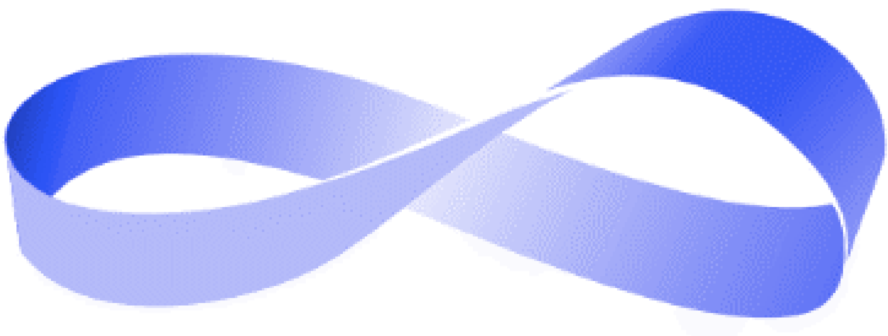
RCML

Proceedings for the 46th Annual Meeting of the Research Council on Mathematics Learning

Leading and Learning: Mathematics Made Accessible for All



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RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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Leading and Learning for Pre-Service and In-Service Teacher Support

Leading and Learning for Professional Development

Leading and Learning for Student Thinking

STUDENTS' PROPORTIONAL REASONING WITH THE PANTOGRAPH

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This paper presents the results of a pilot research study we conducted on how the use of a pantograph promotes seventh grade students' mathematical reasoning and argumentation on proportional relationships. The analysis of the data indicates students' ability to figure out how a pantograph enlarges or shrinks shapes in a certain scale factor, without having this knowledge before. They also found the relation between the scale factor and perimeter and area of rectangles. They estimated that the angles are preserved and verified that they are right. Researchers designed appropriate activities and used them to guide student's thinking process indirectly.

Euclidean geometry and geometric investigations have been a central focus in mathematics from the time of classical Greek culture. According to Healy and Hoyles (2002), although dynamic geometric environments help some students to move from argumentation to logical deduction, they impede some other students to use mathematical argumentations and solve problems. Vincent (2002) reports that the pantograph may facilitate students to explore geometric ideas related to proportional relationships. Mathematical tools allow students visually to connect figures or drawings with relations of geometric elements (through scale in a pantograph). This shows an understanding of the characteristics of proportional reasoning according to TexTeams (Shechtman, et al., 2006, April). The pantograph embodies mathematical properties and relationships as to allow the geometrical transformation, such as, symmetry, reflection, translation and homothety, according to Siopi and Koleza (2016). Students using such tools develop technical skills that allow them to express the kinds of reasoning used in the workplace. Leak et al. (2017) found that topics and tools relate to the curriculum of physicists, engineers, and technicians and can be modified to emphasize mathematical topics and tools needed for the 21st century workplace. This study will focus on how the use of a pantograph promotes proportional reasoning in geometric shapes.

Study Design

Purpose of the study

This pilot study was conducted to determine the viability of using the pantograph as a tool to support 12-15 years old students' proportional reasoning in geometry. Figure 1 shows a picture of a pantograph, a wooden or plastic device that students can manipulate to enlarge or shrink a given original shape. The three co-authors designed activities for four students that attended an after-school program and agreed to work with us for up to seven, 45-minute sessions. The research questions that guided our design, implementation and analysis of the data were:

1. How do students use argumentation as they engage in work with the pantograph?
2. How does the use of the pantograph support students' ideas about proportional relationships?

The four students paired according to similar abilities and worked in two teams. The sessions fit the characteristics of a teaching experiment articulated by Steffe and Thompson (2000) that is used so that researchers can experience, firsthand, students' mathematical learning and reasoning. All sessions were videotaped. The videotapes were transcribed, and pseudonyms used to protect the students' identity. Student artifacts were copied to add to the transcription analysis. The data were analyzed using the constant comparison method of Strauss and Corbin (1990).

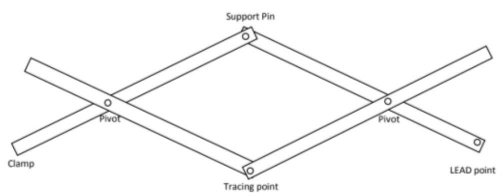


Figure 1. Pantograph

Participants

The pilot research study was conducted during the after-school program at a charter public school after agreement from the Board of Directors. In total seven 7th grade students were enrolled in this program. Four out of seven parents and students signed the consent forms, agreeing to participate in the study.

For these four 7th graders, we use the pseudonyms, Anders, Natania, Navid, and Eamon. Two researchers conducted the interviews, so the four students formed two teams. Anders and Natania formed the one team and Navid and Eamon the other team. At the beginning of the first session, all four students watched a video where a doctor conducted a surgery by viewing the organs of the patient through a digital screen that amplified the image so that he could move robotic arms to conduct the necessary procedures in more precise detail. Students stated their observations about this video, noting that a camera increased the size of the organs which allowed the doctor to make more detailed cuts. Then, the four students were paired according to similar abilities and worked in two teams with pantographs. Anders was currently enrolled in a mathematics course two levels above grade while the other three students were enrolled at grade level. Anders had already been introduced to proportional relationships in school, but the other three students had not. Anders and Natania worked as a team for a total of four sessions. Anders completed two extra sessions, one by himself due to Natania's absence and one in which he joined Navid and Eamon. Navid and Eamon worked together over seven sessions. Due to space constraints, we were only able to describe the reasoning of two students. We chose Natania and Navid because their reasoning was different than their peers and provides interesting findings. This paper presents narratives related to the concepts of *notions of scale factor* and *perimeter change.* In the study, ideas of *changes in angles* and *changes in area* were also investigated.

Findings

Natania

Natania decided to discontinue sessions after the fourth one. Although her contributions during those sessions indicate that she knew very little about scale factor and dilations prior to the project, she developed a multiplicative interpretation at the end, although unstable. In the first two sessions, notably Natania brought the surgical video context into the discussion as a realistic context for magnification (e.g., "*just like the picture [in the video], it got magnified x 2").*

Notions of scale factor. Natania was unaware of the term scale factor until introduced by Anders. This conversation occurred when they were allowed to explore the pantograph by tracing along a straight line on the paper:

> Natania: It's just like a heart tracker [EKG].
>
> Anders: It translates it! It duplicates it! It amplifies! Looks like it's making it larger! And that's the scale factor (points to the number 2 on the pantograph).
>
> Natania: That makes sense!
>
> Researcher: Are you sure it's 2?
>
> Natania: We can measure it (gets a ruler). Should we use inches or cm? Two inches. Here [on the enlarged shape] it is 4. I was right.

This dialogue indicates that although Anders had never used the pantograph, he was excited to discover that it dilates a line segment and even uses the term *scale factor* to describe the growth. Natania, for her part, is unfamiliar with dilations and scale factors, and, when challenged, decides to measure to determine whether the line indeed doubled its length.

Natania’s understanding of scale factor remained unstable as they continue to solve problems. When introduced to the Mystery Club problem (Figure 2 below), they created a poster 10 times larger; however, with a large scale factor, the pantograph is limited in movement. Natania claimed that the new poster would look like the long, thin rectangle in the middle of the pantograph created when they used a scale factor of 10 on the pantograph. This indicates that she did not yet see the scale factors as stretching all sides by the same amount.

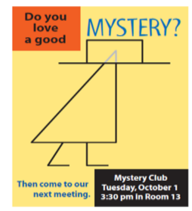
Michelle, Daphne, and Mukesh are the officers of the Socrates Academy Mystery Club. Mukesh designs this flier to attract new members. Daphne wants to make a larger poster to publicize the next meeting. She wants to redraw the club’s logo, “Super Sleuth,” in a larger size. Michelle’s dad is a carpenter and lends them a tool called the *Pantograph*.	

Figure 2. Mystery Figure Activity

As the sessions continued, Natania’s understanding of the impact of the scale factor on the original figure was that it doubled, tripled (multiplied by) the lengths of the sides to create the image. However, when a decimal scale factor was used, Natania’s understanding was shaken. When presented with an original Super Sleuth (see Figure 2) and an image of the Sleuth created by a 150% enlargement, Natania immediately used a ruler. When she measured the original hat length, she determined it was 6cm and the image was 9cm for a scale factor of “3 times.” Natania reverted back to additive reasoning when confronted with a decimal scale factor.

Perimeter change. During the second session, when presented with the Super Sleuth problem, Anders and Natania created an image using a scale factor of 6 on the pantograph. The researcher asked the students to explore how the perimeter changed, if any at all.

Researcher: What’s your first intuition?

Natania: My first is to get a ruler to see if…

Researcher: What’s your first guess? How her perimeter compares to super sleuth’s?

Natania: It’s gonna be, well it seems bigger.

Researcher: It’s definitely bigger. OK. It’s gonna take more [pencil] lead over there.

Natania: Since we said it was going to be about 6 times as bigger, it might take 6 times more? Natania, again grounded her intuition first in the physical act of measuring, but, when pushed, conjectured that the perimeter may be 6 times larger since the pantograph was set on 6. Anders, for his part, argued that the perimeter will be 12 times larger (double the scale factor). Upon measuring, they discovered that Natania's conjecture was correct.

Summary. In summary, for Natania, the pantograph was used as a tool for creating an image rather than a reasoning device. The ruler was a more dependable tool for exploring the relationships between the original and image figure. In fact, she repeatedly used the ruler to verify intuitions or uncertainties and did not provide mathematical reasons for her findings. For example, to justify why the perimeter should increase by the scale factor, she simply said that the shape got larger by the scale factor rather than coordinating the side length growth to the perimeter growth. The same can be said for angle measurement and area; Natania did not come to a stable understanding of those changes; relied on measuring as a way to verify these relationships and attempted to remember Anders' claims rather than understanding why they are true mathematically.

Navid

Navid attended all seven After School sessions. His contributions during those sessions indicated that he did not know anything about scale factor and dilations prior to the project but developed a strong understanding and interpretation of these concepts at the end without naming them. In the first session, Navid commented that the robot in the surgical video must be precise.

Notions of scale factor. During the first session, Navid drew a square and together with Eamon tried to trace it, producing a new one using the pantograph, for first time, without success. Then, Navid drew a line segment equal to 10cm, traced it with Eamon using the pantograph, and produced its image equal to 20cm. He observed that the line segment doubled:

Researcher: Do you think it will be easier if we start with something simpler?

Navid: like a line

Researcher: How long is the line? Could you measure it?

Navid: This here is 10cm (after measuring it using the ruler)

Researcher: And this here?

Navid: It is 20cm.

Researcher: So, what happened?

Navid: It doubled.

Navid's understanding of scale factor, and how the pantograph creates it, became clearer as they continued to solve problems. When introduced to the Mystery Club problem (see Figure 2), they first made the poster two times bigger and then three times bigger. Navid measured the original sides of the rectangle and of the right triangle, conjecturing that corresponding lengths in the produced picture are to scale, and measured to verify his conjecture. He was able to observe the points that move and the stable point in the pantograph, during the tracing-drawing process.

When presented with an original Super Sleuth (the shape in Figure 2 above) and an image created by a 150% enlargement, Navid measured the original hat length (4cm) and the image and determined that the image (about 6cm) is one and half times bigger.

Navid: So, this [pointing at the image] is almost one and a half bigger than that one [pointing at the original]. That's why we say 100 and 50.

After these measurements, they were able to set up the pantograph on 1½, trace the original, produce the new image, and Navid verify that the new image was 1½ times bigger.

Navid: This one, I took the ruler and I measured it and I got 3cm [the shorter side of the right triangle] so the same thing with this one and then I got 4. 4cm so, it is about 1½ times more.

The next feature Navid discovered is how pantographs produce images in scale. Navid and Eamon set up three pantographs, one on 1½, the second on 2, and the third on 3, creating a rectangle in the middle. By measuring the sides of these rectangles formed by the plastic sides of the pantograph, Navid observed that the lengths of sides of the rectangles on 1½ and on 3 are equal the other way around. Focusing on the stable point, the tracing point, and the drawing point (refer to Figure 1), Navid decided to measure these distances. He found that the distance from the clamp point to the tracing point was 14cm and from the tracing point to the lead point was 7cm (Figure 1). Then, he observed that 14 plus 7 is 21 and 21 to 14 is 3 to 2 which is equal to 1½. Therefore, Navid discovered that the ratio 3:2 guided the production of a 1½ scale factor.

The next question was how to shrink the size of the original shape by a scale factor of ½. Anders also joined this session. Anders set up his pantograph on 1½, Navid on 2, and Eamon on 3. Then, they traced the original picture. Observing all three images, Navid noticed that the 1½ times bigger image is half the three times bigger, but he was not able to answer the question. Anders observed that by switching the lead with the tracking point, the image shrinks and if the pantograph is on 2, the image is half of the original. Understanding what Anders explained, Navid conceptualized the process and described it below:

Navid: Put all of them on 2 and then switch these two [the lead and tracker].

Perimeter change. During the second session (Super Sleuth), Navid and Eamon created an image using the default scale factor of 2 on the pantograph. The researcher asked Navid to calculate the perimeter of the original rectangle and Eamon to calculate the perimeter of the image. Navid knew the concept of perimeter and figured out that it is 6cm although Eamon did not have a clear understanding of the concept of perimeter. Navid helped Eamon to calculate the perimeter of the image. Using the calculations, Navid observed that the perimeter doubled.

Researcher: Do you know how much is the perimeter of this rectangle [the original one]?

Navid: How much is the length? It is 2 so 2+1+2+1 so it is 6cm [he writes it down].

Researcher: Eamon how much is the perimeter of this one [the image]?

Eamon: The one side is 2 and the other 4

Researcher: So, what is the perimeter?

Eamon: We are timesing [sic]?

Navid: No, the perimeter is when you add all sides, 4 and …

Eamon: 10 [he writes down 4+2+4+2 and adds to get 12cm]

Researcher: So, Navid what do you think is the relationship of these two perimeters?

Navid: Since this [points at the original picture] no sorry, this was two times bigger [points at the image picture] so these forms [points at the formulas of the perimeters] show that the numbers are two times bigger of each other.

In the third session, Navid generalized the rule about perimeters saying that when the pantograph is on 2, the perimeter of the image rectangle doubles, on 3 it triples, and on 4 it is four times bigger than the perimeter of the original.

Summary. In summary, for Navid, the pantograph was not only a tool to create an image in scale, but also a reasoning device. He used the ruler as a tool to measure line segments and then was able to use mathematical reasoning to apply these measurements using generalized relationships. For example, to justify why the perimeter should increase by the scale factor, he applied the formulas from his prior knowledge. During the last session, he also explored more deeply the mechanics of a pantograph. He described clearly where we need to put the original shape to enlarge an image twice, three times, four times, and 1½ as well to shrink an image by ½. He developed a confidence about the use of a pantograph to create similar shapes without even knowing the work "similar".

