middle level teachers," in *Resources for Preparing Middle School Mathematics Teachers,* eds. C. Beaver, L. Burton, M. Fung, and K. Kruczek, Mathematical Association of America Note Series, pp. 181-201.
Jenkins, Susan; Agamba, Joachim Jack. (2013) "The Missing Link in the CCSS Initiative: Professional Development for Implementation." *Academy of Educational Leadership Journal* 17.2 : 69-79.
Rossman, A., Chance, B., and Medina, E. (2006), "Some Important Comparisons between Mathematics and Statistics, and Why Teachers Should Care" in *Thinking and Reasoning With Data and Chance: Sixty-eighth Yearbook,* eds. G. F. Burrill and P. C. Elliott, Reston, VA: National Council of Teachers of Mathematics, pp. 323-333.
Rossman, A. and Chance, B. (2008). "Concepts of Statistical Inference: A randomization-based curriculum". Available at: http://statweb.calpoly.edu/csi
Schau, C. (2003a). Survey of Attitudes Toward Statistics (SATS-36). [Online: http://evaluationandstatistics.com/]
Schau, C. & Emmioğlu , E. (2012). Do introductory statistics courses in the United States improve students' attitudes? *Statistics Education Research Journal, 11(2), 86-94.* Retrieved from http://iase-web.org/documents/SERJ/SERJ11(2)_Schau.pdf
Scheaffer, R. L., and Jacobbe, T. (2014), "Statistics education in the K-12 schools of the United States: A brief history," Journal of Statistics Education [online], 22, 2, Available at http://www.amstat.org/publications/jse/v22n2/scheaffer.pdf.
Schmid, Kendra K., Blankenship, Erin E., Kerby, April T., Green, Jennifer L., and Smith, Wendy M. (2014). "The Development and Evolution of an Introductory Statistics Course for In-Service Middle-Level Mathematics Teachers", Journal of Statistics Education [online], 22, 3, Available at http://www.amstat.org/publications/jse/v22n3/schmid.pdf
Ziegler, Laura. (2014). Proceedings from International Conference on Teaching Statistics: Reconceptualizing Statistical Literacy: Developing an Assessment for the Modern Introductory Statistics Class. Flagstaff, AZ.

MATHEMATICS KNOWLEDGE FOR PARENTING (MKP): WORKSHOPS TO HELP PARENTS MAKE SENSE OF MATHEMATICS

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The purpose of this study was to investigate the extent to which parents of first, second, and third grade students who attended a two-day workshop on mathematics strategies differed on average and over time as compared to parents who did not attend the workshops regarding beliefs about learning mathematics and comfort level with manipulatives. These findings indicated that workshops for parents were beneficial. By giving parents the opportunity to engage in mathematics in ways similar to the way their children learn in the classroom, and by making sense of mathematics using different manipulatives, beliefs about the way children learn mathematics were challenged.

Related literature

Many parents of elementary school students were likely taught mathematics in a way different from the technique that incorporates a focus on multiple strategies students are learning (Nearney, 2013). Based on what is shared so pervasively on the Internet, on the news, and in many social situations it can be deduced that some parents are frustrated with the excessive amount of time it takes to solve a problem and the variety of strategies students are using to do so (Richards, 2014). The goal of this research was to determine if completing a two-session workshop on mathematics pedagogy and content related to whole number concepts and operations change mathematics knowledge for parenting (MKP), parents' beliefs about learning mathematics, and parents comfort level with manipulatives.

Parental involvement is important because when parents are involved in their child's education, students have a more positive behavior (Sanders, 1998). The part of parental involvement that focuses on parents helping with homework is the basis of this research, and, for the purpose of this research is called mathematics knowledge for parenting (MKP). Workshops might give parents sufficient content knowledge to keep from inhibiting student achievement through the introduction of procedures too early. This is important because research indicates when students are shown the procedures to solving problems without any meaning behind those procedures their understanding is limited and can lead to student errors (Fuson, 1990). Mathematics knowledge for parenting (MKP) identifies the mathematics knowledge necessary to help their child with mathematics homework.

Part of this research relates to belief change. Due to the gap in research on changing parents' beliefs regarding how students should learn mathematics, a connection between parents and teachers is proposed, as the student's parent *is* the child's teacher when the child is at home working on homework. The way preservice teachers think in terms of solving mathematics problems could be similar to the way parents think about solving mathematics problems. Preservice teachers come to their education courses with experiences in learning mathematics procedurally, similar to the way parents help their child and may only rely on their past experiences which could be procedural (Garland, 2014; Richards, 2014). In order to change beliefs about mathematics, teachers need their current beliefs challenged. Teachers can do this by solving problems in ways that call existing beliefs into question or by making mathematical discoveries on their own (Carter & Yackel, 1989). Changing teachers' beliefs about mathematics can be difficult because many preservice teachers (PSTs) think mathematics is about memorizing formulas and procedures (Szydlik, Szydlik, & Benson, 2003). However, when students learn mathematics in a learner-centered environment, they can be actively engaged in the problem in ways that make sense to them. Parents may gain a deeper understanding of this type of learning if they are given opportunities to construct their own knowledge as well (Civil et al., 2002).

Another part of this research relates to parents' comfort level with manipulatives. Successful manipulative use is dependent on how learners understand these manipulatives, because manipulatives should be used to help the learner make sense of the concepts (Nuhrenborger & Steinbring, 2008). In order to help students use manipulatives effectively, connections should be made between the manipulatives and their corresponding concepts (Fischbein, 1977). Research indicates that if teachers have more opportunities to engage in mathematics while using manipulatives that are meaningful, their students may have increased mathematics knowledge (Smith & Montani, 2008). Without providing meaningful activities to help PSTs understand the value of using manipulatives, it might be more difficult for them to successfully incorporate meaningful use of manipulatives in their classrooms (Fischbein, 1977). This could also hold true when parents help their child with mathematics homework because parents may not have had these opportunities, so it would be more difficult to understand how their child is using these manipulatives. This could affect the level of support they could give their child when helping them make sense of mathematics using these manipulatives.

The purpose of this research was to answer the following question: To what extent do parents who attend a mathematics workshop on whole number concepts and operations differ, on average and over time, in their (a) mathematics content knowledge, (b) beliefs about learning mathematics, (c) belief factors (student learning, stages of learning, and teacher practices), and (d) comfort level with different manipulatives, as compared to parents who do not attend?

Methodology

The current study used a quantitative, quasi-experimental, non-equivalent control group design. The research design was chosen because participants self-selected either the control or intervention groups. Any parent or guardian of 1st, 2nd, and 3rd grade students at three neighboring public elementary schools in Central Florida were invited to participate. For the purpose of this research, “parents” will include any person who takes on that role. Additionally, only one parent in each family attended, with the exception of one family where both parents attended. First, second, and third grades were chosen because this is when the mathematics homework represents mathematics that is likely different from instruction most parents received when they were in elementary school (Nearney, 2013). The treatment group included 12 participants who attended both sessions, while the control group included 17 participants who were unable to attend the workshop, but still participated in testing. The majority of the participants: (n=22) identified as being “white,” (n=24) were female, (n=24) were in the 36-45 age range, (n=26) were in a married or domestic partnership, (n=23) worked either part time or full time, and (n=25) earned at least a bachelor's degree.

For both the control and intervention groups, all interactions were conducted face-to-face at the same school site. The school site allowed the researcher to set up a table during school hours to meet with participants in the control group. The treatment group participated in two workshop sessions during spring 2015. The 2 workshop sessions were repeated three times to accommodate different schedules. The material in each 2-day series was identical, but because the focus was on the strategies parents came up with discussion and shared strategies may have been different. In order to achieve some level of fidelity the researcher anticipated some strategies parents might have used. However, the amount of time spent on each strategy may have been different. Additionally, parents shared some strategies not on the power point; so different groups received slightly different treatments. The first series was conducted on two consecutive Tuesdays from 6 - 8 p.m. The second series was conducted on the following two Tuesdays from 6 - 8 p.m. The third series was conducted on Tuesday and Thursday of the same week, from 9 - 11a.m. The workshops were created to help parents engage in mathematics in similar ways their child is expected to learn mathematics, such as allowing their child to come up with multiple strategies that would allow them to think flexibly about mathematics. The first time the researcher met with each participant in the treatment or control group, participants completed the instruments on mathematics content, beliefs, and manipulatives. After completing the pretests, parents in the treatment groups participated in the workshop series, which was the intervention. The current study included a two-day workshop, as opposed to a longer series, to lower the risk of attrition for ongoing PD.

Three instruments were used in this research, the abbreviated Mathematics Beliefs Scales (MBS) (Capraro, 2005), a researcher-created mathematics content instrument, and an instrument that was created to measure parents' comfort level with different manipulatives – specifically base ten blocks, part-part-whole mat, open number line, ten frames, hundred chart, and array. The MBS, originally created by Fennema, Carpenter, and Loef (1990), was developed under a grant funded by the National Science Foundation through the University of Wisconsin, Madison, to measure the mathematical beliefs of teachers. Responses to questions were measured using a five point Likert scale ranging from strongly agree to strongly disagree. The original MBS had 48 items, and because researchers commented that participants complained about the length and repetitiveness of the instrument, an exploratory factor analysis was run on all 48 items (Capraro, 2001). Capraro chose the 18 questions because they explained 46% of the variance, and she determined these 18 items could be split into three factors with six items in each. The three factors were student learning, stages of learning, and teacher practices. Although the original MBS (48 items) had a reliability of .93 in Capraro's study, the abbreviated MBS (18 items) only had coefficient-alpha reliability of .68 for inservice teachers (n = 123). Capraro (2005) administered this shortened version (18 items) to a group of preservice teachers (n = 54), and found reliability of .86.

In the current study a repeated measure ANOVA was run on the same 18 questions that Capraro used, but a separate repeated measures ANOVA was also run on each of the three factors. As previously stated, the researcher proposed a connection between parents and teachers, due to the gap in this type of research on parents. For this reason, the reliability found in Capraro's study will be used in the future to perform confirmatory factor analysis, once the sample size is larger. Participants were given a mathematics content test, which was broken into two sections. One included questions where parents were able to answer them using any strategy they were comfortable, and the other included possible student solutions that participants needed to determine whether or not they were correct, and if incorrect identify the student's error. Additionally, participants were given a one page instrument that asked about their level of comfort with base ten blocks, part-part-whole mats, open number lines, ten frames, hundred charts, and arrays. Responses were coded as not comfortable (1) to very comfortable (4).

Findings

Analyses on the abbreviated MBS (all 18 items) indicated there was a statistically significant within-between subjects interaction effect between group and time for MBS, MBS-factor 3, and comfort with manipulatives (Table 1). This statistically significant result suggested that there were differences, on average, between treatment and control group over time regarding beliefs. When the MBS was split into three factors - student learning (factor 1), stages of learning (factor 2), and teacher practices (factor 3), results indicate no statistically significant difference between groups and over time for student learning and stages of learning. However, there was a statistically significant within-between subject interaction between group and time for teacher practices (see table below). This statistically significant result suggested that there were mean differences, on average between groups over time regarding beliefs about teacher practices.

Table 1. *ANOVA analyses results*

	df_1	df_2	*Mean Control*		*Mean Treatment*		*F*	*p*	Partial η^2	Power
			pre	*post*	*pre*	*post*				
MKP	1	27	9.5	10	9.7	9.7	.837	.368	.030	.143
MBS	1	27	54.4	53.4	50.3	56	6.771	.015	.201	.708
MBS Factor 3	1	27	21.7	20.5	20.5	23.3	7.48	.011	.217	.751
Manipulatives	1	27	18.8	17.7	19.3	22.9	16.441	<.001	.378	.974

Results (using the entire abbreviated MBS) regarding beliefs were statistically significant between groups and over time with a large effect size, with the treatment group changing their beliefs to ones that were more focused on students constructing their own knowledge. Analyses on each of the separate factors indicated no statistically significant change between groups over time for factor 1 (student learning) and factor 2 (stages of learning), but there was a statistically significant change between groups over time for factor 3 (teacher practices) with a large effect size. Parents in the treatment group had beliefs that leaned more towards a learner-centered environment after completing the workshops. Parents in the control group did not change their beliefs from the pretest to the posttest. This indicated the workshops might have changed parents’ beliefs about student learning to beliefs that students should learn in a learner-centered environment. Through participation in the workshops, which were learner focused, parents may have understood the importance of allowing their child to learn mathematics in a student-centered environment instead of one focused on the parent guiding their child to the answer. The findings in the current study are similar to the study by Civil, Guevara, and Allexsaht-Snider (2002) who indicated that parents were better prepared to work with their children when they were given opportunities to construct their own knowledge.