Conclusion

This teaching experiment explored how the use of a pantograph as a tool supported four 12-15 years old students' proportional reasoning and argumentation in geometry. In regard to the research questions, data indicate that students can use the pantograph to develop proportional reasoning and argumentation if they can generalize from the physical act of measuring. Findings indicate that the pantograph can be used effectively as a reasoning device though students, like Natania, may only view the tool as a means for creating an image. Data analysis indicate that Navid was able to reason about how the pantograph enlarges or shrinks images as well how perimeter changes and may facilitate the development of ideas around similarity, through proportional reasoning, without formal development of this concept. However, Natania used the pantograph as a tool for enlarging an image, verifying it by measuring, using little proportional reasoning. Data indicated that during the teaching experiment, Navid's knowledge on proportional reasoning became more abstract while Natania stayed grounded in the physical act of measuring to justify her conjectures. Therefore, the teaching experiment underscored how additional scaffolding is necessary with some students to bridge their procedural views of measuring to proportional relationships using a measuring tool.

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PROBLEM POSING IN A UNIVERSITY DEVELOPMENTAL MATHEMATICS COURSE

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This paper presents a pilot investigation into utilizing problem posing with university developmental mathematics students to better engage with mathematical concepts. Pilot outcomes were measured through questionnaires, student artifacts, post assessment, and an observation journal. Initial findings indicate that problem posing, though challenging for the students, promoted student engagement with the content when personal interests were used. Engagement in problem posing also positively influenced the students' attitudes and beliefs towards the content and their mathematical capabilities.

Introduction

In fall 2015, total undergraduate enrollment in degree-granting postsecondary institutions was 17.0 million students, an increase of 30 percent from 2000 (NCES, 2017). These students are classified as either traditional or non-traditional students. Traditional students enroll at the university directly after secondary school while non-traditional transfer from another post-secondary institution, enroll part time, former military, or have a lapse in time prior to completing their secondary education. A large number of students, both traditional and non-traditional, are entering the university underprepared, especially in mathematics courses (Bader & Hardin, 2002). The National Assessment of Educational Progress (NAEP, 2015), found that only 25% of 12th grade students scored at or above proficient level on the NAEP mathematics assessment so it is not surprising that many of these students enrolled in remedial or developmental mathematics (DM) courses. DM courses are designed to help students increase their proficiency in content and be better prepared for university mathematics coursework (Boylan, Bonham, & White, 1999).

Student attitudes and beliefs are closely tied to student engagement, motivation, and achievement (Aiken, 1970; Alkhateeb & Hammoudi, 2006; Francisco, 2013). For DM students, their negative attitudes and beliefs began as early as elementary school (Aiken, 1970). Due to lack of early success, students' self-efficacy is affected negatively, and students become increasingly disinterested in mathematics, which decreases student motivation and disengages students from mathematics (Walkington & Bernacki, 2015).

DM coursework tends to focus on mathematical procedures. Though an essential part of the students' mathematical development, the students are exposed to the same content repeatedly, leading to disengagement. Many DM courses use word problems to help students engage with the content. However, many DM students do not find or see personal interest with these word problems.

Related Literature

Word problems are a familiar type of problem solving that provide a context for abstract mathematical tasks. Duan, Depaepe, and Verschaffel (2011) state that word problems are useful because they can motivate students and help develop logical thinking. The context in word problems allows students to draw on their lived experiences as they engage with the mathematics, which helps students bridge abstract conceptual mathematics to mathematics applied within a real-world context. However, research from practice indicates that word problems continue to be challenging to students of all ages (Ku & Sullivan, 2000; Walkington & Bernacki, 2015). Much of this challenge is mainly due to the traditional school curriculum, which emphasizes procedural understanding, at the expense of conceptual understanding (Garcia, Jimenez, & Hess, 2006). This focus on computational and procedural skills only allows for surface level understanding (Mji & Glencross, 1999; Schoenfeld, 1989), students pay little attention to the concepts and just try to get the correct answer. This is especially true in DM courses (Boylan et al., 1999; McCoy, 2005).

Ellerton (2013) notes that the formulation of the problem is more essential than its solution. Further, Silver (1997) found that instruction involving word problems and problem posing tasks assisted students in developing more creative approaches to the formulation of problems and understanding the mathematics behind tasks. Problem posing activities allows students to become active learners and provide opportunities to navigate the problems they pose within their areas of interests (Lavy & Bershadsky, 2003). Personalization of word problems contributes to making problems more motivating, and easier to construct a meaningful conceptual representation to relate problem information and solution strategies (Davis-Dorsey, Ross, & Morrison, 1991). Research on problem posing using personalization has shown to be effective as a way to increase student engagement, reduce anxiety, and improve students' attitudes toward mathematics (Lavy & Bershadsky, 2003). A number of research studies have demonstrated the benefits of problem posing on students' understanding of the mathematics (Walkington & Bernacki, 2015; Yee & Bostic 2014). For example, Ku and Sullivan (2000) performed a study with 72 fifth grade elementary Taiwanese students and found that students performed significantly better on the personalized problems based on the student area of interests compared to non-contextual problems.

While there is a significant amount of research on problem posing, few studies address problem posing in DM classes at the university level. Thus, the goal of this study is to engage DM students in problem posing through word problems that draw on their interests. The research questions guiding this study were: How do developmental mathematics students draw on their personal interests as they engage in problem posing? Further, what impact does problem posing have, if any, on their attitudes and beliefs towards mathematics?

Methodology

This research study was conducted during a five-week summer DM course in the southeastern U.S. The first author was the instructor of record for the course with four years experience teaching DM courses. The DM course covered the following concepts: fractions, linear expressions/equations, quadratic equations, rational expressions, algebraic problem solving, and word problems and problem solving. Students enrolled in this course where placed by either having below a 550 SAT, scored below the university placement test benchmark, or self-placed for additional algebra review. The participants were eleven post-secondary students – three females and eight males. There were six freshmen, three sophomores, and two juniors. Two students were classified as non-traditional students while the other nine were classified as traditional.

Data Collection and Analysis

Students submitted work throughout the fifth week and completed a two-part five point Likert scale questionnaire at the beginning and end of the fifth week of instruction. Part one of the questionnaire asked students to rate their agreement of statements about attitudes towards mathematics. A sample question on this scale was: *I do not like mathematics, and it scares me to take it.* The second portion of the scale asked students to rate their agreement with statements about beliefs about mathematics. A sample question was: *Being able to successfully use a rule or formula in mathematics is more important to me than understanding how and why it works.* The attitudes portion of the questionnaire was developed from Aiken (1970, 1972). The beliefs portion of the questionnaire was developed by Yackel (1984). Researchers (Cifarelli, Goodson-Espy, & Chae, 2010; Quillen, 2004) field tested this questionnaire and reported the survey as valid and reliable with a Cronbach alpha of .87. At the end of the questionnaire, students were also asked to list their personal interests outside the classroom. The student problem posing was conducted in week five of the course and built on the mathematical concepts from the previous four weeks. Though word problems were spread out over the entire course, the instructor decided to have a concentrated session around word problems in the fifth week to facilitate data collection. During this time, the students engaged in three scaffolded levels of problem posing - structured, semi structured, and free (Ellerton, 2013). The tasks within each level of problem posing were drawn from the work of Cañadas, Molina, and del Río (2018, Table 1).

Table 1

Examples of Problem-Posing Phases

Problem posing phases	Example: Given	Student Task
Structured	A student has a jar containing 65 coins, all of which are either nickels or dimes. The total value of the coins is $5.30.	Create as many problems using the same subject and values, but with different individuals.
Semi-structured	Consider the statements: $x + y = 65$ and $.05x + .10y = 5.30$.	Create as many problems using your personal interests involving currency
Free	Consider the statements: $x + y = 65$ and $.05x + .10y = 5.30$.	Create as many problems using personal interests based on the given constraints

An observation journal (journal) was maintained by the instructor to note ideas, insights, observations and interactions with students. The journal was used as a data source for triangulation during analysis. Student work was collected and summarized by a scaffold level matrix. An analysis of problems was completed where the problems were categorized based on phase of posing and the point in instruction at which the posing occurred. Students were asked to state personal interests at the beginning of the study. The problems that the students designed were compared to their stated personal interests (see Table 2).

Table 2

Examples of Problems Posed Based on Student Interest

Student	Student Interests	Posed Problem
Student 4	Hunting, fishing, my truck, running, cycling, swimming, camping, my dog and shooting	I got a sign on bonus from the air force (100k) and want to invest it. I decided I want to make $3k off the interest each year. If my interest rate is at 8% how much will I need to invest of my bonus. ---- P=37500 *Note: Student stated that they wanted to have funds to work on their truck.
Student 5	Baseball and anything sports related.	A baseball bat is marked down 20% the discounted price is $400, how much was the bat before the discount? --- The bat was $470.59

Student 8	Fish, Baseball, video games, watching tv, and I love to eat.	I want to buy 20,000 lures to turn around and sell. Roostertails and spoons are the most popular lures. I can buy roostertails for 7 cents a lure and spoons for 9 cents. If I have 1550 dollars, how many do I need of each?

Findings

The class of eleven students posed forty-three structured questions, seventy-eight semi structured, and ninety-two free problems. Of the problems posed, seven structured, three semi-structured, and fifty-six free problems were posed using personal interests. The largest grouping of posed problems within the free posing phase involved money or contexts where products/items were being purchased. Through informal interaction between the students and instructor recorded in the journal, students mentioned they felt more comfortable using money or purchasing as opposed to another context. From student conversations recorded in the journal, the context of money related to the students' daily lives more than the other contexts presented within the course. The context of money provided the flexibility to relate their individual interests as subjects of their posed problems (See Table 2).

In addition to being flexible and designing their own problems in the free problem posing phase, it was noted from informal conversations recorded in the journal that the students were able to correct their own mistakes in algebra and computation while verifying if the problem posed was "sound" or "made sense." For example, a student posed a problem and explained their problem, but switched their variables. In this case, the student was able to correct herself since she was working with a familiar context of money. The student stated, "it did not sound right so I rethought what I had written and switched to two things in the problem to make the numbers right and work [*sic*]." The class discussion helped other students in the course to focus their attention on when their problems did not have correct solutions, and how to address these occurrences. The student also added that she would not have tried unless it was with a problem that interested her. The personal connection was an important component. This helped reaffirm the use of personalization with their interests and reaffirmed the claims that problem posing assists students in becoming better problem solvers through increased engagement (Lavy & Bershadsky, 2003; Lesh & Jewojawksi, 2007). The journal provided better insight in not only on how the students used problem posing, but also how the students thought through problems.

Overall, within the questionnaires, the students' responses varied between positive, negative, and no change (See Table 3). A noted decrease between the pre and post results was seen for

Beliefs #10: Being able to successfully use a rule or formula in mathematics is more important to me than understanding how and why it works. Further, another noted decrease was for the Attitudes #17: I have never liked mathematics, and it is my most dreaded subject.

Table 3

Pre and Post Questionnaire Comparison

Question	Change between Pre and Post Questionnaire: Beliefs Section	Change between Pre and Post Questionnaire: Attitudes Section
Q1	-0.1	0.3
Q2	0.2	0.3
Q3	0.3	0.5
Q4	0.3	0.5
Q5	0.6	0.0
Q6	0.1	0.6
Q7	0.0	0.3
Q8	0.0	0.2
Q9	0.1	0.5
Q10	-0.7	0.4
Q11	-0.3	-0.5
Q12	0.3	0.7
Q13	0.1	-0.2
Q14	-0.3	0.5
Q15	-0.1	0.3
Q16	0.0	0.0
Q17	0.0	-0.8
Q18	-0.3	0.2
Q19	-0.1	0.2
Q20	0.2	0.2

Discussion and Implications

Initial conclusions that can be drawn from the study was that DM students prefer free posing within the problem posing structure. Students within the study openly provided more free posing problems compared to structured or semi-structured. They associated this preference for free problem posing as affording them multiple opportunities to connect with their personal interests, rather than the structured problem posing where they felt more constrained. Observing that the context of money was utilized through much of the problems posed by the students, such context aided in establishing word problems that students found more relatable, and in turn increased their engagement with mathematics. This engagement served to have the students focus more on the creative approach to problem creation and solving process rather than the outcome. As noted in the findings, students recognized when solutions were not viable, implying conceptual understanding. Additionally, students reported through informal conversation that they felt an increase in confidence and interest as they engaged in problem posing. For DM students engaging with problem posing, this shift from a solution centered focus, increased engagement with mathematics, and increased confidence, could improve their mindset to where these students feel they can be successful in a subject that many struggle.

Future Research

Additional research is needed to determine the impact that problem posing has on DM students' attitudes and beliefs and engagement with word problems. An implication for future research would be to conduct a quasi-experimental study in which differences in student engagement, attitudes and beliefs, and student proficiency outcomes could be explored. Such a study would further understanding of DM students' thinking and perceptions of mathematical content and themselves. Moreover, it would also aid in how to better serve students entering underprepared for post-secondary mathematics. Additionally, such research would provide opportunities for pre-service teachers to better prepare themselves for working with students.

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DEVELOPING MODELING CAPACITY BY EXAMINING MIGRANT MORTALITY ACROSS THE SOUTHERN U.S. BORDER

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In this paper we discuss findings from a summer program designed to promote problem solving, critical thinking, and mathematical modeling through the lens of workforce development. As a part of this program our student participants became interested in the context of immigration, so we developed sociocritical modeling tasks that could promote understanding of this context as well as develop the capacities of assumption making, data representation and variable identification. Our results found that students were able to advance in these skills through teacher scaffolding and open discussion.

Background

NCTM (2014) highlights that one of the effective mathematics teaching practices is to “elicit and use evidence of student thinking…to assess progress toward mathematical understanding” (p. 10). Particularly, statistics has the power to reveal understanding to our students in a way that allows them to become critical thinkers that can justify their assumptions and conclusions using data (Makar & Rubin, 2018). This article describes and illustrates a model of teaching that empowers students to reach statistically sound conclusions, develop a personal perspective of the United States-Mexico border context, and better understand hardships immigrants face while trying to cross the southern U.S. border.

To guide our work in helping students understand how statistical decisions are made in real-life contexts. We tasked students with posing their own statistical questions; and collecting, summarizing, representing and interpreting statistical data, all to develop an understanding of immigration issues between the U.S. and Mexico. In other words, students work with real-world problems by using mathematics as a language for understanding, simplifying, and solving these situations in an interdisciplinary method (Bassanezi, 2002). This paper offers examples to illustrate how students’ statistical thinking and the statistical problem-solving process involve the kinds of questions that can be posed in classroom discussions. In particular we set out to examine the following research questions: (1) *How can students’ statistical thinking and statistical problem solving be developed through sociocritical discussions?* and (2) *How can statistical data foster humanizing discussions around sociocritical issues promoting social justice?*

Literature Review

Our work with students draws on sociocritical mathematical modeling (Barbosa, 2006; Rosa, 2012) in an attempt to support learners in solving mathematical problems with a high degree of relevance to their daily lives. In adopting a sociocritical modeling perspective Barbosa (2012) indicates that this relies on addressing the role of mathematical models in society which provides support for understanding a current social situation. Barbosa characterizes mathematical modeling as a process where students take a culturally situated and contextual problem and investigate solutions via mathematics. Barbosa argued that modeling in school involves adopting this sociocritical perspective to promote not only learning but critique as well in order to better understand a social situation or context.

Rosa (2012) expands on this notion of sociocritical modeling by illustrating a method for implementation of sociocritical modeling tasks. He argues that teachers should support students in analyzing problems that surround them in order to promote social justice. Rosa argues for the use of relational discourse (Rosa & Orey, 2007) where all involved parties have rights and duties to evaluate the validity of arguments within a learning environment free of social and political domination. In this way it affords participants the ability to engage in discussion, resolve disputes and collaborate on specific problems relevant to their lives. Rosa (2012) continues by indicating that mathematical modeling is the means by which these local problems can be understood. Rosa characterizes mathematical modeling in this sense as the ability to analyze and interpret data, formulate and evaluate hypotheses, and determine the effectiveness of solutions.

Methods

Data from this study was drawn from the SMART Skills summer program facilitated by The PAST Foundation, an educational non-profit institution in Central Ohio and supported by the Workforce Development Board of Central Ohio. Within this program, authors worked with nineteen youth between the ages of 16 and 18 years old who were in foster care and had been identified as *at risk* by the state. Of these nineteen students, 10 identified as female and 9 as male. The scope of this program was to develop workforce skills through exposure to technology and problem solving through the lens of mathematics. We adopted a perspective of using the mathematical modeling cycle (Blum & Leiss, 2007) where we focused on developing competences across phases. This consisted of developing assumption-making, problem posing, data interpretation and data representation.

Across this program our instructional team, consisting of the first and second author of this paper, developed mathematical modeling tasks that focused on new business development, data interpretation and identification, programming and robotics. The intent was to transition from instructor generated to participant generated questions over the course of time. During the third week of the program, a news story was released indicating the mistreatment of migrant children at a detention center in northern Ohio. In hearing this news story our students indicated that they felt the desire to support these migrant families in their plights. In this light we opted to develop tasks around the context of immigration to help participants better understand the context of immigration and determine a way to support those in need in order to promote sociocritical modeling.

Humanizing Immigration and Migrant Mortality Rates

In adopting a sociocritical perspective in mathematical modeling, we set to support our students on investigating this issue of immigration. We began our exploration by asking students to research information regarding immigration into the United States across the southern border. In their research, our students determined that the primary point of entry was through the Sonoran Desert of Arizona. In contextualizing the problem, as a class we watched the documentary "Who is Dayani Cristal" which follows one migrant's unsuccessful journey across this desert region. Following this documentary, we conducted a notice/wonder with our students. The primary consideration upon viewing was learning more about what major factors inhibited these migrants such as Dayani as they engage in their perilous journey. We then asked that our students explore a website (http://humaneborders.info), that tracks migrant mortality across the desert, to determine the primary cause of death in migrants crossing that region. The Humane Borders website uploads data on a quarterly basis and offers search tools allowing users to query data concerning migrant deaths, view the data using on-line maps and tables, as well as download the data for future use.

The data on migrant mortality is presented in four main categories: gender of migrant, year of death, cause of death, and location where the migrants were found. In our program we asked students to explore the website, collect data, identify any patterns or trends within the data, and determine the primary cause of death. In their research our students determined the primary cause of death to be linked to exposure to the sun. Humane Borders itself offers migration data ranging from 1981 to 2018, and users can select different parameters for the data. Data can be presented on a migrant mortality map, as a bounding box spatial search map, or as a case number-based spatial search map. Each map displays a dot for each individual's remains found and primary cause of death, if able to be determined. If cause of death is unable to be determined these bodies are labeled as being *Skeletal Remains* or *Unknown.* Given these findings we developed the Water Station Task in order to respond to their data-collection research.

> *Water Station Task:*
>
> *Given the primary cause of migrant mortality across the Sonoran Desert is due to exposure to sun and dehydration, how many water stations would be sufficient to minimize migrant mortality?*

Student participants spent the next 5 days investigating this question and determining a suitable model to respond to this posed problem.

Data Results and Analysis

As our students engaged in analyzing the migrant mortality data, we observed growth in their modeling capacity, in particular in identifying variables that impact the solution and in making assumptions, and interpreting relevant data, all of which are aspects of mathematical modeling (Lesh, Hoover, Hoyle, Kelley & Post, 2010). As students navigated through these tasks, we sought to observe those ways that learners engaged in examining and making sense of data, in particular their data collection protocol. Our analysis consisted of transcribing all student work sessions into line-by-line utterances, and conducting a fine-grained discourse analysis (Bloome, Carter, Christian, Otto, & Shuart-Faris., 2010) seeking ways that those aforementioned capacities in mathematical modeling were revealed and advanced.

As students worked, the facilitator asked students to share their findings from the investigation of the Humane Borders website. While the website contained data spanning from 1981 to 2018, our participants opted to use from 2007 to 2018 as a range for analysis as this point lies *in the middle* of the data window. For example, students said "because it was from 1981 to 2018, [2007] was in the middle, so we [wanted to choose the middle]" (Student 6). With respect to the range, examining from 2007 to 2018 offers a recent snapshot of immigration trends, of which these students reported graphically (Figure 1).

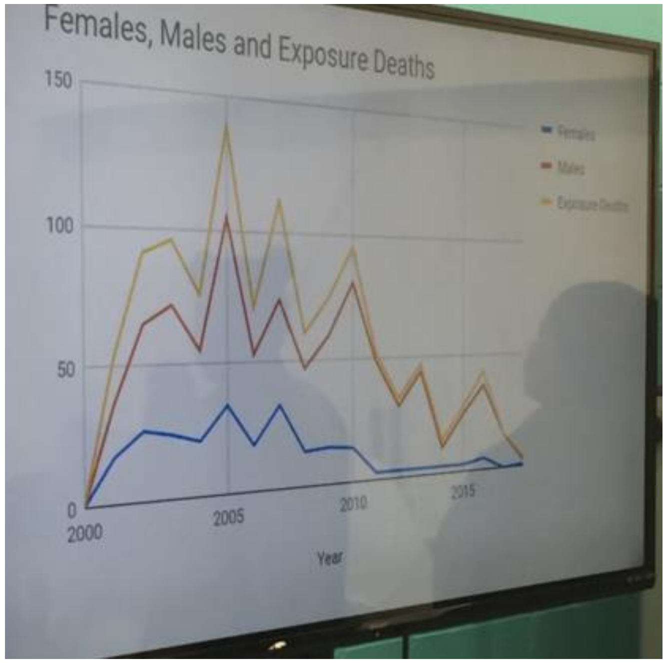

Figure 1. Females and Males Exposure Deaths by Year

Their research uncovered that the age range of individuals attempting to cross the border spans from 19 to 63. When provoked by the facilitator to make an assumption as to why we see this age demographic, our participants indicated that people as young as 14 attempt to cross, which was information retrieved from the documentary and the website, but that this range stemmed from the dangerous of crossing of individuals at extreme ages, i.e. younger than 19 and older than 63 as they would be more susceptible to the elements. In particular students indicated that "people start moving from their home town country when they're at the age of 14, and some people might not want their grandma, their grandfather, or their younger siblings to try to go in the heat to get ways from where they're from, cause they can die quicker" (Student 11).

Barriers that our participants faced were that of interpreting data, primarily due to the vast number of tracked deaths of different types. Many of our students selected particular causes of death and reported on their found results. Primary causes reported were gunshot death, drowning, and exposure to the sun (Figure 2). Each group was then asked to share their findings in a culminating discussion.

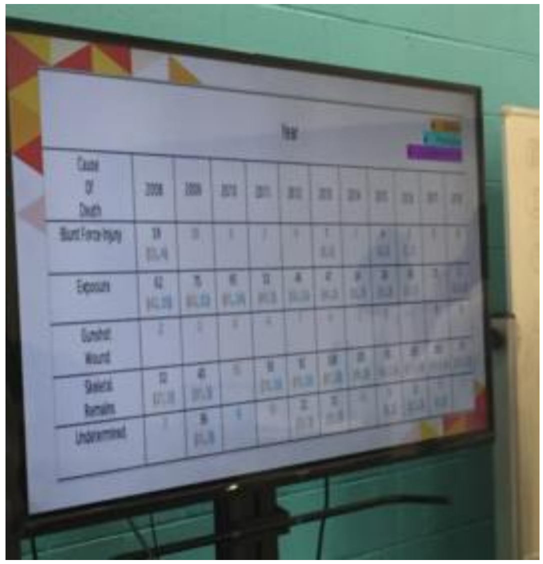

Figure 2. Mortality Frequency by Cause

Through teacher scaffolding and discussion comparing datasets the class as a whole determined the primary cause of migrant mortality was due to exposure to the sun and were then encouraged to make assumptions as to the underlying cause. Again, this wasn't immediately obvious to the students as they openly investigated the data set, however it was through instructor scaffolding and group discussion that yielded learners to determine this primary cause, and the task to be advanced.

Discussion

It was the openness of the instructional team to engage in student generated tasks, and initial goals of developing modeling capacity in learners that afforded our implementation of sociocritical modeling tasks. While we were fortunate not to be constrained by curricular mandates and particular mathematical content, the reality is that in order to effectively engage in sociocritical modeling the need exists to develop a perspective on teaching with modeling at the heart. Gutstein (2006) argues that if students are unable to understand the underlying mathematics of a social issue, that there will be some areas of social justice that they will not be able to completely understand and analyze. In this sense, careful attention to both contextual and mathematics learning is essential in order to provide students with the knowledge needed to "help recognize oppressive aspects of society so that they can participate in creating a more just world" (p. 6). While Common Core Standards (2010) outline the importance of modeling, in particular at the high school level, with ill-defined parameters around developing modeling capacity it is unlikely that tasks of this nature get realized in schools. Standards for modeling need to be further developed that incorporate particular aspects of the modeling cycle, and teachers need to be empowered to facilitate tasks through a sociocritical lens.

Further, particular attention needs to be paid to supporting students in data collection, analysis, and in making assumptions within sociocritical modeling tasks in order to advance instruction in mathematical modeling. In looking at furthering student understanding of the plights that migrants face when attempting to cross the desert, the instructional team had to do careful research and planning in order to find an entry point in this context that would be suitable to the population in which we were working. Additionally, our instructional team had to reconcile multiple data-based investigations and contextual assumptions to land at a common framework capable of advancing the task. Further research is needed in this area.

There was some disagreement in the context of immigration itself across the student group, in that not all members supported the undocumented entry of immigrants into the United States. Our team adopted the stance that regardless of the outcome of those individuals once they entered into the United States, (i.e. assimilation or arrest) we believe that all human beings have a right to live, and that those attempting this perilous journey should have the best chance of survival until they encounter aid. This stance was able to alleviate contrasting view points and better allowed the group as a whole to enter into the task. This brings light to the important point that if we are to facilitate sociocritical tasks with learners that have ties to political ideologies, a need exists to plan for discussions with multiple views and consider ways to humanize victims or issues so that students can work together across differences and engage in the task without defeating other viewpoints.

Lastly, across our data we observed qualitative growth in student assumption-making. At the start of our program students were reluctant to make assumptions and had difficulty getting into the modeling tasks. This we attributed to the presentation of mathematics historically as being solving known problems through procedural mimicry. However, through repeated exposure to open tasks that required assumption-making and identification of variables, students were able to develop in this area. Thus, we argue that deliberate and explicit attention needs to be paid in this area of modeling, and additional research is warranted in further advancing the mathematical modeling process with learners.

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CHILDREN'S CONVEYED MULTIPLICATIVE MEANING ACROSS MODELS

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Evidence suggests that visual models used to represent multiplication are not always interpreted multiplicatively. This study examined elementary students' strategies across different models. Findings suggest students will count by 1s when a model illustrates all ones. By contrast, models illustrating composite units elicited a wider variety of strategies for the multiplication tasks.

Overview & Purpose

Multiplicative reasoning is a foundational concept that facilitates reasoning in later mathematics. In order to teach multiplication and division effectively, many researchers have advocated for the use of different models (Anghileri, 1989; Bolden, Barmby, Raine, & Gardner, 2015; Dabic & Milinkovic, 2015) including arrays, groups-of, number lines, length, area models, and so forth. Complicating such recommendations are findings indicating that some variations of models used to represent multiplication and division are treated by children in ways that are not multiplicative (Anghileri, 1989; Bolden et al., 2015). As noted by Dabic and Milinkovic (2015), children often have "significant difficulties in interpreting representations produced by teachers. They tend to assign meaning unlike the one attempted by teachers" (p. 106). The purpose of the present study is to investigate the mathematical meanings conveyed by elementary students across different models.

Background Literature & Theoretical Perspective

Children's Multiplicative Reasoning

The current study examines evidence of children's unit coordination as an indicator of their multiplicative reasoning on particular tasks. The study is informed by Hackenberg (2010) and other researchers' description of multiplicative concepts (Boyce & Norton, 2016; Kosko, 2018). Thus, children who demonstrate *emergent multiplicative reasoning* (EMR) can coordinate one level of unit in activity. For example, a child with EMR can count by 1s to find the composite number 5. When a child is able to consider a composite number as an object that can be mathematically operated on, they demonstrate an initial multiplicative scheme and have constructed the *first multiplicative concept* (MC1). Students with MC1 can coordinate two levels of units in activity, by anticipating the first level of units. A student with MC1 can anticipate 5 as a unit of five 1s (i.e., they do not need to construct 5 as a unit to use) and skip count 5 six times to find a total of 30. A student that has constructed the *second multiplicative concept* (MC2) anticipates two levels of units to coordinate three levels of units in activity. Tasked with solving 18×5, a student with MC2 may use a known fact such as 10×5 and skip count by 5s eight more times to find the solution of 90. By contrast, a student that has constructed the *third multiplicative concept* (MC3) may solve 18×5 by recognizing there are two 9×5s, and doubling 45 to find the solution of 90. Thus, students with MC3 anticipate three levels of units (i.e., 18×5 includes 2×(9×5), where 9×5 includes nine composite units of 5). The current study uses these descriptions of unit coordination in describing evidence of children's multiplicative reasoning in reference to specific representations.