There was a non-statistically significant within-between subjects interaction effect between group and time for MKP. This non-statistically significant result suggested that there were no differences, on average, between treatment and control group over time regarding content knowledge. The results indicate that two days of learning about addition, subtraction, multiplication, and division is insufficient for parents to increase their content knowledge as measured by the researcher-created instrument. Most participants used the standard algorithm to solve the problems on the pretest, but many participants in the treatment group attempted to use new strategies at the posttest. Having a deep conceptual understanding of the mathematics content and strategies may take more time, which suggest ongoing workshops may be beneficial.

Regarding parents’ comfort level with manipulatives, there was a statistically significant within-between subjects interaction effect between group and time with a large effect size. Self-reported comfort levels for parents in the treatment group were higher at posttest, which indicated these parents were more comfortable using manipulatives after the workshops. The results indicate that these workshops may have helped to increase parent’s comfort level with using these manipulatives. This could be attributed to parents in the workshop being able to engage in using those manipulatives, whereas the control group did not. The results align with research by Knapp, Jefferson, and Landers (2013) who claimed that when parents explored mathematics using manipulatives during a workshop series, the parents were more comfortable with them and use them more often. The workshops in the current study indicate even a short duration, specifically two days, may have some benefits.

One limitation of this research was that participants completed a pre and post belief instrument, where their beliefs were self-reported. Researchers have found that teachers need to be observed multiple times to determine their underlying beliefs, which can be different from their self-reported beliefs (Cross 2009; Leatham, 2006). Due to the similarities of teachers and parents regarding helping a student with a mathematical task, this may be true for parents as well. Additionally, instrumentation validity may be a threat to internal validity of this research study because while there was some evidence supporting the reliability and validity of the abbreviated MBS (Capraro, 2005), this instrument was not tested on parents. Additionally, the small sample size (n=29) in the current study did not support testing statistical validity and reliability evidence for the scores from the instruments. Furthermore, the lack of random selection from the population limits the generalizability of the study findings. Because the sample was small and non-random, results from the current study may be limited in generalizability to similar contexts and populations. With continued research in this area, issues with validity, reliability, and generalizability may be addressed.

Social media outlets indicate parents are frustrated with the way their child is learning mathematics (Garland, 2014; Richards, 2014). This is important because when a child is at home, his or her parents may take on the role of teacher. Some parents want to help their child with homework, but may only have their personal experiences, which could have been very procedural, upon which to base the support they provide (Garland, 2014; Richards, 2014). The results of this study suggest that parents could benefit from workshops regarding belief change towards students constructing their own knowledge and feeling more comfortable with different manipulatives.

References

Capraro, M. M. (2001, November). *Construct validation and a more parsimonious mathematics beliefs scales*. Paper presented at the Annual Meeting of the Mid-South American Educational Research Association. Little Rock, AR.

Capraro, M. M. (2005). A more parsimonious mathematics beliefs scales. *Academic Exchange Quarterly, 9*(3), 83-88.

Carter, C. S., & Yackel, E. (1989, March). *A constructivist perspective on the relationship between mathematical beliefs and emotional acts*. Paper presented at the Annual Meeting of the American Educational Research Association. San Francisco, CA.

Civil, M., Guevara, C., & Allexsaht-Snider, M. (2002, October). *Mathematics for parents: facilitating parents' and children's understanding in mathematics*. Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Athens, GA.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education, 12*(5), 325-346.

Fennema, E., Carpenter, T. P., & Loef, M. (1990). *Mathematics beliefs scales*. Madison, WI: University of Wisconsin, Madison.

Fischbein, E. (1977). Image and concept in learning mathematics. *Educational Studies in Mathematics*, *8*(2), 153-165.
Fuson, K. C. (1990). Issues in place-value and multidigit addition and subtraction learning and teaching. *Journal for Research in Mathematics Education*, *21*(4), 273-280.
Garland, S. (2014). Why is this common core math problem so hard? Supporters respond to quiz that went viral. *The Hechinger Report*. Retrieved from http://www.huffingtonpost.com/2014/03/28/viral-common-core-homework_n_5049829.html
Knapp, A. K., Jefferson, V. M., & Landers, R. (2013). Learning together. *Teaching Children Mathematics*, *19*(7), 432-439.
Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems*. *Journal of Mathematics Teacher Education*, *9*(1), 91-102.
Nerney, K. (2013). *Calming 'core' anxiety of stressed-out students.* Parent Today. Retrieved from http://www.parenttoday.org/client/index.cfm/2013/10/10/Calming-Core-anxiety-of-stressed-out-students
Nührenbörger, M., & Steinbring, H. (2008). Manipulatives as tools in mathematics teacher education. In Tirosh, D. & Woods, T. (Eds.) *International Handbook of Mathematics Teacher Education*: *Vol 2: Tools and Processes in Mathematics Teacher Education.* (pp. 157-181). Rotterdam, The Netherlands: Sense Publishers.
Richards, E. (2014). *Uncommon frustration: Parents puzzled by common core math*. Milwaukee Journal Sentinel. Retrieved from http://www.jsonline.com/news/education/uncommon-frustration-parents-puzzled-by-common-core-math-b99281204z1-261921571.html
Sanders, M. G. (1998). The effects of school, family, and community support on the academic achievement of African American adolescents. *Urban Education*, *33*(3), 385-409.
Smith, L. F., & Montani, T. O. (2008). The effects of instructional consistency: Using manipulatives and teaching strategies to support resource room mathematics instructions. *Learning Disabilities*, *15*(2), 71-76.
Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education*, *6*(3), 253-279.

AN ALTERNATIVE ROUTE TO BYPASS DEVELOPMENTAL MATHEMATICS

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Current trends in developmental courses emphasize repetition and practice as a means to remediate students who enter higher education underprepared for college-level mathematics. However, prior research shows that this developmental mathematics education is not helping students progress to college-level courses. In this paper we share a case study to describe the course materials, instruction, and other characteristics of an experimental, credit-bearing, community college mathematics course. We describe students' prior experiences in mathematics and their experiences in this course, as reported by the individual students. Our findings confirm prior research which show that students face serious challenges with overcoming prior negative learning experiences in mathematics, predominately attributed to past experiences with instructors or other external factors, and internal factors such as perceived innate abilities for succeeding in mathematics (Higbee & Thomas, 1999). Furthermore, our findings show that certain characteristics of the course described in this study generated positive experiences for the students, and in some cases individual students reported feeling more confident in their ability to understand and learn mathematics as a result.

Theoretical Framework and Related Literature

As highlighted in the 2008 multiyear national Achieving the Dream initiative's *Developmental education: Completion status and outcomes* and further substantiated in other research (Bailey, Jeong, & Cho, 2010; Bettinger, Boatman, & Long, 2013), developmental education, particularly in mathematics, has not been an effective practice for advancing students to college credit-bearing courses. This poses a serious threat to accessing STEM careers for which a basic aptitude in college mathematics is a prerequisite. Studies have shown that for developmental mathematics students, attitudes towards mathematics, confidence in mathematics, mathematics anxiety, self-efficacy, and lack of control over external factors affect student goals, as well as performance in mathematics (Hall & Ponton, 2005; Higbee & Thomas, 1999). Prior experiences in mathematics also play a large role in students' opinions of their own perceived abilities in mathematics (Hall & Ponton, 2005). Thus, it is important for educators to have an understanding of students' cognitive barriers in learning mathematics such that they are able to effectively address those barriers through instructional practice (Higbee & Thomas, 1999).

Research Objective and Background

The objectives of this study are to describe (a) the prior experiences in mathematics of students enrolled in a remedial level mathematics course and (b) the impact of the course materials and instructional practices on students learning mathematics. This is a case study of one section of the entry-level algebra course redesigned by the mathematics department at the college was designated to enroll students placed at the developmental level. The course topics included algebraic simplification of polynomial, rational, exponential, and radical expressions; solving equations and inequalities with absolute value, polynomial, rational, exponential and radical expressions; graphing lines and parabolas; and functions. Included in this course section were investigations from *A Modeling Approach to Algebra* or AMAA (Olson, Olson, Slovin, Venenciano, & Zenigami, 2015), a curriculum previously shown to have a positive effect on high school Algebra I students' behavioral and cognitive engagement (Venenciano, Olson, Olson, & Capen, 2015). AMAA follows the premise that learning algebra requires more than memorizing formulas and finding answers. Additional features of the course were not limiting enrollment based on a minimum placement test score and having earned passing grades in prerequisite developmental courses, as well as a wider breadth of topics including those for review, and increased course meeting frequency.

This new college algebra course was offered to students who otherwise would have been placed on a developmental mathematics track based on placement exam scores. The course provided an opportunity for developmental mathematics students to be in a classroom setting with non-developmental mathematics students and to give them an alternative pathway to fulfill their mathematics credit requirements.

The instructor encouraged creative and productive problem solving, collaboration, and discourse among students in small and whole group settings. This course section diverged from the more traditional course, where lecture and independent work are the expected classroom norms. The use of web-based tutorial modules was included in homework assignments to supplement the in-class interaction.

The instructor had prior training and experience using an inquiry-based approach as a secondary mathematics teacher. The instructor used this approach to implement the investigations from the AMAA curriculum materials approximately once every one to two weeks. The instructor encouraged students to engage in classroom discourse to support the development of mathematical practice.

Methodology

Participants. There were 25 students enrolled in the experimental course for the duration of this study. Several students enrolled in the class had previously participated in a summer bridge program intended to support students identified as needing remediation upon admission to college. Given the option of selecting more than one ethnicity, 94% of the students identified themselves as Asian/Pacific Islander, 22% White, 4.5% Hispanic or Latino, 3% Native American or American Indian, 1.5% Black or African American, and 4.5% Other. The mean age of the students was 21 years old and nearly 50% of the students were female. The last mathematics course students reported taking included high school courses, a developmental mathematics course, and a survey of mathematics course.

Research design and data analysis. Part way through the semester, students in the experimental course were assigned a math autobiography and a mid-semester reflection. The writing prompts for the tasks are presented in Figure 1.

Prompt 1: Please write an essay that addresses the following: What have your experiences in math classes been like (other than our class)? How do you feel about math? In what ways have you used math outside of school? Do you learn best from reading, listening or doing? Do you prefer to work alone or in groups? What do you do when you get "stuck"? Do you ask for help? From whom? Describe some of your study habits. Speak more about your previous math experiences. What are your educational and life goals?

Prompt 2: What is your favorite/least favorite part of [this mathematics course]? Explain how you feel about mathematics now compared to the beginning of the semester.

Figure 1. Writing prompts assigned by the instructor.

The responses were gathered and qualitatively analyzed using a grounded theory approach (Strauss & Corbin, 1998). Initial examination of the student responses led to emerging themes in the data and subsequent rounds of analyses and re-examination were conducted for further coding and classification of the themes. To address the reliability of these findings, one research team member performed the initial analysis and reported emerging themes, after which another team member analyzed the data to negotiate and further clarify the themes defined in the initial analysis. After the analyses of the student responses were finalized, observational data and semi-structured interviews with the instructor were used to confirm the thematic findings. Five lessons were observed to capture instruction both with and without AMAA curriculum materials. The interviews focused on the engagement of students in the activities and discourse, their mathematical understanding and development, and other characteristics of the course the instructor felt contributed to student attitudes, self-efficacy, and perceptions of mathematics.

Findings

In response to the students' math autobiography (Prompt 1), the majority reported disliking mathematics or that mathematics was their weakest subject. After multiple rounds of analyses, the responses revealed that students primarily attributed their dislike of mathematics to either external factors or internal factors. We refer to external factors to include the class structure, the influence of a teacher or instructor, and other environmental features over which the student has no control. We use internal factors to refer to attributes or characteristics of which the individual student has control, however, in some cases, the student has perceived little to no control. Using the external/internal lens, a range of themes emerged, yet the data revealed that the predominate external factor to which students attributed success or failure in mathematics was the instructor; whereas the internal factors primarily centered on students' perceptions of their own ability in mathematics. A representative set of responses is included in Table 1.

Table 1. *Sample Responses to Prompt 1, per Theme*

External: Teacher Impact	Internal: Student Perceptions of Ability
• "My experience with math in high school was terrible because of the teachers I had. I don't really like math a lot." • "The pace that teachers go at makes it very easy to not understand some things." • "I remember my ... teacher in high school. ... I felt like I learned absolutely nothing because my teacher literally just sat on her desk the whole time during class. … ."	• "I didn't like high school math classes because I couldn't understand and keep up." • "Math has always been a difficult concept for me to grasp, for some reason my brain just doesn't retain the information. ... I think deep within my subconscious I have a fear of numbers and formulas...they make me uncomfortable." • "Throughout the years … math classes gave me anxiety and a strong feeling of nervousness because I wasn't strong in the subject... in a math class being called on not

	knowing how to do the problem gives me that shamefulness feeling and the fear of not being able to complete what I have started.”