Consistency of Children's Strategies Across Representations

Various researchers have found that children sometimes use different strategies when provided different representations or models of multiplication and division (Anghileri, 1989; Bolden et al., 2015). Interviewing children on single-digit multiplication tasks using arrays, groups-of, and number line models, Anghileri (1989) noted that "very few children…used the same strategy successfully to solve all six tasks" (p. 383) and that such differences may be explained by students' varying interpretations of the tasks. Applying a different approach, Bolden et al. (2015) used eye-tracking software to examine nine fifth grade students' multiplication strategies with arrays, groups-of, and number line representations. They found that attention to iterating groups (i.e., constructing a composite unit using equal sized subunits) within representations was more frequent in the groups-of and array representations than number lines, but none of these frequencies were higher than 50%. Additionally, students' attention in number lines tended to focus on the total number instead of the partitions on the line itself. Findings from studies such as Anghileri's (1989) and Bolden et al.'s (2015) have led various researchers to suggest that students use different schemes when provided different representations. However, such inferences are restricted by the limitation that many of the representations used in these comparison studies differ significantly from one another. For example, Bolden et al. (2015) included groups-of and array models in which a discrete unit of 1 was visible. However, the number line used for comparison did not include visualized units of 1 (i.e., intervals of 1). The representations used by Anghileri (1989) differed in regard to the mathematics conveyed in the task. For example, one task involved considering 4 as a factor of 24 after examining 2 and 3 as factors on a number line, while another task asked students to represent five groups of 3 with colored cubes. Although both tasks relate to multiplicative reasoning, they may relate to different aspects and thus cause difficulty in comparing students' interactions with different models and representations.

Given prior research comparing students' interactions across different models of multiplication, there is a need for more explicit study of this phenomenon. The present study sought to answer the following research question. Do differences in the representations used to model multiplication tasks elicit different strategies to solve them? In the following sections, an approach is described to attend to this purpose, with particular attention to examining differences in the models themselves.

Method

Sample and Measures

This study focuses on a subsample of 55 third and fourth grade students' written work in response to five multiplicative reasoning tasks. Students were enrolled in an elementary school in a small suburban town in a Midwestern state. Participants were part of a larger sample of a study investigating how elementary students responded to multiplicative reasoning tasks that incorporated different models. The larger study uses Rasch modeling to statistically compare the items based on response data, while the present study focuses on analysis of students' written work. Participants completed two test forms a week apart from each other in May 2017. Altogether, the two forms included 19 length model items, 10 set model items, and 11 area model items. Length model items were statistically validated in prior studies (Kosko, 2018; Kosko & Singh, 2018), and the set and area model items were written using the same design guidelines. Specifically, in each item, students are presented with a given quantity in reference to another quantity they are tasked with determining. Item difficulty was differentiated using Hackenberg's (2010) description of multiplicative concepts (Kosko & Singh, 2018).

For purposes of the present study, five of the 40 items were purposefully selected to compare students' written work across items (see Figure 1). Common across all items was that each included the potential for interpreting the given representation as modeling multiplication with the operand 3. For example, Area_01 provided a given unit of 1, but the unknown quantity could be interpreted as 3×8. Likewise, Set_05 can be interpreted as representing 3×4. Set_03 and Length_07 can represent 3×6 and 3×5, respectively, while Length_08 can be interpreted as representing 6×3. Thus, if interpreted through a multiplicative lens, the items share 3 as a common operand, but represent multiplication with this operand differently.

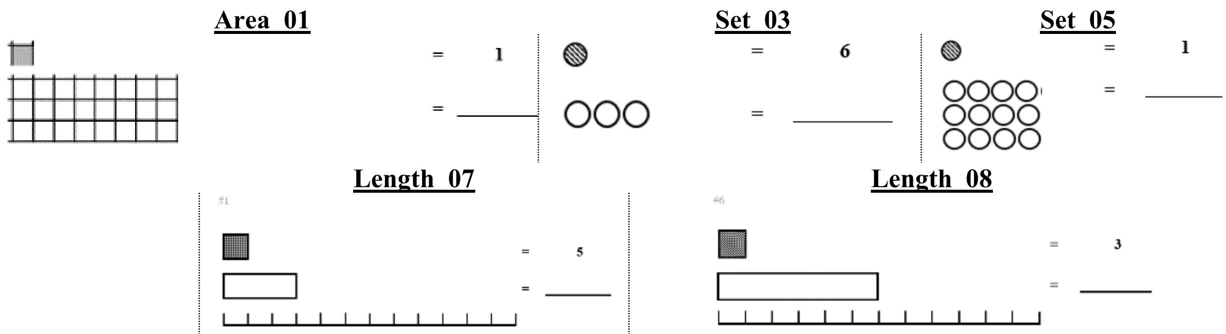

Figure 1. Items used for comparison of strategies, with item designations.

Analysis and Results

In the present study, students' written work was analyzed in terms of reasoning displayed in representing the unknown quantity in reference to the given quantity. Five themes emerged across the items that describe the conveyed meaning: (1) the unknown quantity as a count of ones, (2) the quantity as double/half of the given quantity, (3) the quantity as an iteration, (4) the quantity as an iteration of the given, and (5) the quantity as a multiplicative expression relating to the given quantity. These themes seem to align with the hierarchical stages of the multiplicative concepts previously described and thus, were converted to an ordinal variable with the theme numbers corresponding to the rank. When the written work indicated the first theme (the unknown quantity as a count of ones), this suggests that the student was operating at EMR. The second theme (quantity as double/half of the given quantity) also aligns with EMR but has been identified by prior research as a scheme that facilitates later development of multiplicative reasoning (Empson & Turner, 2006). The third and fourth themes (quantity as an iteration and quantity as an iteration of the given) indicate reasoning associated with MC1, as defined by Hackenberg (2010), and corresponding with the described development of these actions. Theme five (the quantity as a multiplicative expression relating to the given quantity) is also considered as a child demonstrating reasoning at least at MC1 but was considered an abstracted conveyance of such reasoning. In this manner, it was placed highest in the ordinal code. Using the data-driven ordinal coding scheme, the written work was independently coded by each author as described in the following section. A weighted Kappa statistic of .865 was calculated, indicating near-perfect agreement (Landis & Koch, 1977). Disagreements in coding were then reconciled.

Quantity as a count of 1s was inferred when a child's displayed work illustrated pencil markings on individual units or by numerically labeling each unit, suggesting that each unit had been counted. The written markings, in conjunction with the answers, conveyed counting by 1s regardless of whether the stated answer was correct. *Doubling/Halving* was conveyed when the student work showed variations of doubling or halving of the given quantity by partitioning/iterating the illustrated quantity by two, or symbolically in conjunction with the stated answers. *Quantity as an iteration* was inferred when the student work indicated that skip counts were attempted but were either incorrectly applied (i.e., 3, 6, 9, 10, 11), or too many/few counts were included (i.e., using 3 or 6 counts of 3 when solving 5x3). In length models, this was indicated by a misuse of counting on the comparison scale. *Quantity as an iteration of the given* was inferred when the student work indicated visual units named with the iterated unit (i.e., 6, 6, 6), when visual units were named in coordination with skip-counts (i.e., 6, 12, 18), or when length and width (in area models) were identified in a way that accounted for the rows or columns iterated. *Quantity as a multiplicative expression* was inferred from student work, which showed explicit use of multiplication or division with a written expression applied to the task.

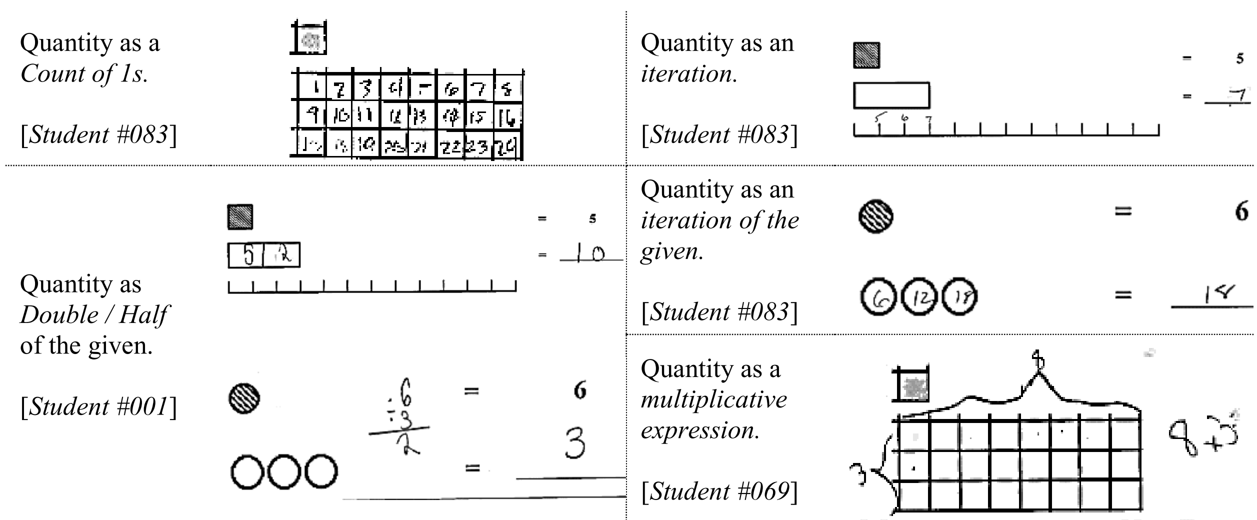

Figure 2. Illustrative examples of three students' written work.

One trend observed across the five themes was that certain ways of conveying mathematical meaning were more prevalent with certain items. Specifically, 90.0% of written work from Set_05, and 83.9% of written work from Area_01 conveyed the unknown quantity as counts of 1s. The prevalence of this theme was much less frequent for Set_03 (17.6%), Length_07 (27.0%) and Length_08 (41.7%). To assess the consistency of students' strategy use, the themes were converted into ordinal data (the theme number coincides with the ordinal value). Spearman Rho correlation coefficients were calculated to examine the relationship between codes across these three latter items (Crocker & Algina, 2006). Results suggested strong relationships between written strategies for Set_03 and Length_07 ($\rho=.83$, $p<.001$), Set_03 and Length_08 ($\rho=.77$, $p=.002$), and Length_07 and Length_08 ($\rho=.80$, $p<.001$). However, students' strategy used within Set_05 and Area_01 did not have statistically significant or meaningful correlations with any other items.

Discussion

Given the observed statistical trends, it appears that students' consistency in written strategy was contingent upon the manner in which multiplication was represented for the items examined. Specifically, Set_05 and Area_01 each includes visualized units of 1, whereas the other three items included visualized composite units. Therefore, a preliminary finding from the present study is that when units are visualized in a similar manner, students' multiplicative strategies are more likely to be consistent. This suggests that prior researchers' observations of varying student strategies (e.g., Anghileri, 1989; Bolden et al., 2015) may be due primarily to differences in the representations.

The present study not only supports earlier findings (Anghileri, 1989; Bolden et al., 2015) that variations in models may be interpreted in ways that are not multiplicative, but also provides evidence in support of Kosko's (2018) extension of those findings that *when representations include visualized units of one*, such representations may or may not elicit multiplicative reasoning. For example, the representation of Set_05, which provides a given unit of one and a three by four array, could be interpreted as three times four and could be solved using a variety of strategies. In particular, it could be solved by iterating a row or column composite unit, which aligns with multiplicative reasoning at the level of MC1 (Hackenberg, 2010). In the present study, however, this representation elicited a count by 1s strategy in 90% of the cases, which demonstrates reasoning at the EMR level (Hackenberg, 2010). Specifically, even though a child may be capable of explaining the multiplicative structure of the item, they may elect to use a simpler strategy because the item does not require a multiplicative strategy to solve it (Kosko, 2018).

In contrast with the representations that depicted discrete counts of 1s, the relationships between the three representations that presented visualized composite units as the given quantity

(Set_03, Length_07, and Length_08), identified by the Spearman Rho statistics, support findings of prior scholars who suggested students will more often utilize strategies other than counting by 1s when discrete counts of one are not visualized in the representation (Downton & Sullivan, 2017; Kosko & Singh, 2018; Kosko, 2018). For example, the representation of Length_07, which presents a composite unit of 5 as the given quantity and can be interpreted as 3 x 5, can be solved using counting strategies, but it elicited a counting by 1s strategy in only 27% of the cases, which may indicate that when the familiar and simpler strategy (counting by 1s) is not readily available, other strategies will be attempted more frequently.

Findings from this study suggest the way that unit is visually conveyed may affect how students interact with a mathematical representation. One implication of this finding is that practitioners should be attentive to the representations they use with students, as some representations may not be eliciting the multiplicative reasoning that teachers intend. Researchers studying children's mathematical reasoning should also exercise caution as data collected through use of certain representations have the potential to be misconstrued in terms of the multiplicative reasoning they elicit (Kosko, 2018).

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THE IMPACT OF MATH TEACHERS ON STUDENT LEARNING AND MOTIVATION

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This study examined the degree to which mathematics teacher qualifications, characteristics, and practices influence high school students' motivational beliefs about mathematics and mathematics learning (assessed by 11th grade mathematics achievement). A nationally representative, large-scale data set—the High School Longitudinal Study 2009 (HSLS:09) was used to conduct hierarchical regression analyses. After controlling for student demographics, results indicated that the degree to which teachers emphasized the development of deeper conceptual understanding of mathematics was a predictor of students' mathematics achievement, identity, and self-efficacy whereas the degree to which teachers emphasized the utility of mathematics predicted students' beliefs about the utility of mathematics.

Introduction

Research has uncovered the vital role that K-12 teachers play in students' academic outcomes (e.g., Blanchard & Muller, 2015). However, little research has focused on the degree to which the characteristics, qualifications, and instructional practices of high school mathematics teachers, particularly ninth grade mathematics teachers, have an effect on their students' motivation and learning in mathematics as they near graduation. Therefore, this study will attempt to fill this gap in research by investigating the role that ninth grade mathematics teachers have on high school students' mathematics learning and motivation towards the end of high school by using a national data set.

Theoretical Frameworks

This study is grounded in two distinct but related frameworks: Lent, Brown, and Hackett's (1994) social cognitive career theory (SCCT) and Goe's (2007) teacher quality framework (TQF). The two frameworks are integrated to understand the extent to which both student level and teacher level factors central to each theory shape students' STEM outcomes (Hattie, Masters, & Birch, 2016; see Figure 1). The two frameworks complement each other by highlighting teacher quality as a contextual (environmental) factor in understanding students' STEM outcomes. Guided by these well-established theories and prior research on student academic outcomes at the secondary level and by utilizing a large-scale data set for analysis, this study enhances our understanding of the relation between teacher quality and student outcomes related to STEM, and specifically both mathematics achievement- and motivation-related outcomes.