Parallel themes seemed to emerge from the few students in the class who reported liking mathematics. Nearly all attributed their enjoyment to either positive experiences with teachers (external), or because they felt they did well in previous mathematics courses (internal). Additionally, however, students who reported liking mathematics also mentioned some type of value or practicality of mathematics in everyday life. For example, one student wrote, “Math is a very important skill to learn and I use it everyday at work.” Another student wrote, “I do like math, because you can apply it to everyday life.”

In response to Prompt 2 we focused our analysis on what students reported as their favorite part of class to address our second research objective. This analysis of student responses led to the identification of course characteristics (see Table 2) that contributed to positive experiences in the class. While we acknowledge that some students wrote about a least favorite part of class (e.g., having class on Fridays), the majority reported not having a least favorite part of class.

Table 2. *Examples of Students' Responses Indicating Positive Experiences*

I. Inclusion of problem solving with real world connections
   - “So far my favorite part of the class is examples that we can adapt to real life.”
   - “The way the instructor uses real life situations to better explain the topic.”
   - “… the math is put into perspective in our own personal lives better to relate than just plain math just slaying away doing problems”

II. Emphasis on student understanding
   - “In previous math classes I felt like a robot … we were expected to know rather to be taught again or a way to see it differently. Rather than this class I am taking shows the foundation of the problem … other classes bombed us with so much problems … it was all about getting the problem done at a certain time rather than fully grasping what I am doing.”

- "I find that I … have a better understanding on how a problem is solved rather than just repeating the same steps over and over again and attempting to memorize the process."
- "…. So many classes (especially math) I've had in the past just want us to answer the way the textbook says to answer a problem."

III. Supportive instructor disposition

- "Knowing my teacher is willing to stop everything to help students out."
- "Having a positive attitude/passion about teaching … helps me want to learn."
- "I thought it would be a lot harder and complicating, but the instructor understands us and doesn't rush us to finish our work."

IV. Frequent group work and class discourse

- "Math isn't as scary now because having constant conversations about it with other students reminds me that we are all on the same page. This allows me to open up my mind to the lessons and actually store the new information."
- "My favorite part of [this course] is working with each other by this we learn different views of other peers that may work easier for ourselves."
- "My favorite part is that we get to work in groups. This class helps me to be more confident and try to grasp the topics ... it made me aware of where people struggle in the certain topics and that topic I might be more better at and I can help them."

Evident from the data in Table 2 is that many of the students perceived benefits from the instruction in which student-to-student interaction was maximized. Students reported learning by seeing others' ways of thinking or approaching a problem. In some cases, seeing other students struggle with a problem helped some students feel less isolated in the struggle and motivated them to work together to find a solution.

Discussion and Implications

Designing courses to support students identified at the developmental level is a pressing concern. Research suggests that the fewer developmental math courses students are required to take, the more likely they are to enroll in and pass a credit bearing mathematics course (Howell & Walkington, 2015). For this study, an experimental credit-bearing course was designed by the course instructor to address introductory college algebra topics while providing additional support for reviewing and/or reteaching of fundamental topics. Our findings indicate that students identified at the developmental mathematics level face significant challenges with overcoming prior negative learning experiences and perceived abilities for succeeding in mathematics. This study confirms prior research which describe how the mode and design of instruction support students' content knowledge as well as perceptions of their ability in mathematics (Hall & Ponton, 2005). Our findings suggest that if students are enrolled in a course that promotes mathematical understanding over memorization, encourages frequent group work and class discourse, includes problem solving activities with real-world connections, and is taught by a positive instructor who facilitates rather than lectures to support the development of students' independent reasoning, then we can expect to create positive experiences for students' and hypothesize that these experiences contribute to positive gains in course attendance, attitudes toward learning mathematics, and perceptions of own their abilities.

The observational and interview data show that the instructor made adaptations to best meet the needs of her students. These included shifting away from web-based tutorial programs as homework and dedicating more class time to working out practice problems as a class. During a post-course interview with the instructor, she reported a course-passing rate of 64% and that the overwhelming reason for failures was attributed to poor attendance throughout the semester. Prior research has shown a strong positive correlation of poor class attendance with low grades among developmental education students (Moore, 2003). Of the students who had attended class regularly, the instructor noted changes in how students talked about mathematics. Where initially the students hesitated and asked for early confirmation about their work, toward the end of the semester, students were more confident in sharing feedback and attempting solution strategies.

This study highlights the potential for overcoming barriers presented by a developmental mathematics course. We hypothesize that creating positive experiences through instructional practice and course design, student attendance, as well as students' confidence in mathematics may improve. We recognize the complexity of class dynamics and that a single factor, such as curriculum, is not an isolated contributor to reforming a course. Rather, it is the integration and cohesion of several aspects such as mathematics course structure, content, and instructional delivery, that we believe support improved motivation and confidence.

References

Bailey, T., Jeong, D., & Cho, S. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review, 29*(2), 255–270.

Bettinger, E. P. & Boatman, A. & Long, B. T. (2013). Student supports: Developmental education and other academic programs. *The Future of Children*, *23*(1), 93–115.

Developmental education: Completion status and outcomes. (2008, July/August). *Data Notes, keeping informed about Achieving the Dream data*, *3*(4). Retrieved from http://files.eric.ed.gov/fulltext/ED521310.pdf

Hall, J. M., & Ponton, M. K. (2005). Mathematics self-efficacy of college freshman. *Journal of Developmental Education*, *28*(3), 26-33.

Higbee, J. L., & Thomas, P. V. (1999). Affective and cognitive factors related to mathematics achievement. *Journal of Developmental Education*, *23*(1), 8.

Howell, E., & Walkington, C. (2015). An examination of factors impacting college algebra readiness: Pathways through developmental mathematics. In Che, S. M. and Adolphson, K. A. (Eds.). *Proceedings of the 42nd Annual Meeting of the Research Council on Mathematics Learning*. Las Vegas, NV.

Moore, R. (2003). Students' choices in developmental education: Is it really important to attend class? *Research and Teaching in Developmental Education, 20*(1), 42-52.

Olson, J., Olson, M., Slovin, H., Venenciano, L., & Zenigami, F. (2015*). A modeling approach to algebra, Student edition.* (2nd ed.). Honolulu, HI: Curriculum Research & Development Group, Marketing and Publishing Services, University of Hawai‘i.

Strauss, A.L. and Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (2nd ed.). Thousand Oaks, CA: Sage.

Venenciano, L., Olson, J., Olson, M., & Capen, S. (February 2015). *Students' engagement, confidence, and self-efficacy for learning algebra*. Presented at the 42nd Annual Meeting of the Research Council on Mathematical Learning. Las Vegas, NV.

MATH DANCE: A STUDY OF EFFECTIVENESS

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Pattern Play is an interdisciplinary college course exploring pattern recognition, creation, and analysis through simultaneous study of mathematics and dance. The effectiveness of this method was evaluated through the use of pre/post mathematics tests, attitudinal surveys, and drawing prompts. These data were compared to those of students in a traditional general education mathematics course covering the same mathematics topics. The results show that the students in Pattern Play outperformed the control group students in the areas of mathematical content knowledge, attitudes towards mathematics, and persistence in problem solving.

Despite the fact that about 70% of students enrolled in mathematics and statistics university classes are non-majors fulfilling general education requirements (Blair, Kirkman, & Maxwell, 2013), traditional mathematics courses are "primarily serving the needs of potential science majors" (Laws,1999, p. 217). For students not majoring in the sciences, fulfilling general education requirements for mathematics sometimes becomes a seemingly insurmountable hurdle preventing graduation. The resulting attrition is causing educators to look for more effective ways to engage and inspire students in their study of mathematics in order to communicate the beauty and relevance of the subject. While a fresh look at non-lecture based teaching practices is useful for all postsecondary mathematics classrooms, liberal arts majors who often do not identify as strong traditional mathematics learners stand to gain the most from such innovations (Laws, 1999; Steen, 2000). The pilot section of Pattern Play: Mathematics and Creative Arts taught at Weber State University provides an intriguing model for delivering mathematics content to non-majors using an integrated study of mathematics and dance. This quasi-experimental study investigated the effectiveness of the Pattern Play course.

Related Literature

For decades, calls have been made to change the teaching of general education mathematics classes (Laws, 1999; Steen, 2000). These calls suggested giving students more opportunities to collaborate with instructors and peers, encouraging multiple solution strategies, exploring fewer topics in more depth, reasoning critically and conceptually, relating science and math topics to everyday life, and developing communication skills (Laws, 1999). Arts integration methods provide one model for deepening the learning of mathematics through stimulating self-thinking, self-expression, and problem solving (Hanna, 2000; Rinne, Gregory, Yarmolinskaya, & Hardiman, 2011; Schaffer, Stern, & Kim, 2001). In the article "Learning Through Dance," Hanna (2000) refers to a study of dance that encouraged the explorations of various mathematical concepts such as space, time, and phrasing. The article also cites a ten-year study of low-income youth in which a regular study of the arts improved the youths' academic performance and increased their abilities in self-assessment. Yakimanskaya (1991) documented the important relationship between spatial thinking and mathematics education.

A progression from the palpable to the abstract served as a foothold to understanding the mathematical topics in the Pattern Play course. This progression aligns with the principles of cultural-historical activity theory that emphasize educational tasks that cause students to first investigate a concept through the study of real objects and activities and then to follow up on these experiences by carefully scaffolding understanding to a more abstracted, symbolic representation of a given concept (Davydov, 2008; Kozulin, 1990).

Instructional Methodology

The pilot section of Pattern Play: Mathematics and Creative Arts provided an innovative approach to diversifying the delivery of general education mathematics course content. Co-taught by professors from mathematics and performing arts, Pattern Play satisfied university general education requirements for mathematics and creative arts. The course was open to all students at the university and was advertised widely across campus.

The mathematics content aligned with the university liberal arts mathematics course and included the topics of algebraic functions, geometry, and basic probability and statistics. The accessible dance activities focused on choreographic, collaborative problems that students "solved" by creating short movement studies within given parameters. Students were encouraged to push themselves physically and conceptually, but there was no pre-requisite dance experience or standard of dance ability. The movement activities were based on the performances, writings and workshops of Schaffer, Stern and Kim (2001) and adapted by the second and third authors to suit the required topics and level of this university level mathematics for liberal arts course. In addition to the study of pattern in arts and traditional mathematics contexts, students were introduced to movement forms and breath regulation as a method for dealing with anxiety often associated with learning mathematics.

Significant class time was devoted to the transition from the exploration of patterns in dance to the symbolic representation of these patterns on paper. Instructors devised extensive worksheets designed to develop student understanding and to trace in a rung-by-rung manner the overlapping critical and creative thinking important to both the arts and mathematical problem solving. Multiple pathways to a solution in both creative arts and mathematics were encouraged in an effort to build persistence in problem solving.

The topic of permutations will serve as an example of the method employed in Pattern Play. Students begin with a warm up activity moving in and out of various positions with their bodies. Then students are led through approaches to inventing a succinct and clear dance "move" that is repeatable with a beginning and ending. Placed into trios and quartets, each student teaches her dance move to the rest of the group. Each group decides on a new order for the moves and are tasked with developing transitions between the moves, rehearsing and performing their combined group dance for the rest of the class. After the performance, the class reflects on the activity in terms of aesthetics; they also list the orders of the moves on the board, leading to a discussion of how many total four-move dances could be made. Following a class discussion of permutations, students collaboratively or individually begin worksheets that connect movement problem-solving skills to permutation problems of increasing complexity.

It must be emphasized that the kinesthetic mode of instruction used in the course creates an entirely different environment for learning. Student explorations were informed by the collaborative, close association inherent in dance classes. Moreover, being in large open spaces without desks encouraged students to work in small groups or individually, to find comfortable spaces and positions, and to follow their own learning needs when solving problems and completing worksheets. This allowed for focus on breath control, awareness of returning anxiety, and development of ways to cope with that anxiety via discussion, yoga, changing problem solving approaches, walking around, and talking to others. Instructors de-emphasized lecture, and students were encouraged to focus on gathering what they knew before considering what the answer might be. Sometimes the instructors even instructed students to *avoid the answer* in order to focus on the process of problem solving.

Research Methodology

The purpose of this study was to assess the effectiveness of this interdisciplinary general education mathematics and dance course. There were 12 general education students in the Pattern Play course. The mathematics professor from Pattern Play also taught a traditional section of liberal arts mathematics in the spring following the pilot of Pattern Play. This section served as the control group for the study, and there were 34 students that completed this course. Several assessment tools were used to analyze the effectiveness of Pattern Play in an attempt to capture the full influence of the multimodal approach to learning featured in this course. These assessments included pre and post mathematics tests, pre and post drawing prompts, and pre and post attitudinal surveys. Students were not given any extra credit for completing any of the assessments and these assessment were not included in the grades for the course. Also the course instructors were never in the room at the time of assessment.

Mathematical Content Knowledge

The mathematical pretest/posttest was composed of twelve questions. The students were given 30 minutes to complete the exam and were allowed to use a scientific calculator. The questions were either taken from the course textbook or from a past exam used in another section of mathematics for the liberal arts. The questions included analysis of a linear function, a counting problem, a Venn diagram probability question, an applied Pythagorean Theorem problem, a proportionality arc length question, and a scaling problem. The exams were graded according to a rubric. For example, for a five point question involving multi-step calculations, students received all five points for correct responses with clearly communicated work, four points for mostly correct work involving an arithmetic error, three points for mostly correct work with multiple arithmetic or algebraic errors, two points for citing relevant formulas or trying valid methods but failing to arrive at a complete solution, one point for any work with some correct component, and zero points for blank or fully incorrect work. The solutions on this exam were also analyzed qualitatively to discern any patterns in the type of solutions produced.

Drawing Prompt

The drawing prompt asked students to "Draw yourself doing mathematics." The students in both the treatment and control sections were given the drawing prompt on the first and last day of their respective classes. The students had about ten minutes to complete the drawing. The researchers explained that the quality of the drawing was not important and instructed the students to draw what came to mind when they thought of themselves doing mathematics.

This drawing prompt was adapted from previous work with the Draw-Yourself-Learning/Teaching-Mathematics test (Mcdermott, & Tchoshanov, 2014) and the Draw a Mathematics Teacher test (Utley, Reeder, & Redmond-Sanago, 2015). Instead of focusing on developing teachers' view of mathematics, this adaptation focuses on the student view of learning mathematics. A rubric was developed based on prior work by Farland-Smith (2012) and Utley et al. (2015). The rubric categorized drawings as extremely negative (1), negative (2), unpleasant (3), neutral (4), pleasant (5), positive (6), and extremely positive (7). The coding process is fully described in Bachman, Berezay, and Tripp (2016).

Attitudinal Survey

The attitudinal survey featured seven statements about mathematics: "Math is confusing," "Math involves a lot of experimentation," "I fantasized of a world without math," "Math involves a lot of memorization," "If I get stuck on a math problem on my first try, I try a different approach," "Math is a social activity," and "I get anxious when I have to do math." Students were asked to select a choice that best fit their response to each statement from the following options: Strongly Disagree, Disagree, Neither Agree nor Disagree, Agree, and Strongly Agree. All the students in the study answered the attitudinal survey on the first and last days of their respective classes. They were given about five minutes to complete the survey.

Findings

Table 1: *Quantitative Results*

	Treatment			Control				
	n	Mean Change	Median Change	n	Mean Change	Median Change	Conan's *d*	Mann-Whitney *U*
Math Content	10	26	19.5	23	9	2	0.831	1.000**
Drawing Prompt	11	2.25	2	17	0.22	0	1.106	45.500*
Attitudinal Survey	12	6.33	6.5	19	1.82	0	0.9471	58.500*

*$p< 0.05$. **$p < 0.01$

Table 1 summarizes the pre and post measures used to assess treatment and control groups. The Mann-Whitney *U* test was chosen due to the small sample sizes in this study. All three assessments showed a statistically significant difference between the pre/post gains made by the treatment group compared to the control group. Also, the effect size for each measure (Conan's *d*) was determined to be large according to Conan's classification of effect sizes.

Mathematical Content Knowledge

On average, the treatment group score 26 points higher on the posttest than the pretest while the control group only improved by 9 points on average. Eighty-three percent of the treatment group students scored higher on the posttest than the pretest; only 71% of the control group scored higher on the posttest. Throughout the assessment, the treatment group displayed more persistence in problem solving than the control as evidenced by fewer blank problems. For example, when determining arc length, 54% of the control either left the problem blank or wrote "Can't remember the formula" compared to only 25% of students responding this way in the treatment group.

Drawing Prompt

On average, Pattern Play students increased their rubric score on the drawing prompt by 2.25 points (out of a maximum positive score of 7). Students in the control group increased their score by 0.22 points. Overall, 75% of the treatment group drew more positive pictures about themselves doing mathematics on the posttest. Only 39% of student drawings from the control group were more positive on the posttest. Furthermore, none of the posttest treatment drawings were considered "strongly negative" while five students (28%) from the control group received this score on the posttest. Also, students from Pattern Play were more likely than the control group students to include other people in their pictures, draw a nontraditional setting for learning, and not focus their drawings on panic or confusion on the posttest drawings.

Attitudinal Survey

The responses from the attitudinal survey were combined to give each student a mathematics attitudinal score. The maximum score was 35 and signaled a perfectly positive attitude toward mathematics. On average, the treatment group improved their attitudinal survey score by 6.33 points while the control group improved only an average of 1.82 points.

Implications

Students in Pattern Play outperformed the traditional control group students in the areas of mathematical content knowledge, attitudes toward mathematics, and persistence in problem solving. Also, every student that took Pattern Play successfully completed the course. In the control group, three students (9%) failed to successfully complete the course for general education mathematics credit. Furthermore, Pattern Play offers an exciting, innovative option for learning mathematics in an understandable, interesting, and relevant setting. This method has the potential of reaching students who struggle with traditional textbook mathematics. While this pilot section was offered in a college level liberal arts mathematics course, the methods in this class are pertinent to K-12 and developmental mathematics classrooms as well to foster understanding and use of effective mathematical practices of teaching and learning.

Every Pattern Play class was videotaped to allow for a case study of the methods used in the course. Key lessons were also videotaped from the control group to provide a contrast of the Pattern Play lessons to approaches used in more traditional mathematics classrooms. Initial analysis of the Pattern Play classroom videos has piqued interest in the research team about the methods used in that course to reduce math anxiety, foster class participation, elicit persistence and perseverance in problem solving, and foster a collaborative class environment. Future study of this course includes a detailed analysis of these videotaped classes. Furthermore, the research team plans to offer the course again in the future to refine the instructional methods used in the course and to design materials to be shared with others desiring to implement such techniques in their own classrooms.

References

Bachman, R. M., Berezay, K., and Tripp, L. (2016). Draw yourself doing mathematics: Assessing a mathematics and dance class. In Adolphson, K. A. and Olson, T. (Eds.). *Proceedings of the 43rd Annual Meeting of the Research Council on Mathematics Learning.* Orlando, FL.

Blair, R., Kirkman, E. E., Maxwell, J. W. (2013). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2010 CBMS survey*. Providence, RI: American Mathematical Society

Davydov, V. V. (2008). *Problems of developmental instruction: A theoretical and experimental psychological study*. New York: Nova Science Publishers, Inc.

Farland-Smith, D. (2012). Development and field test of the modified draw-a-scientist test and the draw-a-scientist rubric. *School Science and Mathematics, 112*(2), 109-116.

Hanna, J. L. (2000). Learning through dance: Why your schools should teach dance. *American School Board Journal, 187*(6), 47-48.

Kinard, J. T., & Kozulin, A. (2005). Rigorous mathematical thinking: mediated learning and psychological tools. *Focus on Learning Problems in Mathematics, 27*(3).

Kozulin, A. (1990). *Vygotsky's psychology: A biography of ideas*. Cambridge, MA: Harvard University Press.

Laws, P. W. (1999). New approaches to science and mathematics teaching at liberal arts colleges. *Daedalus, 128*(1), 217-240.

Mcdermott, B., & Tchoshanov, M. (2014). Draw yourself learning and teaching mathematics: A collaborative analysis. In Matney, G. T. and Che, S. M. (Eds.). *Proceedings of the 41th Annual Meeting of the Research Council on Mathematics Learning*. San Antonio, TX

Olson, T., Olson, M., Venenciano, L, & Slovin, H. (2014). Task alignment to the common core: How our solution lens matters. In Matney, G. T. and Che, S. M. (Eds.). *Proceedings of the 41th Annual Meeting of the Research Council on Mathematics Learning*. San Antonio, TX

Rinne, L, Gregory, E., Yarmolinskaya, J., & Hardiman, M. (2011). Why Arts Integration Improves Long-Term Retention of Content. *Mind, Brain and Education, 5*(2), 89-96

Schaffer, K., Stern, E., & Kim, S. (2001). *Math dance with Dr. Schaffer and Mr. Stern*. Santa Cruz, CA: MoveSpeakSpin.

Steen, L. A. (2000). Reading, writing, and numeracy. *Liberal Education*, *86*(2).

Utley, J., Reeder, S., & Redmond-Sanago. (2015). *Development of the draw a mathematics teacher test and rubric*. 42nd annual meeting of the Research Council on Mathematics Learning.

Yakimanskaya, I. (1991). *The development of spatial thinking in schoolchildren.* (Soviet studies in mathematics education, Vol. 3). Reston, VA: National Council of Teachers of Mathematics.

THE ROLE OF SUPPORT STRUCTURES IN THE SUCCESS OF DEVELOPMENTAL MATHEMATICS PROGRAMS

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The success of developmental mathematics programs to prepare students for college-level coursework is questionable. The current study examines a five-year longitudinal data set of community college students in order to illuminate factors associated with successful outcomes for developmental mathematics students. Two analyses are considered: the role of tutoring for developmental mathematics students, and the combination of tutoring with enrollment in a student success course or in developmental reading/writing for developmental mathematics students. The likelihood of a student completing the developmental sequence and subsequently passing a credit-level mathematics course is discussed in relation to these analyses.

Introduction

There is no larger issue in higher education with regards to college readiness in mathematics than that of the role of developmental mathematics and the dismal results these courses have had in fostering student success. Developmental mathematics courses are typically non-credit, semester-long courses that do not satisfy any mathematics credential but serve to reinforce and strengthen algebraic skills for students with placement scores indicating a lack of college preparedness in mathematics. Recent reports show that only 39% of high school seniors are prepared for entry-level college courses in mathematics (Heitin, 2014), and more than half of community college students are required to enroll in at least one developmental course for remediation (Bailey, Jeong, & Cho, 2010). These students are struggling to persist and successfully complete the required coursework to meet college readiness benchmarks and become eligible for enrollment in credit-level mathematics courses such as College Algebra or Statistics. Despite the intention of developmental math programs to provide alternative access to higher education, evidence suggests that developmental mathematics has instead been serving as a barrier (Bonham & Boylan, 2011; Edgecombe, 2011). In a study examining a cohort of developmental mathematics students, a paltry 30% completed the prescribed sequence of developmental classes, and of those completers, only half were able to successfully complete a credit-bearing mathematics course within three years (Bailey, Jeong, & Cho, 2010). In 2012, the national non-profit organization Complete College America released a report surveying the landscape of developmental education with a grim title that echoed the sentiments of many in academia, *Remediation: Higher education's bridge to nowhere* (Jones, 2012).

The need to identify effective and innovative methods for successfully remediating students in mathematics is critical, and there is currently a reform movement to reinvent how these students are assisted and supported at colleges and universities. Previous studies have examined alternative developmental course format options, such as acceleration, self-paced, online/hybrid, or mainstreaming (Edgecombe, 2011; Rutschow & Schneider, 2011; Dana Center, et al., 2012; MDC, 2012; Keller, 2013) as well as student characteristics such as gender, full-time status, and financial need (Boylan & Saxon, 1999; Bailey, Jeong, & Cho, 2010; MDC, 2012). In prior work, we examined the impact of course format and demographic characteristics for a cohort of first-time-in-college students, tracked for five academic years through developmental and credit mathematics courses (Howell & Walkington, 2015). In the present paper, we extend this analysis to additionally consider the role of institutional support factors (tutoring, taking a student success course, and taking developmental coursework in reading or writing) in students' success rates on their path to a credit-bearing mathematics course. In the next section, we briefly discuss prior research in each of these areas in turn.

Related Literature

Developmental education encompasses a broad range of programs and services that are designed as a means of access for students that are underprepared for postsecondary coursework (NADE, n.d). The structured sequence of semester-long mathematics classes to be taken prior to enrolling in an introductory credit course was built on the common sense premise that underprepared students needed additional time in order to build prerequisite skills, and has typically been composed of three courses - Pre-Algebra, Beginning Algebra, and Intermediate Algebra (Dana Center, et al., 2012). The institutional support structures which surround developmental math students, such as on-campus tutoring services, student success courses, and other developmental coursework, play a key role in the retention and success of at-risk students (Dennis, Phinney, & Chuateco, 2005; Zalaquett, 2006).

The role of tutors is essential in providing feedback and support to emerging mathematics learners (Kenner & Weinerman, 2011). Learning assistance centers (also known as tutoring centers or mathematics labs) are common, and students can visit the center for aid on mathematics coursework without prior scheduling. Research on the impact of tutoring for developmental students is limited and provides mixed results (Rutschow & Schneider, 2011). Boylan, Bliss, and Bonham (1997) found that tutoring was only impactful in schools where tutors had received extensive training of high quality, and depends on a tutor's ability to apply individualized strategies tailored to each student's situation (Boylan & Saxon, 1999).