Social cognitive career theory (SCCT). SCCT posits that one's career choice is influenced by beliefs an individual develops and refines through the complex interaction among the individual, environment, and behavior (Lent et al., 1994; Yu, Corkin, & Martin, 2016). According to SCCT, the most important factors influencing career decisions relate to student motivation (i.e., task value, self-efficacy, interest, outcome expectations). Individuals' behavior and actions are influenced primarily by their sense of personal capability (self-efficacy; Bandura, 1986), their beliefs about the likely consequences of performing particular actions (outcome expectancy; Bandura, 1986; Lent et al., 1994), and the extent they find certain academic domains useful (utility value; Eccles & Wigfield, 2002) and/or interesting (interest/intrinsic value; Eccles & Wigfield, 2002). Empirical research has shown that students with higher math and/or science self-efficacy, outcome expectations, and value for engaging in math and science are more likely to persist and be successful in these areas (e.g., Andersen & Ward, 2014).

In addition to personal motivation, the SCCT framework recognizes several contextual factors including socializing agents such as parents and teachers that influence a person's academic and career aspirations and choices (Yu et al., 2016). Teachers, however, have been found to be the most significant contextual factor accounting for student achievement (Hattie et al., 2016). SCCT mainly focuses on learning experiences (e.g. perceptions of their past performance and vicarious learning experiences) that are sources of self-efficacy (Navarro, Flores, & Worthington, 2007). SCCT does not particularly focus on the role that teacher qualifications, characteristics, and practices have on students' learning experiences. The TQF supplements SCCT by broadening its conception of learning experiences to include a more specific understanding of the teacher characteristics, qualifications, and practices that inherently affect K-12 learning experiences, which in turn, may influence students' academic outcomes.

Teacher quality framework (TQF). The TQF (Goe, 2007) provides the most comprehensive framework to date based on a review and synthesis of research regarding the impact teachers have on student achievement-related outcomes. TQF comprises three strands that are distinct but interrelated: inputs, processes, and outcomes. Inputs focus on two different but related ways of looking at teacher quality: teacher qualifications and teacher characteristics. Teacher qualifications include teachers' degrees, coursework, and grades in higher education as well as teacher preparation routes, certification types, years of experience, and continuing education such as internships, induction, coaching support, and professional development (Goe, 2007; Rice, 2010). TQF also conceptualizes teacher quality as encompassing soft attributes (teacher characteristics) such as subjective judgements, organization skills, critical thinking skills, and attitudes and beliefs (e.g., self-efficacy, beliefs about teaching and learning; Pajares, 1992). The processes strand of the teacher quality framework focuses on factors related to teacher practices—i.e., what teachers actually enact in the classroom including instructional practices and classroom management practices. This study will be guided by the first two strands of the teacher quality framework (teacher qualifications and characteristics and teacher practices) and not the outcomes strand because this strand attributes teacher effectiveness to students' achievement test scores, which has received much criticism (i.e., Darling-Hammond, 2016).

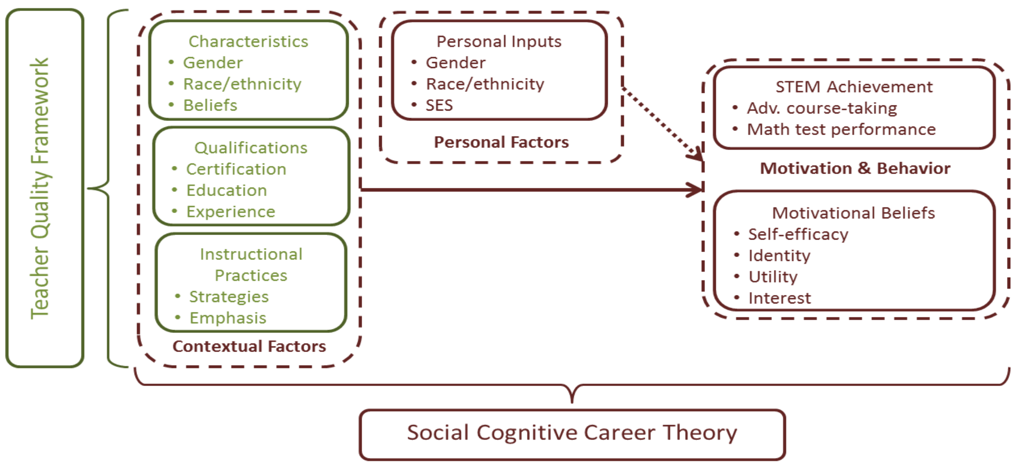

Figure 1. Conceptual framework explaining the connection of TQF and SCCT

Research Questions

1. To what extent do 9th grade math teacher characteristics, qualifications, and instructional practices contribute to high school students' math achievement and motivation?
2. To what extent do 9th grade math teacher characteristics, qualifications, and instructional practices contribute to high school students' math advanced course-taking behavior?

Method

Data Set. HSLS:09 is a study of more than 23,000 ninth grade students as of 2009. Conducted by the Institute of Education Sciences, HSLS:09 includes demographic information and survey responses from nationally representative students and their ninth grade mathematics teachers.

Variables. Student demographic information included gender (binary), underrepresented-minority (URM; African American, Hispanic, American Indian, and Native Alaskan)-status (binary), and socioeconomic status (continuous composite of several indicators; Ingels et al., 2011). Student motivation outcomes (self-efficacy, identity, utility, and interest) were continuous variables measured by several related items that are reliable and validated (Ingels et al., 2011). Mathematics achievement variable was the standardized theta score for a mathematics test taken by all the participants. Student demographic, achievement, and motivation data were retrieved from follow-up data collection cycle (11^{th} grade). The other achievement-related student outcome was advanced course-taking data retrieved from high school transcripts and coded as 1 if students had completed any AP, IB, or dual-credit mathematics courses and 0 if none. Teacher variables were retrieved from base year data (ninth grade) and included students' ninth grade mathematics teachers' demographic characteristics (gender—binary, and URM-status—binary), high school teaching experience (years), mathematics teaching certification (binary—standard vs. alternative), mathematics teaching self-efficacy (continuous composite variable), and mathematics degree (binary—undergraduate/graduate vs. none). The two teaching practice variables included in the study were teachers' emphasis on developing students' deeper conceptual understanding of mathematics (understand) and teachers' emphasis on developing students' interest in mathematics and an understanding of the utility of mathematics (connect). These two variables emerged through the factor analysis of several teacher practice variables asking teachers, for example, how much emphasis they were placing on (in their fall 2009 math course) "*teaching students to reason mathematically*" (understand) and "*teaching students how to apply mathematics in business and industry*" (connect).

Analytic Techniques. First, hierarchical linear regression analyses for continuous outcome variables (e.g., mathematics performance and motivational beliefs) were conducted. Second, binary logistic regression analysis for the advanced mathematics course-taking behavior was completed. The complex sampling design of HSLS:09 required the use of weights and design effects to properly calculate standard error terms for each variable (Ingels et al., 2011). In essence, the use of weights and design effects in a sample allows generalization of the results of statistical models to a wider range of the population (whole high school students in the U.S. in this case) and was a critical step in developing causal hypotheses and inferences. Appropriate BRR weights were incorporated in all analyses using STATA.

Findings

To answer the first research question, a series of hierarchical linear regression analyses were conducted predicting mathematics achievement and four motivational beliefs pertaining to mathematics. The motivational beliefs selected as outcomes are predictors of STEM achievement and persistence according to SCCT theory and research (see Yu et al., 2016). Table 1 presents hierarchical linear regression and logistic regression analyses results.

Table 1

Summary of Hierarchical Linear Regression Analyses (Predicting Mathematics Achievement and Motivational Beliefs about Mathematics) and Binary Logistic Regression Analysis (Predicting Advance Math).

Variable	Achievement β	Self-efficacy β	Identity β	Utility β	Interest β	Advance math[a] $Exp(\beta)$
Step 1						
Male	.01	.11***	.09***	.04***	.02*	0.92*
URM	-.13***	.04***	-.01	.05***	.06***	0.53***
SES	.38***	.12***	.11***	.02**	.06***	1.30***
R-square	.18	.02	.02	.01	.01	.04[b]
Step 2						
Male	.01	.10***	.09***	.04***	.01	0.84**
URM	-.11***	.06***	.00	.05***	.05***	0.56***
SES	.36***	.12***	.11***	.02*	.08***	1.28***
Teacher male	-.02	-.01	-.01	-.01	.01	0.86**
Teacher URM	-.04***	-.01	-.02*	.01	.00	0.95
Teacher self-eff.	.01	.03**	.02	-.00	.03*	1.01
Teacher cert.	.03**	.01	.03**	-.01	.01	1.26**
Teacher degree	.03**	.00	-.02	-.01	-.02	1.18**
Teacher exp.	.05***	.01	.04***	.02	.01	1.02***
Understand	.14***	.03*	.06***	.00	.02	2.32***
Connection	-.02	.02	.00	.04**	.03	1.16*
R-square	.21	.03	.03	.01	.01	.07[b]

Note. n = 18,600. β indicates standardized regression coefficient. *Exp(B)* is the odds ratio for the logistic regression. [a]Binary logistic regression. [b]Pseudo R-square for binary logistic regression. $*p < .01$ $**p < .01$. $***p < .001$.

In the first step of the regression analyses, personal student demographic variables were entered, followed by entry of teacher characteristics, qualifications, and instructional practices. Given the brevity of this report, we only highlighted the teacher factors that had the strongest effects on students' math achievement and motivation. All of the hierarchical linear regression analyses were statistically significant. However, of the five linear regression analyses conducted, the model with the greatest variance explained by student and teacher factors in the ninth grade was math achievement (R^2 = .21). The teacher factor that emerged as having the strongest effect on 11th grade math achievement was the degree to which ninth grade mathematics teachers emphasized a deeper conceptual understanding of mathematics (β = .14, p < .001). In other words, students who received instruction from teachers that emphasized connecting mathematics ideas, developing mathematics reasoning and problem solving skills, and understanding mathematical concepts performed better on a math achievement test in the 11th grade compared to students who received instruction from teachers who did not place emphasis in these areas. This finding provided further support for student-centered teaching approaches (informed by constructivist philosophy) that are foundational to reform-based teaching within the mathematics education community (National Council of Teachers of Mathematics [NCTM], 2014). The emphasis on deeper conceptual understanding also had a significant effect on the degree to which students saw themselves identifying with mathematics and being a mathematician (identity; β = .06, p < .001). In terms of whether students perceived mathematics as a useful subject, the strongest teacher factor predictor that emerged was the degree to which teachers emphasized increasing students' interest in math which may have included discussing the applications of mathematics in different academic disciplines as well as emphasizing the history of mathematics (β = .04, p < .01).

To answer the second research question, a binary logistic regression analysis was conducted predicting advanced mathematics course-taking behavior. The percent odds were reported to provide the reader with a clear understanding of the effect size that a variable had on advanced math course-taking behavior. For the odds ratio values presented in the last column of Table 1 that were greater than one, they were calculated by subtracting one from the odds ratio values and multiplying by 100. The odds percentage results reported refer to the effect of every one-unit increase in the given predictor on the odds of advanced math course-taking behavior. Again, the degree to which teachers emphasized a deeper conceptual understanding of mathematics was the strongest predictor of advanced math course-taking behavior. Specifically, when holding all other variables constant, greater levels of emphasis in deeper conceptual understanding of mathematics by ninth grade teachers increased the odds of their students taking advanced math courses in high school by 132 percent.

Discussion

The main aim of the current study was to understand the degree to which the characteristics, qualifications, and instructional practices of ninth grade mathematics teachers predict students' mathematics motivation and learning outcomes as they near graduation. Overall, our findings supported prior SCCT-informed research suggesting that teachers are important socializing agents that promote positive beliefs towards STEM fields (Yu et al., 2016). Specifically, our findings were consistent with prior individual classroom studies indicating that teachers' self-efficacy for teaching mathematics and the extent to which they emphasize understanding of mathematics are positively associated with students' self-efficacy for mathematics and achievement (Stipek, Givvin, Salmon, & MacGyvers, 2001). Furthermore, current findings were consistent with teacher education research that demonstrates the importance of teachers having mathematics degrees and a certification in mathematics in promoting greater student mathematics achievement (Rice, 2010). Our findings contributed to this line of research by showing that teacher qualifications have a positive association with both students' mathematics achievement and motivation over time. The findings were significant given NCTM's (2014) math practice standards, math teacher practice standards, and push towards a conceptual understanding for all students.

Results of this study may inform policies and promote additional research in areas that help broaden participation in mathematics. If we understand which malleable teacher factors most strongly contribute to students' mathematics learning and motivation outcomes over time, we can develop policies to address these important factors, including but not limited to producing and retaining teachers with desired qualifications and supporting professional development. Finally, we encourage readers to consider limitations while interpreting results. First, a limited number of variables in the HSLS:09 relate to teacher practices and are self-reported rather than observational. Second, HSLS:09 includes only ninth grade teacher data and student outcomes from 11th and 12th grades. It may be the case that after the ninth grade, students were taught by teachers who also impacted their STEM outcomes, a common limitation among longitudinal studies attempting to understand the long-term effects of teachers on students (e.g. Bradshaw, Zmuda, Kellam, & Iolango, 2009).

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Leading and Learning for Measurement and Assessment Practices

VALIDATION: A BURGEONING METHODOLOGY FOR MATHEMATICS EDUCATION SCHOLARSHIP

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Validity-related issues are a growing topic within the mathematics education community. Until recently, validation has been treated as something to gather when convenient or is rarely reported in ways that conform to current standards for assessment development. This theoretically-focused proceeding adds to a burgeoning theoretical argument that validation should be considered a methodology within mathematics education scholarship. We connect to design-science research, which is a well-established framework within mathematics education. The goal for this proceeding is to foster the conversation about validation using examples and to communicate information about validation in ways that are broadly accessible.

Introduction

In the last four years, validity issues are taking a greater focus within assessment and measurement using quantitative instruments. This is evidenced through a special issue of *Investigations in Mathematics Learning*, National Science Foundation-funded conferences on validity issues within mathematics education contexts, and peer-reviewed manuscripts and books addressing validity and validation issues within the scope of mathematics education scholarship. These works are springing from mathematics education researchers working collaboratively with others from different disciplines such as learning scientists, psychometricians, research methodologists, and special educators. Grounding ideas in theoretical and methodological frameworks is central to generalizable research that has broader impacts (Confrey, 2018). While there are procedures for validation (e.g., Kane, 2012; Schilling & Hill, 2007), there are few that frame validation as a methodology with its own nuances (e.g., Jacobsen & Borowski, in press). There may be many reasons for why validation has not been treated as a methodology and some of those include but are not limited to (a) pressures not to conduct validation studies, (b) challenges in publishing validation arguments (Bostic, Krupa, Carney, & Shih, in press), and (c) decreased emphasis in methodological training of doctoral students in the disciplines (Shih, Reys, Reys, & Engledowl, in press). To that end, this paper aims to augment recent work by

Jacobsen and Borowski (in press) to ground validation work in mathematics education as a methodology akin to design science.