Most new college students lack academic skills, and all could benefit from the extra support that developmental students require (Dana Center, et al., 2012). Student success courses exist to provide this additional support by explicitly teaching underprepared students about studying for college courses, time management, and other necessary but missing academic study skills (Zeidenberg, Jenkins, & Calcagno, 2007). Cho and Mechur Karp (2013) found that these student success courses are especially impactful for developmental mathematic students that co-enroll in a success course within the first year. Offered parallel to other innovations in course delivery formats, support structures can positively increase student engagement with content as well as with classmates (Bonham & Boylan, 2011), leading to positive learning experiences and improved attitudes toward math and achievement in math. Collaborative efforts build support, a sense of community, and a safe and welcoming educational environment.

Limited research exists examining the potential impact of non-mathematics developmental courses (i.e., developmental courses in reading and writing) on long-term mathematics outcomes. Bremer, Center, Opsal, Medhanie, Jang, and Geise (2013) found that the beneficial impact of developmental reading and writing classes was short-lived, limited to improving retention to the second semester. No long term impact on retention – such as persistence to graduation – was reported. Further, enrollment in developmental reading and/or writing courses in the first term was negatively associated with GPA for subsequent credit math-related courses. These findings contradict those of Fike and Fike (2008), who found that enrollment and successful completion of a developmental reading course was a strong predictor of student retention. More generally, research suggests that reading ability is related to students successfully solving math word problems (Walkington, Clinton, Ritter, & Nathan, in press).

In the present study, we seek to add to the incomplete understanding of these support structures. We investigate the following research questions: How are options intended to academically support community college developmental mathematics students, including tutoring, enrollment in a student success course, and co-enrollment in a developmental reading/writing course, associated with students' likelihood of *completing the developmental* *mathematics sequence*? How are these support structures associated with the student's likelihood of *passing a credit-bearing math* course?

Methodology

A cohort of 595 first-time-in-college (FTIC) students (61% female, 26% full time) who enrolled in Fall 2009 in any level of developmental mathematics was selected from a mid-sized community college system in a suburban/metropolitan area in the Southern U.S. The study tracked the students for the 2009-2014 academic years. Students in the identified cohort are predominantly Caucasian (43%), and approximately one-third (35%) are of low socio-economic status, as measured by their eligibility for Title IV grant funds. Nearly half of the students in this cohort began the developmental sequence in Pre-Algebra, and approximately one-third attended tutoring on campus during the five-year study. Additionally, about 31% of the cohort was also required to take developmental coursework in Reading or Writing, and almost 40% of the cohort also enrolled in a student success course. Students' developmental course placement was determined by a placement test administered by the college (THEA, Compass, or Accuplacer).

To analyze students' progress through the developmental sequence, the successful completion of all required developmental mathematics courses was coded as a 0/1 dependent variable. Additionally, the successful completion an entry-level credit-bearing mathematics course was also assigned a 0/1 value. Logistic regression techniques were used to predict completion of the developmental sequence as well as completion of a credit mathematics course. Grades of A, B, C, or D were considered passing, while withdrawals and grades of F were considered failures. Several predictor variables were defined. First, variables related to developmental course format were developed. Accelerated courses are those taught in a condensed time frame, including 8 week "fast track" courses and 5 week summer classes. Self-paced classes are those which allow students to progress through a mastery-based series of assigned content modules. Hybrid or online courses formed the third course format category, capturing courses with at least 50% of course delivery using technology. Dichotomous coding occurred for each category: if a student took at least one developmental math class that was of a certain format, the student would receive a code of 1 for that predictor; otherwise the student received a 0. A variable describing the first course in a student's developmental pathway was also created, along with the students' placement test score which was a proxy for each student's mathematical background knowledge. Student demographic characteristics were also added to the model as predictor variables. Finally, variables related to our three focal support structures, tutoring, enrollment in a student success course, and enrollment in developmental reading/writing, were added. Throughout the paper, we focus on the effects related to these three factors, as findings related to course format and demographic variables were reported previously (Howell & Walkington, 2015).

Findings

The overall outcomes for the cohort are provided in Table 1. Semester to semester retention led to less than half of the cohort students successfully completing the developmental mathematics sequence and only a quarter successfully passing a credit-mathematics course in the five-year study.

Table 1. *Overall course taking outcomes for 2009-2014 (n = 595)*

Criteria	**Number of Students (% of cohort)**	**Number Progressing to Next Course (Pass Rate %)**	**Number Progressing to Next Course (Pass Rate %)**	**Number Progressing to Next Course (Pass Rate %)**	**Number Completing Developmental Math (% of cohort)**
% starting in Pre-Algebra	258 (43%)	186 of 258 (72%)	94 of 186 (51%)	82 of 94 (76%)	82 of 595 (14%)
% starting in Beginning Algebra	154 (26%)	104 of 154 (68%)	79 of 104 (76%)		79 of 595 (13%)
% starting in Intermediate Algebra	183 (31%)	124 of 183 (68%)			124 of 595 (21%)
% that complete developmental math sequence	285 of 595 (48%)				
% pass first credit-bearing mathematics course	157 of 595 (26%)				

The first logistic regression model examined whether successful *completion of the developmental mathematics sequence* was predicted by course format, first developmental class taken, demographic variables, and our three support structures. In order to reduce the false discovery rate for multiple comparisons, the FDR *p*-value correction method was utilized

(Benjamini & Hochberg, 1995). Odds ratios calculated in the logistic regression model were transformed to D-type effect sizes (standardized mean differences) using an approximation that allows for ease of comparison among logistic models (Chinn, 2000). Students enrolled in an accelerated developmental math course were more likely to complete the developmental sequence (Odds = 11.10, $d = 1.33$, $p < .001$), along with students required to enroll in only one level (the highest) of developmental mathematics – Intermediate Algebra (Odds = 2.93, $d = 0.59$, $p < 0.001$). These findings are not surprising given the rigorous nature of accelerated courses and the shortened sequence length for students beginning in Intermediate Algebra, compared to the more traditional timeframe of other course formats, or the additional required courses for students placing into Pre-Algebra or Beginning Algebra. Gender was statistically significant ($p = .045$) in that males were less likely to complete the developmental sequence than females (Odds = 0.62, $d = -0.26$). A student's standardized placement test score was a statistically significant positive predictor of completion ($p = .006$), as was full-time enrollment (Odds = 7.33, $d = 1.10$, $p < 0.001$), and whether or not a student attended on-campus tutoring (Odds = 5.03, $d = 0.89$, $p < 0.001$). When restricting tutoring only to math-specific tutoring, the odds were comparable (Odds = 4.78, $d = 0.86$, $p <.0001$). Other support services considered (student success course or developmental reading/writing coursework) were not statistically significant in the model.

We also considered a subset of the cohort of only students starting the developmental sequence in Pre-Algebra ($n = 258$), in order to see how effects varied for the students most in need of support. For these students, the impact of accelerated coursework and math tutoring increased dramatically. Pre-Algebra students that enrolled in at least one accelerated course format were over 18 times (Odds = 18.15, $d = 1.60$, $p < 0.001$) more likely to complete the developmental math sequence, and students beginning the sequence in Pre-Algebra that attended on campus math-specific tutoring were almost 10 times more likely to successfully complete remediation (Odds = 9.35, $d = 1.23$, $p < 0.001$). Note that these analyses control for prior math ability through the placement test score variable, so it is likely not the case that these effects are driven by the tendency of stronger students to seek tutoring or take accelerated courses. However, when allowing for interactions, students with lower placement test scores (as standardized with a z-score conversion) were less likely to be successful in an accelerated course format (Odds = 0.20, $d = -0.90$, $p = .02$).

The second logistic model considered the impact of course format, student characteristics, and support structures on *passing a credit math* course. In the main effects model (n = 212), no predictor variables were statistically significant in increasing the likelihood of passing a credit math course. An interaction did occur for online/hybrid course format and tutoring: on-campus tutoring greatly increases the likelihood of an online developmental math student eventually passing a credit math course (Odds = 6.97, d = 1.07, p < 0.05). When considering not just any on-campus tutoring but math-specific tutoring, the odds almost doubled. Developmental students taking an online/hybrid format that went to at least one on-campus math tutoring session were over 12 times more likely to eventually pass a credit-level math course (Odds = 12.05, d = 1.38, p = 0.03). It is important to note that online/hybrid formats were not offered for the first developmental course, Pre-Algebra. In order to control for a possible confound due to this issue, an interaction term between initial course and the tutoring variables was included in these models, and it was nonsignificant and results did not change with its inclusion.

Discussion & Conclusion

The current study considered the association of several variables on whether developmental math students successfully complete the required developmental math sequence and eventually pass a credit math course. Our focus is on the examination of support structures such as the role of tutoring, enrollment in a student success course, or enrollment in other developmental courses in Reading/Writing. Participation in any on-campus tutoring positively predicted *completion of developmental math* coursework, echoing previous research findings (Kenner & Weinerman, 2011). Furthermore, math-specific tutoring is critical for the weakest students and dramatically increased their likelihood of successfully completing the developmental math sequence. When examining a student's likelihood of eventually *passing a credit mathematics* course, no single variable was a strong indicator of success. However, interactions revealed that tutoring for students that enrolled in at least one online/hybrid developmental math class appeared to greatly increase the odds of passing a credit math course. Math-specific tutoring was especially important to the success of this group of students in passing a credit math course. Surprisingly, literacy support measures (e.g., enrollment in developmental reading or writing) as well as enrollment in a student success course had little impact in any of the models examined. While these courses may provide needed academic skills for emerging math learners, these findings support previous research that indicates the benefits of student success and developmental reading/writing course may be short-lived (Bremer, et al., 2013; Cho & Mechur Karp, 2013). This outcome is important when considering current efforts to reform developmental mathematics through mainstreaming underprepared students into credit-bearing math courses along with additional support coursework.

In current work, we are conducting interviews with students to identify variables of interest that may shed light on how to best help students struggling with mathematics at the community college. One limitation not addressed in the current data is information regarding the impact of the instructor's teaching on outcomes. While additional analysis is needed, we are excited by the valuable insight into the complex and perplexing issues of developmental mathematics students that this study provides. This study, along with similar research and reform efforts occurring nationally, contributes to the exchange of ideas that ultimately will allow educators to better serve struggling mathematics learners in higher education. We will continue to examine this issue and work towards the expansion of this body of knowledge regarding students who need mathematics support at all levels of student development.

References

Bailey, T., Jeong, D., & Cho, S. (2010). Referral, enrollment, and completion in developmental education sequences in community colleges. *Economics of Education Review*, *29*(2), 255-270.

Benjamini, Y., and Hochberg, Y. (1995). Controlling the false discovery rate: a practical and powerful approach to multiple testing. *Journal of the Royal Statistical Society Series B* **57**, 289–300.

Bonham, B., & Boylan, H. (2011). Developmental mathematics: Challenges, promising practices, and recent initiatives. *Journal of Developmental Education*, *34*(3), 2-10.

Boylan, H., Bliss, L., & Bonham, B. (1997). Program components and their relationship to student success. *Journal of Developmental Education*, *20*(3), 2-4, 6, 8.

Boylan, H. R., & Saxon, D. P. (1999). *What works in remediation: Lessons from 30 years of research*. Unpublished report. Retrieved from http://www.beaufortccc.edu/progrm/developmental/assets/documents/whatworks.pdf.

Bremer, C. D., Center, B. A., Opsal, C. L., Medhanie, A., Jang, Y. J., & Geise, A. C. (2013). Outcome trajectories of developmental students in community colleges. *Community College Review*, *41*(2), 154-175. doi:10.1177/0091552113484963

Chinn, S. (2000). A simple method for converting an odds ratio to effect size for use in meta-analysis. *Statistics in Medicine*, *19*(22), 3127-3131.