Relevant Literature

What is a Methodological Framework?

For this proceeding, we characterize a methodological framework as one that allows a researcher to apply analytical tools to respond to a research question (Creswell, 2012). For our purposes here, methodology implies ways to conduct research in a manner that synergizes with a chosen theoretical, philosophical, or epistemological framework.

One Approach to Design-science as a Methodology

Design science research was developed to address central questions about learning (Collins, Joseph, & Bielaczyc, 2004). A central component of design research is a "temporal process flowing roughly from conceptualization to realization" (Middleton, Gorard, Taylor, & Bannan-Ritland, 2003, p. 63). Design research can: (a) address theoretical questions about the nature of learning in context, (b) provide a methodological approach for studying learning phenomena in an authentic setting as opposed to laboratory settings, (c) go beyond a singular measure of learning, and (d) derive justifiable findings from formative evaluation (Collins et al., 2004). Design research thus serves scholars as a methodological tool. There are multiple ways to frame design-science methodologies. In sum, a design-science based methodology (e.g., Middleton et al., 2003; Schwartz, Change, & Martin, 2003) fosters "a focus on instruments that both precipitate and measure effects has historically been effective at supporting innovation" (Schwartz et al., 2003, p. 63); in our own research, a test in diverse classroom settings.

One design-science methodological approach has seven phases within its design cycle: (1) grounded models, (2) artifact development, (3) feasibility study, (4) prototyping and trials, (5) field study, (6) testing, and (7) dissemination and impact (Middleton et al., 2003). For phase 1, reviews of literature and interfacing with experts helps to ground work on assessment development. It begins to answer questions such as: What will this instrument do? What has already been done in this area of assessment development? How will the interpretations/outcomes from the assessment be used? In phase 2, a rough draft assessment is produced based upon responses to these questions and others. For phase 3, data are gathered to evaluate the quality of the initial draft and make revisions. Cognitive interviews with a measure or real-time observations with an assessment might be used to explore response processes evidence. In phase 4, revisions are made, and a new artifact is produced. A content review committee (i.e., expert panel) or potential typical respondents might then examine the instrument for content, response processes, and/or internal structure validity evidence. In phase 5, implementation studies with a larger sample are conducted to examine the assessment for facets related to internal structure and usability. This sets up for phase 6, when psychometric studies are conducted because there are sufficient (i.e., size and type) data. Finally, at phase 7, the developed assessment is disseminated for broad use. This is also the stage where effectiveness studies are conducted to engage questions such as: How sensitive is the assessment to the desired phenomena? Are there quantitative similarities between the assessment and similar instruments? What are the contexts for which might the instrument not be appropriate? Through these seven steps, researchers are able to reify an idea into an actionable product, like an assessment.

Validity and Validation: Definitions and practice

Validity is "the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (AERA, APA, & NCME, 2014, p. 11). Because peer-reviewed manuscripts have historically tied validity to an instrument (see Bostic et al., in press), it must re-state that validity is linked to the interpretations and outcomes - not the assessment. Validity gives scholars confidence that the interpretations from quantitative scores derived from an assessment are the intended ones and not associated with a different construct. The *Standards for Educational and Psychological Testing* (AERA et al., 2014) frame five validity sources for assessment developers and users: test content, response processes, relations to other variables, internal structure, and consequences from testing.

The validation process is cyclical (see Figure 1) in nature and requires iterative loops before an assessment is ready for broad-scale use. The first step is to determine what an assessment will do and what it will measure.

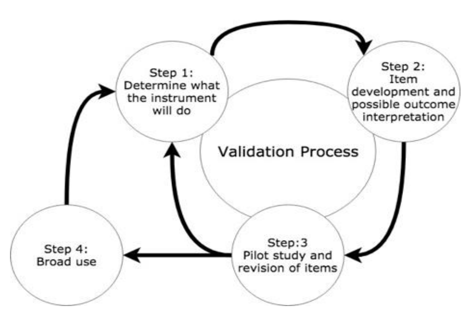

Figure 1. Validation process. See Gerber, Bostic, & Lavery (2018) for further information.

This typically requires determining a construct, defined here as "the concept or characteristic that a test is designed to measure" (AERA et al., 2014, p. 11). The second step is developing items and reflecting on ways of interpreting results. During this step, assessment developers think deeply about validity evidence. Drawing across validation frameworks (e.g., Kane, 2012; Schilling & Hill, 2007), this step is likely the most arduous but also the most important. In step three, an assessment is piloted to gather data, inform revisions, and a return to examining the construct that was selected. The reason for returning to step one is that it is possible to move away from the intended construct; therefore, a formative check is warranted. If there is sufficient evidence for the assessment developers suggesting it is functioning adequately, then broader use is acceptable (step 4). Previously, presentations at RCML focused on assessment development address this validation process (e.g., Bostic & Matney, 2018) but didn't connect them to validation as a methodology within mathematics education scholarship. Digging into previous work by this team, Bostic and Matney (2018) present and foster discussions at RCML annual meetings around the *Standards* (AERA et al., 2014) and how they enact across three assessments ready for broad use in scholarship. This paper picks up where that one ended and extends work to be more educative and approachable to scholars with a wide range of experience in measurement. In what follows, we connect the validation process, one design-science framework (Middleton et al., 2003), with one problem-solving measure (e.g., PSM6; see Bostic & Sondergeld, 2015) that is a component of a series of measures available for grades 3-8 in Table 1.

Table 1

Connecting validation and design-science stages with PSMs

Validation	Actions completed in PSM development	Design-science
(1) Determine what the instrument will do	Examine relevant lit, review assessments, conduct interviews with expert panel	(1) Grounded models
(2) Item development and possible outcome interpretation	Conduct expert panel review, cognitive interviews, small-scale pilot with one class of students	(2) Artifact development
(3) Pilot study and revision of items	Perform small-scale study (~ 100 respondents), analyze with Rasch modeling, revise items appropriately.	(3) Feasibility study, (4) Prototyping, trials, (5) Field study
(4) Broad use	Perform large-scale study with 300+ respondents.	(6) Testing, (7) Dissemination

It is evident that there are clear connections between validation stages and one design-science framework. Where validation may be a broader term and include many aspects, the design-science framework breaks it down into subcomponents in much the same way sources of validity are categorized in the *Standards* (AERA et al., 2014).

A central piece of the validation process is a methodological (i.e., procedural) aim - that is, how to accomplish specific goals. There are specific decisions to be made, which are tied to a desired outcome and chosen theoretical framework (e.g., AERA et al., 2014; Kane, 2012; Schilling & Hill, 2007). These decisions involve when, how, and from whom to collect data - and what manner to analyze those data and for what purpose. Ways to communicate choices for those decisions to potential users is not as simple as a manuscript section labelled participants, instrumentation, data collection, and data analysis. Because the involvement of participants varies at different stages in both design science and validation, it becomes complicated to convey this information. Moreover, the ways information is gathered during those stages are analyzed can vary. For instance, assessment developers might choose to analyze a few samples of assessment data at first using one approach and digging deeply into it (e.g., grounded theory; see Charmaz, 2006). Later (i.e., broad use) they might require a different analytical approach in which they look to confirm broad themes through inductive analysis (Hatch, 2002) earlier in the validation process. Another challenge is that the goal (i.e., assessment being developed) is not validated but its outcomes are. Thus, a central focus on conveying information must be a clear, convincing argument that the outcomes from using an assessment are logically drawn and not that it is merely sound psychometrically.

Current Discussions of Validation as a Methodology

Jacobsen and Borowski (in press) argue that validation acts as a methodological tool that has been underutilized. They and others (e.g., Bostic, 2017, Bostic et al., in press) note the lack of validation work within mathematics education scholarship. Albeit, gathering validity evidence and constructing a validity argument during the design and use phases for an assessment are central to generating generalizable research (AERA et al., 2014; Kane, 2012). Without a validation argument for the interpretations of scores from an assessment, it is uncertain how the scores on that assessment are accurate reflections of an individual or group's attributes (Bostic et al., in press; Kane, 2012). Thus, validation ought to have a central place in mathematics education research that uses quantitative assessments if an aim is to understand factors related to teaching and learning in their authentic settings. Design research draws upon authentic (real world) settings of research and not lab settings. Therefore, validation and design research share a mutual interest in understanding "what is" rather than "what might be".

Implications for Current Assessment Development: A Brief Example

A current National Science Foundation-funded project titled Developing and Evaluating Assessments of Problem Solving (DEAP; NSF #1720646, 1720661) is using the validation stages and a design-science framework (see Middleton et al., 2003) simultaneously to develop a series of measures that assess elementary (i.e., grades 3, 4, and 5) students' problem-solving ability within the context of math content and practices addressed in the Common Core State Standards for Mathematics (2010). This series connects to previously developed measures for grades 6-8. The development team is currently in stage 3 of the validation cycle and is preparing to re-enter the cycle after conducting the initial product and pilot testing. Concomitantly, the team's work might be classified at stage 5 of the design-science framework. More information about current assessment development activities are available (see Bostic, Matney, Sondergeld, & Stone, 2018).

Conclusions and Next Steps

As a result of using validation as a methodology within mathematics education scholarship, assessment developers are better equipped to converse with potential users (e.g., teachers, district representatives, scholars) and those closely associated with test-takers (e.g., students, parents/guardians, school personnel). Data gathering takes a practical approach to inform product development and validate outcomes/interpretations of the assessments. Assessment is central to sound research and without valid outcomes from assessment – the field cannot truly trust their implications. An issue coming from the fervor among mathematics education scholars is that validity must become part of the critical conversation about scholarship that aims to have high impact (Williams & Latham, 2017). As a result of a growing focus on validity issues, methodological framing of such scholarship becomes a bigger issue. Applying traditional quantitative or qualitative methodologies to communicate scholarship on validity issues and validation arguments presents unnecessary challenges to both authors and readers. Hence, validation should be considered as a viable methodological tool in empirical mathematics education research. We argue that validation as a methodology in mathematics education scholarship has utility. Validation bears striking similarities to design science, which is an established methodology. We recognize that this work and Jacobsen and Borowski (in press) are at the leading edge and more scholarship is needed to better ground validation as a methodology within mathematics education scholarship. Continued validation projects within mathematics education and discussions with diverse scholars will ultimately derive a powerful means for scholars to have broad impact and substantiate intellectual merit for work examining assessment and measurement within mathematics education contexts.

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TEACHERS' KNOWLEDGE OF MATHEMATICAL MODELING: A SCALE DEVELOPMENT WITH EXPLORATORY FACTOR ANALYSIS

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This pilot study examined the initial development of a scale to measure teachers' mathematical knowledge of teaching mathematical modeling from a Midwestern state. The phases used in this initial scale development included item generation, expert reviews, item analysis, and exploratory factor analysis. The study results suggest a psychometrically reliable and useful scale for measuring teachers' knowledge of mathematical modeling, which has implications for teacher preparation and professional development.

Introduction

Concern for the need to address the skills and understanding of mathematical modeling (modeling or model with mathematics) in the teaching and learning of mathematics has increased in recent years (Blum, 2015; Blum & Ferri, 2009; Lesh, 2012; National Council of Teachers of Mathematics [NCTM], 2009; National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010; Pollak, 2011). Additionally, modeling has gained increased focus in assessments for school mathematics—both nationally and internationally. However, implementing modeling in the classroom has challenged most teachers (English, 2009) and there was limited research regarding teachers' knowledge of modeling. Therefore, the purpose of this study was to develop a scale that assesses practicing teachers' knowledge of the nature of mathematical modeling.

To successfully implement mathematical modeling, teachers need strong content knowledge and pedagogical strategies of teaching mathematical modeling. However, there are no existing measures of teachers' knowledge of mathematical modeling. Therefore, this study provides an opportunity to design an appropriate quantitative tool to measure teachers' knowledge of teaching mathematical modeling guided by the following research questions: (a) do the items included on the mathematical modeling knowledge scale (MMKS) provide reliable measures of teachers' knowledge of the nature of mathematical modeling? and (b) how do teachers conceptualize the nature of mathematical modeling?

Conceptual Framework and Related Literature

Mathematical modeling is a content in its own right and as a tool to teach mathematics (Blum, 2015; Lesh, 2012). The research that perceive modeling as a content in its own right focuses on the modeling process, the phases in modeling, and modeling abilities and competencies (Blum, 2015; Blum & Ferri, 2009). Alternatively, most research identifies mathematical modeling as a tool to teach mathematics by considering modeling eliciting activities (MEAs) as productive problem solving situations for teaching mathematics in a meaningful way (Blum, 2015; Lesh, 2012). In this study, teachers' knowledge of mathematical modeling refers to the background knowledge about the nature of mathematical modeling and its process (Lesh, 2012), which serves as the theoretical foundation that delineates the content domain for this new measure. Therefore, teachers' knowledge of modeling is conceptualized as their familiarity with mathematical modeling practices and pedagogies.

Teacher's content knowledge is a central aspect of teachers' professional capabilities (Shulman, 1986). Ball, Thames, and Phelps (2008) explain that teachers of mathematics need certain knowledge domains to teach mathematics effectively, which includes mathematical modeling. Mathematical modeling knowledge provides much more "powerful and effective ways to help students become (a) better problem solvers, and (b) better able to use mathematics in real life situations beyond school" (Lesh, 2012, p. 197). However, evidence shows that knowledge of mathematical modeling among practicing teachers is limited (Blum & Ferri, 2009; Spandaw & Zwaneveld, 2010), and measurement tools related to modeling is woefully lacking (Kaiser, Schwarz, & Tiedmann, 2010). Therefore, this study is necessary and important.

Methodology

Based on the analyses of several sources such as the Common Core Standards, NCTM standards, and research articles (Blum & Ferri, 2009; DeVellis, 2012; Fowler, 2014; Gould, 2013; Lesh, 2012; NCTM, 2009; NGA Center & CCSSO, 2010; Pollak, 2011; Ziebarth, Fonger, & Kratky 2014) an initial MMKS was developed, which included true or false items and an open-ended question. The true and false option was used in order to obtain quantitative data, make the questions consistent and easier to answer, and encourage the respondent to provide the exact information being sought. Using cognitive interviews, reviews with content experts and practicing teachers, as well as item analysis and exploratory factor analysis, the initial items on the scale were modified and honed to a 13-item scale. The 13-items for this pilot study comprised of 12 true or false items and an open-ended question. The survey items represented one concept—the nature of mathematical modeling (see https://bit.ly/2DnvMAX)—and included questions about the modeling process and practices. Items answered correctly on the 12–item true or false questions were coded a score of "1," and those answered incorrectly were coded a score of "0. An average score of eight and above indicated satisfactory knowledge of the nature of mathematical modeling.

A descriptive cross-sectional survey design (DeVellis, 2012; Fowler, 2014), with a purposeful sampling technique was used to self-select teachers from a Midwestern state for this study. The data collection was a web-based self-administered survey. Because this was a pilot study, a sample size of between 25 and 75 was considered to be adequate (Converse & Presser, 1986; Johanson & Brooks, 2010). The sample in this study consisted of 71 teachers of mathematics from public school districts, but only 62 participants responded to the demographic questions. Of these 62 teachers, about 60% of the sample were White or Caucasian, 51% were K–5 elementary teachers, and 35% were master's degree holders. Concerning gender, 85% of the sample self-identified as female, and 15% as male. Statistical procedures used to demonstrate the usefulness and reliability of the scale included item analysis, Cronbach's alpha internal consistency reliability analysis, and exploratory factor analysis. The SPSS statistical software was used for all the analyses. All analyses were considered statistically significant with $p < .05$.

Results

Item Analysis of the MMKS

This study provided a quantitative analysis of the development of a scale to measure teachers' knowledge of the nature of mathematical modeling and how they conceptualize modeling. The overall mean score on the MMKS was 10.20 ($SD = 2.34$). Although two of the 12 true or false items on the MMKS had item-total correlations less than .30 but greater than .25, all of the items were retained in the analysis because of their correlations ($r \geq .30$) and theoretical relevance (Nunnally & Bernstein, 1994; Osterlind, 2010). The overall internal consistency reliability of the MMKS for this sample was $\alpha = .80$, indicating a reliable scale (DeVellis, 2012; Fowler, 2014). (Table 1 provides information on the item–total correlations and alpha values on the MMKS.)

Exploratory Factor Analysis of MMKS

Exploratory factor analysis (EFA) was used to assess the interrelationships and internal structure of the items. The EFA was appropriate because the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy of .70 was acceptable, the Bartlett's test was statistically significant ($p < .001$) and subjects–to–variable ratio of no lower than five was appropriate

(Bryant & Yarnold, 1995; Kaiser, 1974). Principal axis factoring (PAF) with a varimax rotation was used in the exploratory factor analysis. PAF explores the inter-relationship of items, provides a basis for the removal of redundant items, and can identify the associated underlying construct (DeVellis, 2012). The factor loadings on the MMKS with only one-factor extracted based on theoretical relevance explained about 29% of the shared variance. The factor loadings values (> .30) indicated the items correlated well with the whole scale and was labeled—knowledge on modeling.

Table 1

Item–Total Correlations of all the 12 Items on the MMKS

Items	*M*	*SD*	*SE*	ITC	α if item deleted
Item 1	.92	0.28	.03	.27	.80
Item 2	.90	0.30	.04	.59	.77
Item 3	.80	0.40	.05	.48	.78
Item 4	.72	0.45	.05	.28	.80
Item 5	.86	0.35	.04	.43	.79
Item 6	.86	0.35	.04	.54	.78
Item 7	.77	0.42	.05	.27	.80
Item 8	.90	0.30	.04	.44	.79
Item 9	.82	0.39	.05	.63	.77
Item 10	.94	0.23	.03	.39	.79
Item 11	.93	0.26	.03	.31	.80
Item 12	.77	0.42	.05	.69	.76

Note: $n = 71$; ITC = item–total correlation.

Analysis of the Open-ended Item

The open-ended question assessed how teachers conceptualize the nature of mathematical modeling. Specifically, they were asked to write a brief definition for the phrase mathematical modeling. A total of 54 teachers responded to this item and their responses were categorized by two mathematics educators and the researcher based on a rubric as shown in Table 2. The ratings were coded as 4 = excellent, 3 = good, 2 = fair, and 1 = poor. The inter-rater reliability based on the intra-class correlation (ICC) was calculated to be .86 for single measures and .95 for average measures. The resulting ICC values indicated that the raters had a high degree of agreement.

An examination of teachers' responses about their understanding of the phrase mathematical modeling revealed interesting results. Of the 54 teachers who responded to this question, about

83% were female (n = 45), 55% were White (n = 30), and 53% (n = 29) taught elementary grades (K–5). Based on the rubric in Table 2, only 7% of the responses were excellent.

Table 2

A Rubric for Evaluating the Definition of Mathematical Modeling

Category			
Excellent = 4	***Good = 3***	***Fair = 2***	***Poor = 1***
Definition demonstrates complete understanding and provides detail explanation. It states almost all steps involved in the modeling process. Links mathematics, real world situations, and the translation between the two.	Definition demonstrates basic understanding and provides minimal explanation. It does mention the steps involved in the modeling process. There is no link between mathematics and the real world.	Definition demonstrates little understanding and little to no explanation. It doesn't mention the steps involved in the modeling process. There is no link between mathematics and real world situations.	Definition shows no evidence of understanding of the phrase mathematical modeling.

Most of the teachers had misconceptions about mathematical modeling and they confused *mathematical modeling* with *modeling mathematics*. Thus, they literally thought mathematical modeling was to show or model a step-by-step process of solving a math problem (Figure 1 provides the distribution of respondents' responses about the meaning of the phrase mathematical modeling).

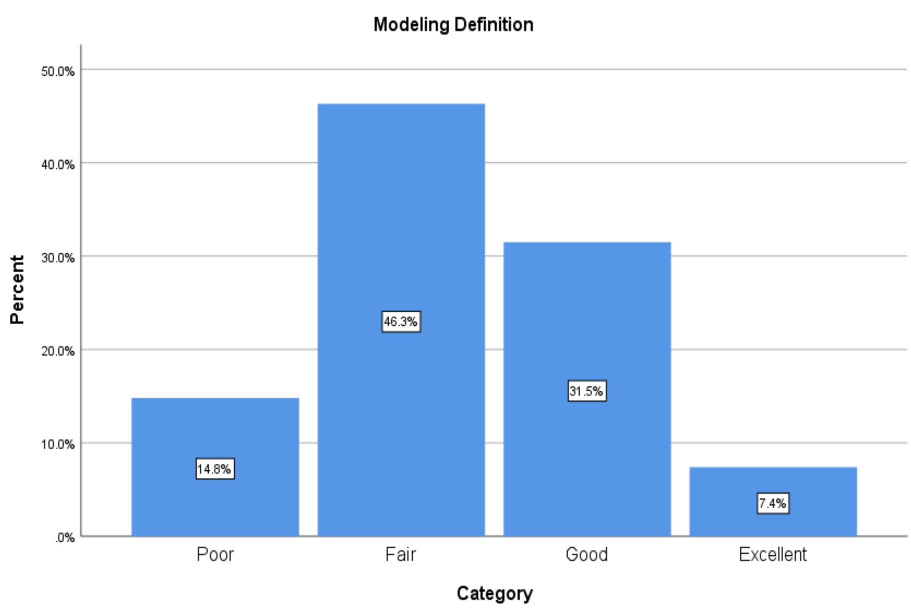

Figure 1. A bar chart showing teachers' responses about the phrase mathematical modeling

Most of the teachers' explanations incorrectly assumed mathematical modeling as using only physical objects, manipulatives, or representations to solve mathematics problems. Additionally, their explanations failed to recognize mathematical modeling as an iterative process, which involves choices and assumptions by the modeler. Although respondents had a satisfactory knowledge of the nature of mathematical modeling, their responses on the open-ended question, showed most of the teachers had misconceptions about the phrase mathematical modeling. Experiences shared by the teachers indicated that the phrases *mathematical modeling* and *modeling process* were new terminology to most of them, and they had little or no experience with mathematical modeling practices.

Conclusion and Implications

This pilot study examined the psychometric properties of the initial development of a scale. Results showed that all the items performed well, and taken together, they cover a comprehensive range of the domain of interest as defined by the researcher. The method of collecting data from only public school districts might have introduced volunteer bias and social desirability—a tendency to respond in a manner that will be viewed favorably by others (Tourangeau, Rips, & Rasinski, 2000). Although this study had some limitations in terms of the sample size, selection, and generalizability, it has added to the existing and growing body of literature on teachers' knowledge of mathematical modeling. The development and testing of a scale was a complex process, but the results from this study showed that the MMKS has the potential to generate a single score representing teachers' knowledge of the nature of mathematical modeling.

Despite the limitations of this study, it is important that we develop a quantitative tool that measures teachers' knowledge of mathematical modeling to help students do mathematics as set forth by the Common Core and NCTM standards. Having a new scale that measures teachers' mathematical knowledge of teaching mathematical modeling could benefit professional development and teacher preparation, by providing researchers and educators with an assessment tool in their training and practices. Results from this study and other published materials (e.g., Kaiser et al., 2010; Spandaw & Zwaneveld, 2010) indicate a need exists for mathematical modeling training standards or courses to be integrated in teacher preparation programs. Finally, the psychometric results suggest the scale is a promising quantitative tool for advancing research on teachers' mathematical knowledge for teaching mathematical modeling. The researcher hopes the MMKS will benefit mathematics educators, researchers, and advance mathematical modeling education in school mathematics.

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SECONDARY REHEARSAL:
ANALYSIS OF A NEW MODEL FOR INSTRUCTIONAL ACTIVITIES

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We present a new instructional activity (IA) designed to be used for the pedagogy of rehearsals to support teachers in engendering understanding of a complex mathematical topic. We report on affordances and limitations of the IA by analyzing instruction of an experienced middle school teacher who implemented it after participating in multiple rehearsals. Results indicate that while the IA provided a structure to create potentially productive student-teacher interactions, professional development experiences to support the implementation of the IA need to include a greater attention to the connections between ways of reasoning about the particular topic the IA addresses.

Introduction and Literature Review

In recent years, as the research community has established a strong link between student understanding and teachers' attention to students' mathematical thinking (e.g. Jacobs, Franke, Carpenter, Levi, & Battey, 2007), a new student-centered form of instruction has emerged in which teachers elicit and leverage students' conceptualizations to connect their burgeoning understanding to more refined mathematical ways of thinking (e.g. Stein, Engle, Smith, & Hughes, 2008). Such a conception of instruction is no doubt challenging and requires teachers to engage in complex and improvisational practices.

Acknowledging the challenges associated with this form of responsive teaching, various teacher preparation programs have begun to adopt a new practice-based model of professional training in which the details of specific "core practices" that target the skills associated with such a model of instruction, are identified, studied, and performed. A central component of this approach were purposefully crafted *approximations* of teaching (Grossman, Hammerness, & McDonald, 2009), in which the work of a teacher is decomposed and pre-service teachers engage in smaller, more manageable components of practice. One form of approximations of practice that mathematics teacher educators have begun to use are rehearsals, which simulate teaching situations by having peers play the role of students (Kazemi, Franke, & Lampert, 2009). This pedagogy provided a context where the teacher educators can introduce particular problems of practice and offer in-the-moment suggestions about the details of instructional moves and decisions. These enactments reduced the complexity of teaching, enabling teachers to develop an understanding of the nuanced practices associated with such adaptive and responsive instruction.

Such a structure not only embeds the training of teachers in the actual work of the profession but makes the details of professional actions and reasoning more visible.

Typically, rehearsals are organized around planned instructional sequences called “instructional activities” (IAs), which target particular high-leverage practices (e.g. eliciting and responding to student contributions). These authentic yet controlled instructional activities “structure the relationship between the teacher, students, and the content in order to put a teacher in position to engage in and develop skill with interactive practices” (Campbell & Elliot, 2015, p. 150). A key feature of IAs is that they contain some elements that become routinized which teachers can carry out with little to no thought. Such routines reduce the complexity of teaching by limiting certain instructional choices, and thus create a space for teachers to engage in the much higher cognitive task of focusing on how to respond to student thinking. To date the majority of IAs used by teacher educators have been designed for instruction at the elementary level and focused on supporting teachers to develop designated teaching practices (e.g. Kazemi, Ghousseini, Cunard, & Turrou, 2016). These IAs have been intentionally created to allow a wide range of mathematical content to be inserted into the format. With such models, while pedagogical decisions around students’ mathematical contributions are at the forefront, the focus is a designated core practice, with the mathematical content serving as an instructional context.

In contrast, we wondered about the use of rehearsals to hone skills associated with teaching particular content, specifically topics that involve complex mathematical understandings that inherently require more time to develop. Rather than creating an IA which left the mathematics open to include a range of appropriate topics but targeted particular instructional practices, we wanted to investigate the use of an IA that was designed with a rich mathematical goal in mind and involved those practices, possibly content specific, which related to developing a deep understanding of the mathematical concept. This shift in focus resulted in an emphasis of different types of practices. For example, instead of focusing on how to elicit student thinking (a practice where very similar moves could be used in teaching various topics), our work tended to target practices like the selecting and sequencing of mathematical ideas to lead to a particular content goal (Stein et al., 2008). This latter practice was particularly difficult with a complex mathematical topic as it inevitably involves the negotiation of multiple student conceptions to engender a deeper understanding across the class. To explore such challenges, we examined how a teacher implemented the IA in her classroom to identify areas where we could have better supported her during the professional development. This led to the following research question: What does one teacher's implementation of an IA in her classroom reveal about the nature of the rehearsal experience that teachers need to productively teach a complex mathematical topic?"

Instructional Activity

We chose the mathematical goal of fostering a quantitative understanding of algebraic notation as the basis for our IA. We wanted students to be able to interpret the contextual quantities represented by the algebraic symbols (Knuth et al., 2005). To target such an understanding, we decided to use the context of figural patterns (see Figure 1 for an example). In such a context the quantities represented by the alphanumeric symbols involve concrete and discrete objects that can be physically identified and easily counted, qualities we believed would facilitate the connection to symbolic form (The nuances of this type of understanding as it pertains to figural patterns is in the IA below, see Table 1). We then leveraged the instructional trajectory by Hawthorne (2016) to identify a sequence of key conceptualizations involved in developing such an understanding.

In designing our IA, we wanted it to have components of a lesson plan, in that it aligned with an instructional trajectory that leveraged student thinking to converge to a rich understanding of a mathematical topic. Moreover, because we planned to work with in-service teachers, we strove to embrace authenticity. While such a design might inherently add complexity, we believed a format that more closely resembled the instructional flow of a typical secondary lesson would be more suitable. An affordance of the instructional trajectory was that the conceptualizations identified are organized around making explicit connections between various increasingly abstract representations. Therefore, to create a manageable IA, we decomposed this sequence into smaller iterative phases organized around different mathematical representations. The structure of each phase was replicated, with the different representations supporting an increasingly sophisticated understanding of the topic.