Cho, S., & Mechur Karp, M. (2013). Student success courses in the community college: Early enrollment and educational outcomes. *Community College Review*, 41(1), 86-103. doi:10.1177/0091552112472227

Dana Center, Complete College America, Education Commission of the States, & Jobs for the Future. (2012). *Core principles for transforming remedial education: A joint statement*. Retrieved from http://www.completecollege.org/docs/CCA_joint_report-printer.pdf.
Dennis, J.M., Phinney, J.S., & Chuateco, L.I. (2005). The role of motivation, parental support, and peer support in the academic success of ethnic minority first-generation college students. *Journal of College Student Development, 46*(3), May/June 2005, 223-236. doi*:* 10.1353/csd.2005.0023
Edgecombe, N. (2011). *Accelerating the academic achievement of students referred to developmental education* (CCRC Working Paper No. 30). New York, NY: Community College Research Center, Teachers College, Columbia University.
Fike, D. S., & Fike, R. (2008). Predictors of first-year student retention in the community college. *Community college review, 36*(2), 68-88.
Heitin, L. (2014). Fewer than 40 percent of seniors are prepared for college, NAEP analysis finds. *Education Week.* 5/21/2014, 33(32), 4.
Howell, E. & Walkington, C. (2015). An examination of factors impacting college algebra readiness: Pathways through developmental mathematics. In Che, S. M. and Adolphson, K. A. (Eds.). *Proceedings of the 42nd Annual Meeting of the Research Council on Mathematics Learning*. Las Vegas, NV.
Jones, S. (2012). Remediation: Higher education's bridge to nowhere. *Complete College America*. Retrieved from http://completecollege.org/docs/CCA-Remediation-final.pdf.
Keller, J. (2013). *Community college student success in developmental mathematics courses: A comparison of four instructional methods*. Unpublished doctoral dissertation, University of North Texas.
Kenner, C., & Weinerman, J. (2011). Adult learning theory: Applications to non-traditional college students. *Journal of College Reading and Learning*, *41*(2), 87-96.
MDC. (2012). What We Know: Lessons From the Developmental Education Initiative. Raleigh, NC. Retrieved from http://www.mdcinc.org/sites/default/files/resources/DEI_WhatWeKnow_final.pdf .
National Association for Developmental Education (NADE). (n.d.). Definition of developmental education. Retrieved from http://www.nade.net/aboutdeved.html.
Rutschow, E.Z., & Schneider, E. (2011). *Unlocking the gate: What we know about improving developmental education*. MDRC Report. National Center for Postsecondary Research, Columbia University.
Walkington, C., Clinton, V., Ritter, S., & Nathan, M. J. (in press). How readability and topic incidence relate to performance on mathematics story problems in computer-based curricula. *Journal of Educational Psychology, 107*(2).
Zalaquett, C. P. (2006). Study of successful Latina/o students. *Journal of Hispanic Higher Education*, *5*(1), 35-47.
Zeidenberg, M., Jenkins, D., & Calcagno, J. C. (2007). *Do student success courses actually help community college students succeed?* New York: Columbia University, Teachers College, Community College Research Center.

INCREASING STUDENT ENGAGEMENT IN MATH WITH ONLINE GAMES AND ELEMENTS OF GAME THEORY

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Specific strategies for increasing student engagement in math classes by using online games and aspects from game theory will be presented. Gamification is not a new concept, but its application to education is in a nascent developmental stage. Research is clear: we are more interested in learning when it doesn't feel like drudgery, when it is fun, and when there is a significant degree of control and autonomy. Game designers have defined the "sweet spot" between hard fun and an almost addictive level of engagement. Aspects of game theory that can be applied to the typical math classroom are discussed.

Introduction

Every math teacher knows the struggle around student engagement. Countless studies, both formal and informal, have shown that student performance is enhanced when student engagement is increased. How many of us have wished for a way to harness the time and energy our students spend in trying to master an online game like Angry Birds and refocus it to more productive pursuits like mastering mathematical content? As it turns out, elements of game theory *can* be applied to useful, even mundane, tasks in order to exploit aspects of human nature that control engagement and motivation.

Gamification Defined & Common Terminology Introduced

For the purposes of this paper, “gamification” is defined as “the craft of deriving all the fun and addicting elements found in games and applying them” to education, specifically the math classroom. (Chou, 2015) Because game designers “have spent decades learning how to master engagement and motivation, we are now learning from games, and that is why we call it *gamification*.” (Chou, 2015, italics added)

What we are learning is that certain “game mechanics” like feedback loops, interconnectedness (“system of systems”), problem solving (“black boxes”) which involves exploration and mastery, and rewards, often in the form of levels or scores can be added to the learning environment in order to increase student engagement and motivation and to encourage desired behavior. (Cook, 2006) The good news is that these elements of game theory are proven and well-defined; thus, can be adapted to areas other than mere entertainment. The not-so-good news is that applying them to the traditional classroom environment often involves significant re-thinking and re-structuring. Like most things worthwhile, it will involve some work on both your part and the part of your students. However, the payoff in terms of increased student engagement, motivation, and enjoyment make the investment well worth it.

Gamification in the Classroom

Crafting a math class that effectively uses elements of game theory to increase student engagement and motivation requires understanding the human being the way game designers do.

We know that humans are hardwired to solve problems. Cook (2006) colorfully describes it like this:

> **Humans are infovores**
>
> Humans are wired to solve black boxes. It is a fundamental aspect of our neurological learning wetware. We get real chemical rewards when we grok a problem or gain information that we suspect will help in grokking a black box. Evolution has selected for this behavior over thousands of generations since it is the biological reward system that encourages tool use and technological adoption. Without this built in addiction to problem solving, we would lack agriculture, medicine, architecture and other fundamental survival techniques that make the human species such a remarkably successful animal.

Elements of game theory can be applied to the math classroom in order to exploit this human tendency and result in students who are as motivated and engaged to learn how to solve quadratic equations, as they are to level up in *World of Warcraft*. The elements go beyond leader boards, points, and levels though – it involves asking the question, "How do we want our students to *feel* when they are in class, engaged in solving a math problem, and learning?" This feeling can be described using 8 core drives according to Cook. Of these, four are most applicable to the math classroom: (1) development and accomplishment (2) ownership and possession (3) scarcity and impatience (4) unpredictability and curiosity. To motivate and engage students, you need to craft a math experience in which the quest is clear, the reward is significant, and there are choices, unexpectedness, and ownership.

One way many of these elements can be incorporated into a math classroom is by using gamification software like *Classcraft* (www.classcraft.com) to help develop the quests, organize the structure, and track progress via points and rewards. Within this structure, we can redefine failure and reward mastery: "In gaming, failure is not a negative, but rather an opportunity to learn from mistakes and correct them. Set the gaming software to allow students to repeat quests without penalty until they have mastered the skill." (Kolb, 2015)

Feedback Loops Explained

Using gamification software allows the teacher to set up an appropriate learning environment that includes and implements game design and game elements in ways that encourage the desired student behaviors. The feedback loop can be developed and controlled from the *Classcraft* dashboard as shown in Figure 1 below.

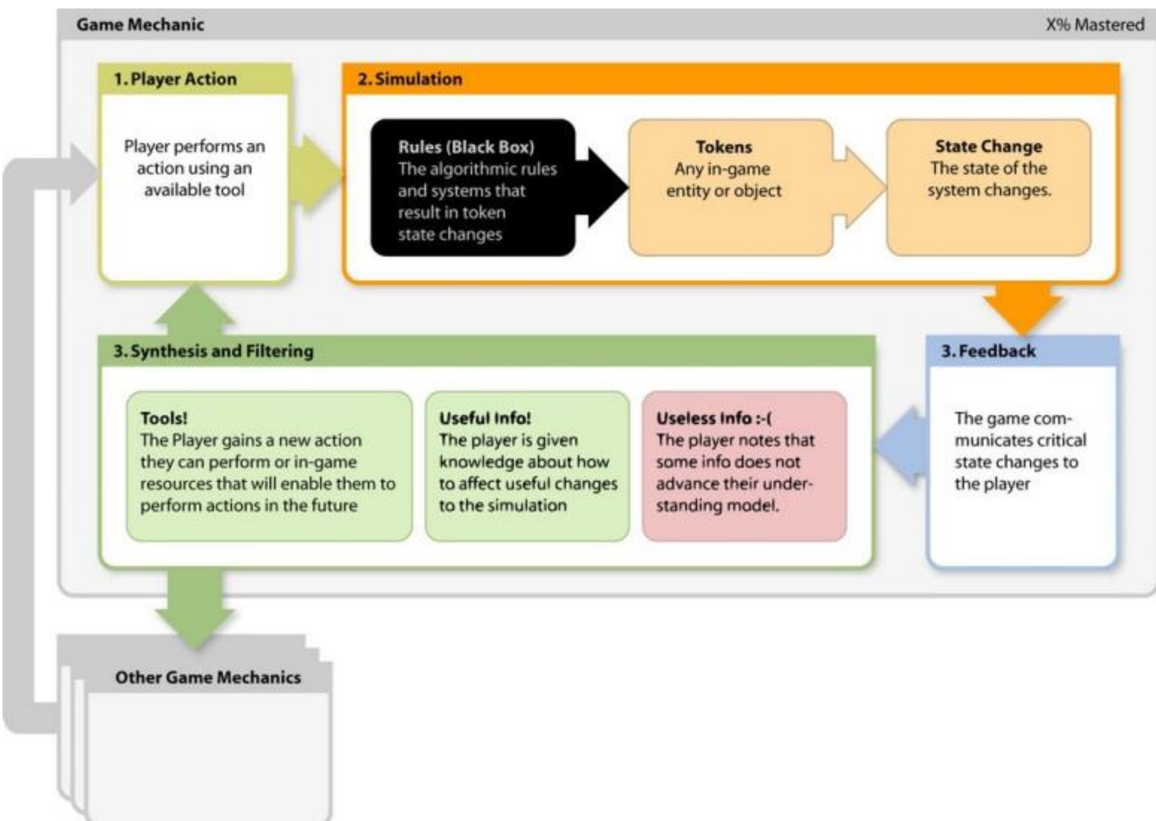

Figure 1. Classcraft feedback loop (Cook, 2006)

This loop can be used in a mathematics classroom in many ways. Here is an example of what this might look like:

1. Student performs an action: correctly answering a question in class earns them 60 xp (experience points)
2. The action causes an effect within the gamified classroom environment: that amount of xp was enough to allow the student to “level up” (progress in the game)
3. The student receives feedback: as a result of the increased level, the student receives the ability to “learn a power” like “counter attack” which allows the student to receive a hint on a quiz.
4. The student performs another action: the loop repeats but now with new information, skills, powers, and abilities.

An Example of Gamification in Mathematics

One particular practitioner, Kate Fanelli, developed “MathLand” as part of her gamified math classroom. She describes her attempt at trying out gamification as occurring only “after abandoning hope that her standard teaching method could deliver her students from the seemingly-futile loop of boredom, stagnant scores, and chronic absenteeism.” (Ross 2011) Fanelli states her “aha” moment came from hearing about the motivation behind why students like computer games: the thrill that they get from “leveling up”. Here is a nice description of what leveling is to a gamer:

> Leveling indicates a graduation day in the world of games. Players level each time they achieve some in-game goal or some standard of experience. Leveling is both a celebration and a rite of passage. New honors and powers are heaped upon gamers each time they level up. (Ross, 2011, p. 2)

She then incorporated this into her math class by creating 20 levels, based on the curriculum, that students may achieve by performing certain tasks and successfully completing mastery tests in order to earn points.

> “Points are awarded only for successfully completing the mastery test”, says Fanelli. “I tell the kids, the lesson and practice are for learning, the mastery test is for showing you've learned it. That's what they earn points for: actually demonstrating they've learned something.”

> The other key component of MathLand is a visual tracking system for the students – what Fanelli's students call the MathLand Board. Each student creates an avatar on the first day of class. (Ross, 2011, p. 3)

Like other successful gamification applications, MathLand includes choice, feedback loops, the acceptance of failure as a required part of the leveling process, and the ability to earn rewards, both tangible and cosmetic (being able to upgrade to a fancier avatar, for example). The results Fanelli achieved with MathLand are impressive: 17% increase in standardized test scores and 13% improvement in attendance.

Personal Experiences

My personal experience with including online games and elements of game theory has been more mixed. I have successfully incorporated some game theory elements, like converting to a point-based system, into my university-level mathematics classes but not yet taken it to the level of MathLand with a game board and avatars or an online version using *Classcraft*, mostly because of the upfront time required for the planning and creation phases necessary to gamify a classroom mathematics experience.

I have also been successful in including smaller components of online games into my mathematics classrooms, specifically in the form of using online games like *Kahoot!* (https://getkahoot.com) an open-source, online tool that allows you to create your own "game" and then play it with your students using almost any device (computer, tablet, smart phone) with nothing to download or install. I have personally witnessed the power of *Kahoot!* to transform a class of lifeless, lethargic, zombie-like students into an excited, energized, motivated group of people competing for mastery of a topic. THAT is the true power of gamification!

You have the choice to make your *Kahoot!*s public and when you do, they are searchable. This results in a nice resource if you want to simply use someone else's Kahoot as-is or to get a head start by modifying an existing game. Figure 2 is a screen shot of a search I did for Kahoots on "algebra" with the caveat to only show games that were made by teachers:

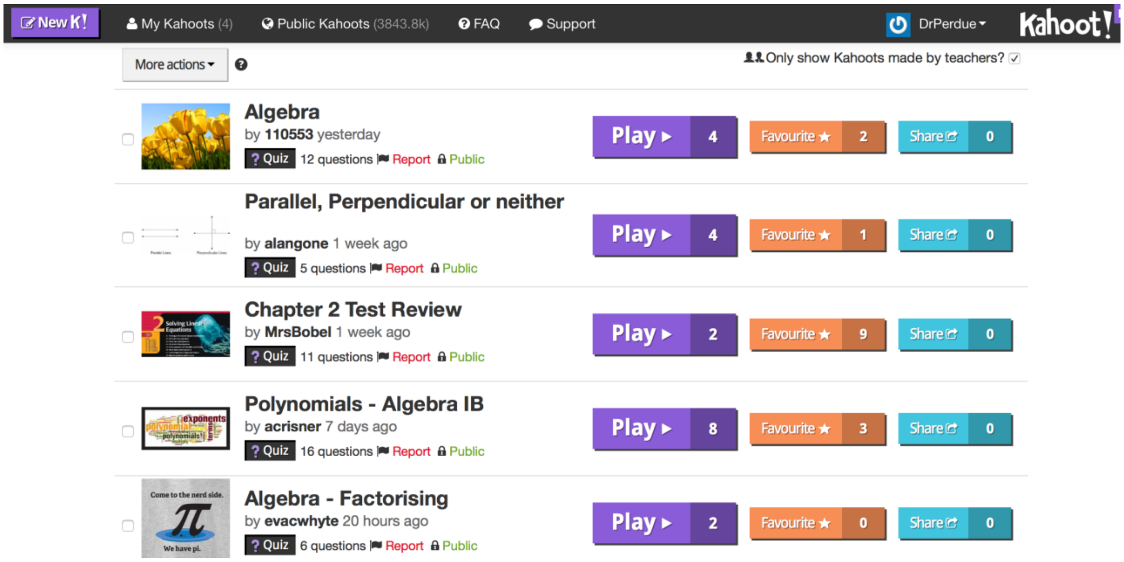

Figure 2. An example of *Kahoot!* teacher-made game search.

I like *Kahoot!* as a starting place for teachers new to gamification. It is easy to create a game, simple for the students to play, doesn't require anything at all (no plug-ins or drivers), has no issues with school firewalls, and it's free. It also illustrates some of the best features of effective game theory: the feedback loop is very fast and very intuitive; the design is beautiful, simple, and engaging; it destigmatizes failure and encourages mastery, particularly with the newest feature, "ghost mode", that allows students to play a *Kahoot!* again and try to beat their last score played as a ghost; and it is highly motivational to students! Figure 3 is a summary from *Kahoot!*'s website of why it's a great tool to use in the classroom:

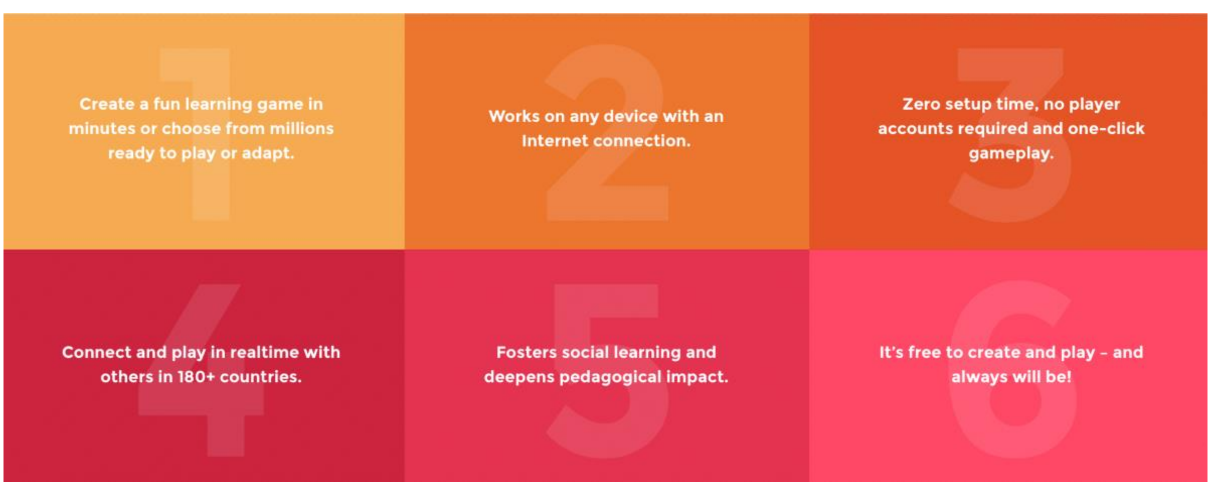

Figure 3. *Kahoot!*'s advantages

In my pre-service teacher courses, I have demonstrated incorporating elements of game theory using Kahoot and, without exception, the students told me it was one of their favorite activities and one of the main “take-aways” from class that they implemented in their own classrooms. It also lends itself nicely to other aspects of gamification like the social aspect, ownership, and autonomy if it is used as part of a group project or presentation: students must “teach” a topic, then create and play a game using Kahoot to assess how well their audience understood the material presented.

Online games like Kahoot can, of course, be incorporated into a larger gamified structure in the mathematics classroom as well. For example, within Classcraft, you could create a custom item for earning a place on the Kahoot leaderboard that week that could allow the student to earn extra xp or the ability to learn a new power.

Potential Pitfalls

Thus far, there is not enough long-term quantitative (or qualitative) research on the use of gamification and aspects of game theory in the mathematics classroom to give a thorough description of results and findings. However, smaller, action-based practitioner research has shown promise and the field certainly could benefit from additional study in this area. One particular area of danger that I have experienced personally is what Seaborn & Fels (2014) refer to as “pointsification”: the phenomenon whereby the concepts of gamification are reduced to points only; or, as other critics Seaborn and Fels referenced put it, the thing that is least important is made the only thing. More research on these pitfalls and how they can be avoided is certainly merited.

Other Applications and What the Future May Hold

There are excellent applications of game theory to areas outside of the mathematics classroom as well. (Chou, 2013) One of these is the *World Peace Game* (http://worldpeacegame.org), created by an educator from Virginia named John Hunter who wanted to find a way to teach peace in a world often filled with violence. According to the website for the game, its mission is stated below.

> *The World Peace Game* is a hands-on political simulation that gives players the opportunity to explore the connectedness of the global community through the lens of the

economic, social, and environmental crises and the imminent threat of war. The goal of the game is to extricate each country from dangerous circumstances and achieve global prosperity with the least amount of military intervention. As "nation teams," students will gain greater understanding of the critical impact of information and how it is used.

As their teams venture further into this interactive social setting laced with highly charged philosophical issues, the skills needed to identify ambiguity and bias in the information they receive will be enhanced and more specifically they will rapidly perceive that reactive behavior not only provokes antagonism, it can leave them alone and isolated in the face of powerful enemies. Beliefs and values will evolve or completely unravel as they begin to experience the positive impact and windows of opportunity that emerge through effective collaboration and refined communication.

In essence, as meaning is constructed out of chaos and new creative solutions are proposed, *World Peace Game* players will learn to live and work comfortably at the frontiers of the unknown. (http://worldpeacegame.org/world-peacegame-foundation/about-the-game)

Some of the elements of game theory are very related to behavior analysis and management. This can be seen in some of the applications developed for education; for example, the online classroom management tool, *Class Dojo* (www.classdojo.com) which incorporates visual design in the form of student avatars, reward systems for desired behavior, and very fast feedback loops to increase student engagement and encourage good behaviors, particularly among young learners.

In summary, online games and aspects from game theory can be added to a mathematics classroom to craft a learning environment that increases student engagement, improves student performance, encourages mastery and persistence, and rewards productive behaviors and habits.

References

Chou, Y. (2013). *Gamification in Education: Top 10 Gamification Case Studies that will Change our Future*. Retrieved from http://www.yukaichou.com/gamification-examples/top-10-education-gamification-examples/#.Vio-7YQ5nnh

Chou, Y. (2015). *What is Gamification?* Retrieved from http://www.yukaichou.com/gamification-examples/what-is-gamification/#.Vill1tY5nng
Classcraft, (n.d.). www.classcraft.com
Class Dojo, (n.d.). www.classdojo.com
Cook, D. (2006). *What are Game Mechanics?* Retrieved from http://www.lostgarden.com/2006/10/what-are-game-mechanics.html
Hunter, J., (n.d.). *World Peace Game,* http://worldpeacegame.org
Kahoot! (n.d.). https://getkahoot.com
Kolb, L. (2015). *Epic Fail or Win? Gamifying Learning in my Classroom*. Retrieved from http://www.edutopia.org/blog/epic-fail-win-gamifying-learning-liz-kolb
Ross, P. (2011). *Case Study – Gamification*. Retrieved from http://www.goventureoasis.com/en/learner/resources/t009/_case-study-gamification-one-simple-and-390.aspx
Seaborn, K. & Fels, D. (2014). Gamification in Theory and Action: A survey. *International Journal of Human – Computer Studies*, *74*, 14-31.
World of Warcraft, (n.d.). https://us.battle.net/account/creation/wow/signup/

USING TECHNOLOGY TO ENGAGE STUDENTS IN INTRODUCTORY STATISTICS

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One way to engage students in their own learning is through the use of technology. Two sections of collegiate introductory statistics were compared. Online applets were used to investigate sampling distributions of the sample mean and the Central Limit Theorem. One section was taught using traditional lecture, where the instructor demonstrated the use of these applets while the students watched. In the second section, students actively used the applets in the computer lab. The results indicated that the students who were engaged in using the technology outperformed those that merely observed the use of the technology by the instructor.

Introduction

There is a belief that when students engage in their own learning their understanding and retention of the content is greater. The purpose of this study is to explore how the use of technology impacts student performance and retention of content knowledge. Historically, sampling distributions is a difficult concept for students to understand (Zerbolio, 1989; Turner & Dabney, 2014). However, it is an important topic since it lays the foundation for many statistical inferential procedures. If students are to truly understand statistical inference, then they need to have a firm grasp on the concept of sampling distributions. Many instructors use technology to illustrate the sampling distribution of the sample mean. This study focuses on the way in which this technology is utilized. A comparison was made between two different implementations of an online applet. In one implementation, the students were actively engaged in using the technology to form initial conjectures about properties of the sampling distribution of the sample mean and then these conjectures were formalized via a class discussion. In the other, the students watched the teacher use the technology during lecture, while she explicitly pointed out these properties. The students were then assessed on this content three different times during the semester.

Background

A 2005 report from the Guidelines for Assessment and Instruction in Statistics Education (GAISE) Project funded by the American Statistical Association (ASA) offers recommendations for aiding in the development of statistically literate students via introductory statistics courses. Three of the six recommendations inform the research that was undertaken in this study. These are: 1) “Stress conceptual understanding, rather than mere knowledge of procedures,” 2) “Foster active learning in the classroom,” and 3) “Use technology for developing conceptual understanding and analyzing data” (GAISE, 2005, p. 4). Overlapping themes among these recommendations are that students need to be actively engaged in the learning process, and there needs to be less memorization of procedures and more conceptual understanding. In order to accomplish this, the GAISE report recommends more projects and lab activities that are centered around problem solving and discussion.

When considering the use of instructional technology in the classroom, one must consider whether the technology is enhancing the learning process for instructional purposes or for computational purposes. When using technology to enhance in the process of learning, there is often a change in pedagogy so that students may make sense of abstract concepts by exploring various situations (Chance et al., 2007). Due in part to increased accessibility to these instructional technologies, the content of introductory statistics courses has also started to change from a focus on just computation to that of decision making and interpretation (Chance et al., 2007). A recent review of literature about the teaching and learning of statistics by Tishkovskaya and Lancaster (2012) indicates that pedagogy needs to shift to a focus on conceptual understanding and that technology needs to be integrated as an essential part of that pedagogy. This review indicates that in order to accomplish these goals, there needs to be a shift from teacher-centered lecture to student-centered instructional methods.

Methodology

This study was a classic quasi-experimental design with a control and experimental group, where content in two sections of Introductory Statistics was taught using different instructional methods. The content that was the focus of these lessons was sampling distributions of the sample mean and the Central Limit Theorem (CLT). The control section, referred to as teacher-led technology (TLT), was taught this content using traditional lecture, where the instructor demonstrated the use of computer applets while the students watched. In the experimental section, referred to as student-led technology (SLT), students actively used the applets in the computer lab via a student-centered activity. This research compared the results of three different assessments during the semester.

This particular course satisfies the general education mathematics requirement at a regional university in the Southeastern United States. This course also fulfills the statistics requirement for entry into many professional programs, such as pre-pharmacy, pre-nursing, pre-veterinary, etc., and is also one of the statistics courses that counts towards a Middle Grades Mathematics degree. In the TLT section, 44% of the students were pre-nursing majors, and 20% were STEM majors, whereas 50% of the students in the SLT section were pre-nursing majors, and 28% were STEM majors. In the SLT section, 75% of the students were female, and in the TLT section 88% were female. The breakdown by academic rank can be seen in Table 1.

Table 1. *Academic Rank by Section*

	TLT	SLT
Freshman	7	3
Sophomore	11	15
Junior	4	9
Senior	3	5

The same instructor taught both sections of the course, which met three times per week. The TLT section was very traditional in that the instructor lectured while students took notes and then completed similar problems on their own. For all the lessons in this class, the instructor provided guided notes that students could print out prior to class. They could then add to them during the lecture and work examples that were included in the lecture. For the purposes of this research, the lecture notes that covered sampling distributions and the CLT were structured in the same way that the notes had been for the entire semester. To demonstrate the properties of the sampling distribution of the sample mean, the instructor used the StatCrunch sampling distributions applet with various population distributions and sample sizes (Integrated Analytics LLC, 2015). As she was doing this, she would ask the class what they noticed about the sampling distribution, but in the end the instructor identified the properties being demonstrated.

In the SLT section, for the lessons that were part of this research, students met in the computer lab which was not the class's normal meeting place. For this particular class meeting, students were given an activity packet that included the same material and examples that the TLT section had in the lecture. However, students were expected to utilize the StatCrunch applets to uncover the properties of the sampling distribution of the sample mean themselves instead of having the instructor explicitly state them. It is worth noting that the instructor does have training in facilitating a student-centered classroom. As a result, as the instructor was circulating around the lab to answer questions and address any issues that might arise. She was careful not give answers away, but instead answered questions with guiding questions. Once the activity was complete, the instructor facilitated a whole-class discussion over the activity. It is important to note that other than this one class meeting, the SLT and TLT sections were taught in the same way (using guided lecture notes) for the entire semester.

Upon completion of the content, students took a quiz during the next class meeting time. Then at the end of the unit, students took a test that included this content once again. Students were assessed for a third time over this material at the end of the semester when they took the final exam. Both sections had identical assessments.

Results and Discussion

The data analyzed for this research comes from only the students who gave consent and were present for this particular lesson and all three assessments; this resulted in 25 TLT students and 32 SLT students. Summary statistics of the results from the three assessments are shown in Table 2. The data used was the score (percent correct) on only the questions that pertained to sampling distributions of the sample mean and the CLT. The students who were actively engaged in the SLT section had higher mean and median scores on the applicable questions. These features are also seen in the boxplots in Figure 1. Additionally, the SLT students had less variability in their scores on the three assessments than did the TLT students.

Table 2. *Summary Statistics*

		TLT	SLT
Quiz	Mean	51.89	62.85
	Median	55.56	65.28
	Std. Dev.	29.12	20.81
Unit Exam	Mean	65.00	76.22
	Median	66.67	77.78
	Std. Dev.	18.41	16.43
Final Exam	Mean	71.60	83.28
	Median	70.00	90.00
	Std. Dev.	20.35	17.07

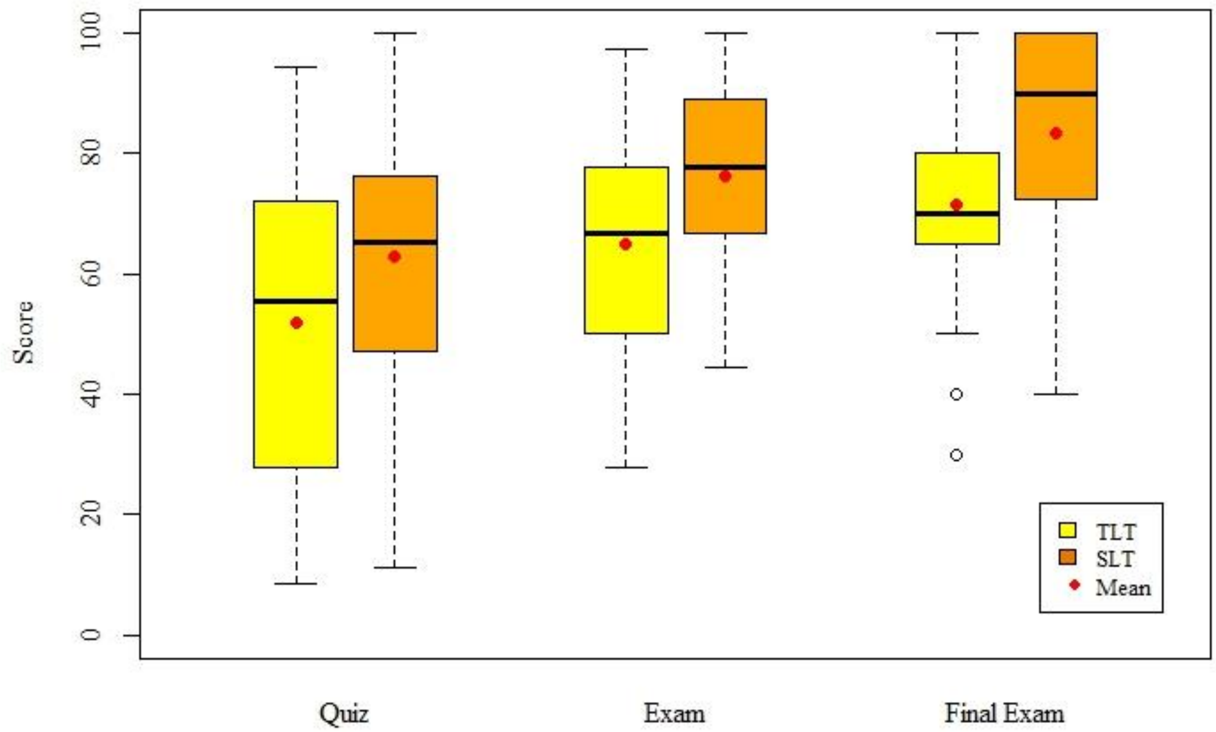


Figure 1. Boxplots of student scores on assessments

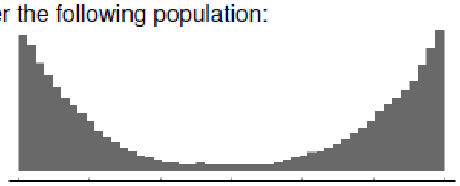
These results suggest that being actively engaged in using the applets is associated with higher performance on assessments. It is important to note that the assessments were evaluating the students' conceptual and procedural knowledge of the content. Examples of each type of question are shown in Figures 2 and 3. Since the medians and means are higher for the SLT while the standard deviations are smaller, this suggests that the SLT students consistently performed better on the three assessments than the TLT students.

Americans watch an average of 39.4 hours of TV per week with a standard deviation of 8.7 hours. **If possible, find each of the following. If not, explain why.**

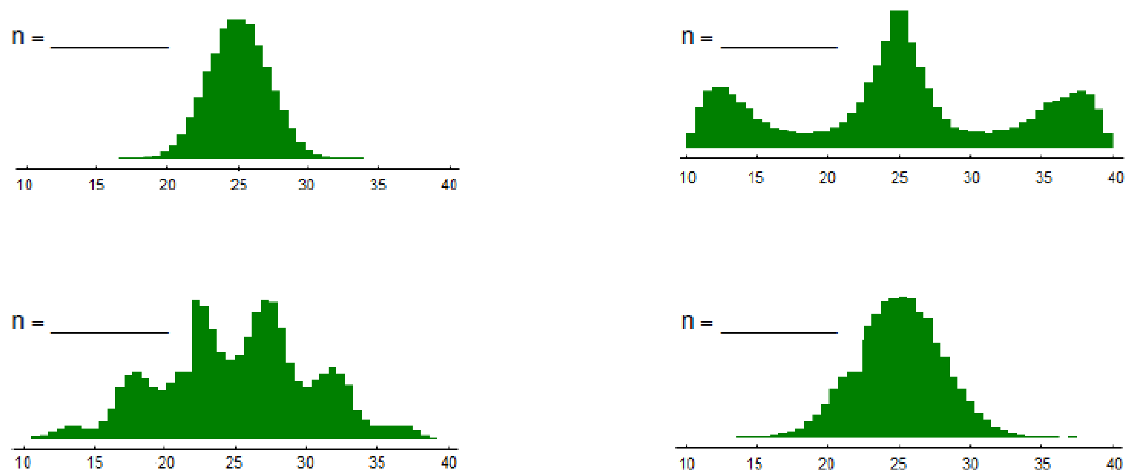
a. Find the probability that a randomly selected American watches less than 30 hours of TV per week.
b. Find the probability that 45 randomly selected Americans watch an average of more than 40 hours of TV per week.

Figure 2. Procedural question example

Consider the following population:



a. Each graph below represents the (approximate) sampling distribution of the sample mean, $\overline{x}$, based on various sample sizes. Please note that the horizontal axis is the same for all graphs, but the vertical axis is changing. For each sampling distribution graph, indicate the sample size that was most likely used for each sample: **2, 5, 15, or 30**. Please note that each sample size should be used only once as an answer.



b. What two facts about the sampling distribution of $\overline{x}$ helped you determine the answer to part a?

c. Give a reasonable value for the mean of the *original* population. μ = ____________

Figure 3. Conceptual question example

Since the assumptions for standard parametric tests for the mean were not met, the nonparametric Mann-Whitney test was used to analyze the median scores. For each assessment, the median scores were compared to determine if the SLT section median was significantly higher than that of the TLT section. The resulting p-values are presented in Table 3. Although the median quiz score for the SLT section was higher than that of the TLT section, this was not found to be statistically significant. However, the median scores for the SLT section were significantly higher than those of the TLT section on both the unit exam and the final exam.

Table 3. *Mann-Whitney Test Results*

Assessment	p-value
Quiz	0.0985
Unit Exam	0.0010
Final Exam	0.0122

The students in the SLT section who were actively engaged in hands-on use of the technology scored significantly higher on the unit exam and final exam questions about sampling distributions of the sample mean and the CLT than did those who were passively engaged in watching the instructor use the technology in the TLT section. This suggests that the SLT students better retained the information than did the TLT students.

To shed light on the students' performance in this course prior to the change in format for this particular lesson for the SLT section, students' scores on the previous two unit exams were compared. For each student, their two unit exam scores were averaged. Then, a Mann-Whitney test was conducted to determine if there was a significant difference in median scores for the two sections. These results indicated that there was not a significant difference between the two sections (p-value = 0.2308). Thus, one would not expect a significant difference in the performances on the questions related to the content being explored for this study on the three assessments unless something in the two sections was different.

Implications

The results of this research indicate that student-led technology has a greater impact on students' understanding of sampling distributions of the sample mean than teacher-led technology. While the same technology was used in both sections, the students who actively engaged in statistics by using the technology themselves better retained the content. Since both sections received the same review materials posted online and class time was not spent revisiting this specific content, one would assume that both sections would perform similarly on the final exam. However, there was a significant difference in their performance over this content on the final exam, where the only real difference was how the material for sampling distributions of the sample mean was covered. Further investigation revealed that while there was a significant difference on the content that was taught using SLT versus TLT, there was not a significant difference in overall final exam scores (Mann-Whitney p-value = 0.2848). This indicates that using student-led technology has the ability to positively impact students' performance in the both the short-term (i.e., unit exam) and the long-term (i.e., final exam).

While this small study advocates for the use of student-led technology, further research is needed to ascertain whether these positive outcomes were a result of just the student-centeredness, the combination of student-centeredness with technology, or the specific content studied. Given the importance for everyone to be statistically literate in today's society, steps should be taken to use instructional methods that foster retention of the concepts that are important for living in today's data-rich world.

References

Chance, B., Ben-Zvi, D., Garfield, J., and Medina, E. (2007). The role of technology in improving student learning of statistics. *Technology Innovations in Statistics Education*, 1(1). Retrieved from http://escholarship.org/uc/item/8sd2t4rr

GAISE (2005). *Guidelines for assessment and instruction in statistics education college report*, American Statistical Association. Retrieved from http://www.amstat.org/education/gaise/GaiseCollege_Full.pdf

Integrated Analytics LLC. (2015). StatCrunch – Data analysis on the Web [Software]. Available from http://www.statcrunch.com

Tishkovskaya, S. and Lancaster, G.A. (2012). Statistical education in the 21st century: A review of challenges, teaching innovations and strategies for reform. *Journal of Statistics Education*, 20(2). Retrieved from http://www.amstat.org/publications/jse/v20n2/tishkovskaya.pdf

Turner, S. and Dabney, A.R. (2014). A story-based simulation for teaching sampling distributions. *Teaching Statistics*, 37(1), 23-25.

Zerbolio, D.J. (1989). A "bag of tricks" for teaching about sampling distributions. *Teaching of Psychology*, 16(4), 207-209.