During instruction, each phase consists of the same figural pattern, but the task posed to the students shifts by the type of representation they are to use in their response. This serves to deepen the students' understanding of the quantities in the figure and embed this understanding in each subsequent representation. Similarly, in each phase, the teacher engages in the same cycle of instructional practices. While each phase consists of many more detailed practices, in general, the teacher first poses the task, then walks around the classroom probing student thinking to select and sequence student ideas, and finally leads a discussion, using discourse moves and board work to support students in making their thinking public and encouraging the class to attend to each other's thinking. With each phase, while the teacher solicits and engages in a variety of ideas, she must actively work to elevate particular ways of thinking and ensure that others in the class participate in these forms of reasoning. This delicate negotiation of ideas provides a foundation for students to deepen their understanding from phase to phase. Table 1 describes each phase of the IA. The *Teacher Actions* column highlights the iterative nature of the IA, describing what elements stay the same and which are unique to each phase as students' understanding becomes more robust. The *Goals* and *Student Thinking* provide an exemplar of the type of reasoning that is anticipated for the specific figural pattern below.

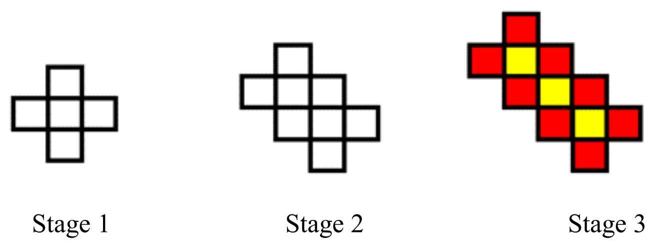

Figure 1: Example of figural pattern

Table 1

Phases of Instructional Activity

Phase	Representation	Teacher Actions	Goals	Student Thinking
1	Pictorial Image	*Introduce Problem:* Ask students to draw stages 4 & 5. Select 2-3 students to share their picture and explain how they knew to draw it as such.	Allow students to begin to generalize the pattern, decompose the figure in quantities that make sense to them, and identify how these quantities change.	Most likely recursive, each stage increases by an L with 3 squares above/ below or each diagonal is growing by 1 square.
2	Verbal Description	*Explicate quantitative relationships:* Ask students to describe in words what stage 6 looks like. Select 2-3 students to share their understanding.	Allow students time to continue to generalize the pattern and to use verbal descriptions to help make their understanding of the quantities, their values, and relationships explicit.	Recursive view: Stage 6 is stage 5 with one more L. Explicit view: Stage 6 has two diagonals with 7 squares and one diagonal with 6 squares.
3	Numerical expression	*Transition to explicit thinking*: Choose a particular student's decomposition and ask class to write numerical expression for a near (e.g. 6) and far stage (e.g. 37) that captures this interpretation. Ask	Align class's thinking around a particular decomposition. Ensure to choose one that is *not* in form mx + b. Transition recursive interpretations to an explicit way of thinking. Connect values of quantities in the figure to the stage number. Relate quantities in the figure	Stage 10: 2x11+ 10 2 diagonals of 11, plus 1 diagonal of 10 2 refers to the number of longer diagonals, 11 represents the number of squares in the outside diagonals

		students to identify what quantity each number represents.	to specific symbols in numerical expression.	10 represents the number of squares in the middle diagonal
4	Algebraic Expression	*Link quantities to literal symbols*: Ask students to write an algebraic expression for the nth stage. Ask students to identify what quantity each literal symbol represents.	Relate the quantities in the figure to specific symbols in the numerical expression. Make meaning of variables as quantities whose role is consistent, but value is changing.	Stage n: $2(n+1) + n$ $n+1$ represents size of the outside diagonals. The quantity is constant but value is always 1 more than the stage number. n represents the size of the middle diagonal. Quantity is again constant but value equals the stage number.

Methods

To explore how to support teachers implementing the lesson described in the IA, a 20-hour, 5-day professional development session was designed with 14 middle school teachers participating (9 eighth grade and 5 seventh grade teachers). During the week, teachers were familiarized with the IA and provided instruction about using discourse moves and other instructional practices to elevate particular student understandings. Over the last two days of the PD (4 hours each day) the teachers participated in 6 different teacher led rehearsals. This included working through phases 1-3 with two different figural patterns. Each of the fourteen teachers was involved in planning at least one of the rehearsals, but only six teachers had the opportunity to serve as the acting teacher in a rehearsal. This planning and implementation were supported by the work we did earlier in the week where we explored the nature of the conceptual goals associated with the IA, discussed research on student thinking in this area, and modeled the teaching of this lesson and the rehearsal process.

To develop an understanding of what support teachers need to implement the IA to productively leverage nuances of student thinking and engender the targeted understanding, a detailed case study was considered to be the most appropriate approach (Yin, 2003). While we recognize that a short one-week PD experience is necessarily limited in what it can accomplish in terms of changing teachers' practice, we wanted to explore how a teacher implemented the lesson after an introduction to the material. We believed that such a case study analysis would highlight additional structures teachers need if this IA were to be used as part of a more extended professional development experience. For the case study, we selected Dawn, a teacher who had been a vocal participant throughout the PD, had led one rehearsal, and whose comments during the week indicated that she had internalized the structure and intent of the IA in general. We followed her into the class as she engaged in a 3-day unit built around this IA and analyzed her teaching by examining all teacher-student interactions during this unit. We first made descriptive and interpretive notes for each of these moments. In an attempt to understand the teacher's motivation throughout the lesson, we identified and transcribed notable exchanges to focus on the details. Analyzing this collection of interactions using a grounded theory method (Strauss & Corbin, 1998), different themes emerged that provided a consistent rationale for her instructional decisions. Synthesizing these themes, we developed an understanding of the specific ways in which the goals of the IA and subsequent rehearsals were evident and influenced her teaching.

Results

Analysis indicated that Dawn's instructional decisions formed two distinct and disconnected stages of teaching. Phase 1 and 2 of the IA aligned with an exploratory approach in which students were provided space and scaffolding to explore patterns and create their own mathematical understanding. The opportunity to verbalize their mathematical thinking supported students in deepening their own understanding of the figure and aided other classmates to share in their thinking. That being said, during this phase, Dawn struggled to leverage the variety of student contributions to support students in fostering a more sophisticated understanding. Rather than productively selecting and sequencing student ideas to elevate a particular way of thinking, she called on volunteers and gave everyone equal sharing time. At this point, unable to bridge the gap between students' disjointed ideas and productive mathematical understanding, her instruction changed notably as she moved to the next phases of the instruction. In particular, as she engaged in phase 3 and 4 of the IA, her mode of discourse changed from open questioning to funneling (Herbal-Eisenmann & Breyfogle, 2005). Rather than eliciting students' thinking, her questioning became very answer-oriented as she targeted specific ways of thinking. Furthermore, when she received answers she wanted, she would quickly write them on the board and move on. Alternatively, when students did not offer up her desired answer, she supplied it herself and again quickly presented it without supporting other students in the class to grapple with these ideas. This was in contrast to the instructional methods she used earlier when she promoted other members in the classroom to engage in the ideas presented.

Furthermore, four specific areas emerged where the IA and associated rehearsal work provided support for Dawn to engage in responsive teaching within the observed unit. First, the IA structured the lesson flow in such a way that allowed students to work independently and to create their own mathematical thinking. Students were actively engaged throughout all phases and seemed to notably enjoy the task. Second, the IA outlined specific instances for the students to verbalize their mathematical thinking. The influence was most notable in phase 2 when Dawn had students provide a verbal description of their understanding, while others attempted to draw and make sense of their depiction. Third, the students were exposed to a more meaningful view of algebraic expressions. While the teacher did proceduralize the generalization process, eventually guiding students to a particular way of thinking, the quantities associated with each symbol were elevated during the lesson, allowing the opportunity for students to make the connection. Fourth, as a result of working with the IA, Dawn developed her specialized content knowledge associated with teaching this unit. She demonstrated an increased ability to decompose figural patterns and write algebraic expressions capturing the quantities associated.

While the IA created a pattern of potentially productive student-teacher interactions, it did not provide the necessary scaffolding to effectively support navigating the diversity of student thinking that was generated as a result. Analyzing Dawn's difficulties, three particular areas emerged that indicate ways in which we could have supported use of the IA more effectively. First, more attention needed to be given to purposefully selecting responses to leverage student thinking. Dawn tended to treat all contributions equal, struggling to find a balance between honoring all student thinking and elevating particular productive ideas. Overwhelmed by multiple ideas, she either called on any and all volunteers or targeted very specific answers. Second, a robust and conceptual understanding of the targeted mathematical topic must be developed. Without such deep understanding of the topic, Dawn guided students towards a procedural view of the generalization process and to some degree the associated algebraic symbols. Third, each phase of the IA must be accompanied by a clear understanding of the type of reasoning associated. While Dawn possessed a general idea of the progression of thinking, she did not have a well-defined idea of the exact reasoning she was looking for in each phase or how different ways of thinking related. Consequently, she was not able to effectively select and sequence student ideas.

Conclusion

Reflecting on the IA and how the rehearsal supported Dawn to teach more responsively, it seems clear that she would have benefitted from a deeper understanding of the instructional practices necessary to navigate diverse thinking as well as nuances of the mathematical goals and the associated student thinking. Her instructional decisions and responses focused on a particular way of thinking rather than cultivating and connecting multiple ways of reasoning. The IA seemed to function more as a detailed lesson plan, structuring the overall organization of the class, but our rehearsal work did not provide an understanding of these nuances. While the IA supported potentially productive student-teacher interactions, our rehearsal work did not provide a nuanced understanding to enable her to fully leverage these. There were many instances that the teacher engaged in actions elevated by the IA, but with a slightly different rationale.

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RANKING THE COGNITIVE DEMAND OF TASKS ACROSS MATHEMATICAL DOMAINS

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As students begin working with more advanced mathematical tasks, many are faced with challenges in understanding concepts and keeping track of units when solving systems. A better understanding of how students aged 7-12 solve a task and their limitations will provide insight into how lesson plans can better fit student needs and pinpoint the connections that are lost in the process. This article details a student-independent diagraming and ranking method of tasks within three mathematical domains; laying the groundwork for how classroom studies and student interactions can be analyzed. Student limitations can be better explained by making a connection between mental attention capacity and units coordination. Defining a clear ranking system of mathematical tasks and student capabilities provides more structure for the comparison of interactions.

Introduction

For the purpose of this research, I will be referencing the definition of units coordination stages one, two, and three as compiled by (Ulrich, 2015, 2016). Much of units coordination is also referred to as 'shortcuts' or 'mental math'. Once a student truly understands a units coordination concept, they can take these shortcuts, explain their process, and infer what would happen to the solution if a simple change was made within the task.

Additive Units Coordination

A stage one student relies on an initial number sequence when performing addition tasks. When asked to add 2 to 7, a stage one student's actions would look like: 7…8, 9. They would need to rely on figurative material to keep track of how many steps have been taken and when they need to stop counting. A stage two student is able to take advantage of a tactically nested number sequence, answering a task of 16 minus 7 by counting up from 7 to 16 to arrive at an answer of 9; the student is able to recognize the seven as 'nested' within the sixteen (Ulrich, 2015). A stage three student also operates using a tactically nested number sequence to solve additive tasks.

Multiplicative Units Coordination

Stage one and stage two students use initial number sequences and tactically nested number sequences to solve addition problems. A stage three and some stage two students, however, are able to construct iterable units, streamlining their work by condensing information (Ulrich, 2015). Referred to as an explicitly nested number sequence, a student operating on this level can recognize in the problem mentioned above that 7 is a subset of 16 (Ulrich, 2015). A stage three student may also operate with a generalized number sequence, allowing them to perform tasks involving three units (Ulrich, 2015). Most fractions and algebraic reasoning tasks require the use of a generalized number sequence.

Interiorized Units and Composite Units

Internalization or interiorized units is a student's ability to store a mental action of units coordination, allowing them to refer to this stored memory at a later time to avoid additional work to solve a task (Steffe & Wiegle, 1992; Ulrich, 2015). When working with two or more units at once, a composite unit can be formed. A stage one student is not able to form composite units, as they can only focus on one unit as a time and lack the ability to group. A stage two student (operating with a tactical number sequence) is able to form composites of two units, meaning they are working with two different units but their mental actions allow them to group it as one unit. A stage three student is able to make a composite of three units (Ulrich, 2015).

Cognitive Demand

Working Memory and M-Capacity

A student's units coordination stage can impact the level of mathematical tasks they are able to successfully complete. However, research shows this is not the only limiting factor to a student's success; the cognitive demand of a problem taxes the student's mental abilities (Agostino, Johnson, & Pascual-Leone, 2010). Baddeley's model of working memory connects the central executive with the visuospatial sketchpad, phonological loop, and episodic buffer (Baddeley & Hitch, 2000). The visuospatial sketchpad and phonological loop work as 'slave systems', operating with short-term storage. The episodic buffer makes a connection between visual and spatial information (Baddeley & Hitch, 2000). The capacity of the central executive is taxed when performing more than one task simultaneously. The theory of constructive operators predicts the capacity of the central executive function throughout age development (Pascual-Leone et al., 2010). It is defined that children of ages 7-12 years old generally operate with an m-capacity ranging from three to five (Pascual-Leone et al., 2010). It is estimated that from the ages of 3-15, m-capacity increases by one unit every-other year (Pascual-Leone et al., 2010). Figure 1 diagrams the mental action of processing five tasks at once in the central executive. According to research referenced above, a student aged 11 or 12 (approximately a sixth grader) would be able to process information at this level of m-demand.

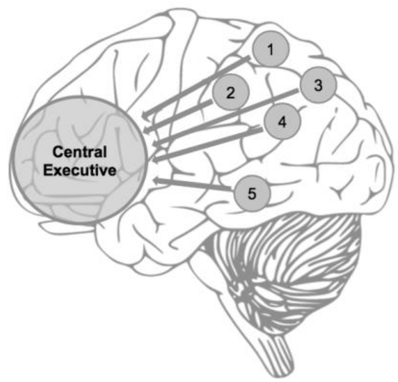

Figure 1. M-Capacity

Units Coordination and M-Capacity

Figure 2 represents the rank of a task a student would be able to successfully solve in relation to their units coordination stage and m-capacity. A student with units coordination stage one is not able to work with any composite units to solve a mathematical task; their m-capacity is taxed for each unit and relationship. Therefore, a student with UC stage one and m-capacity 5 would be able to solve a task ranked three. It should be noted that a student with UC stage one and m-capacity 2 is only able to solve a task ranked one. This would represent a unit and a relationship or two units. No connection can be made with an m-capacity of one without using composite units, so it is fair to say this student would not successfully arrive at an answer. A student with UC stage two, is able to construct a composite unit that represents two singular units, thus their m-capacity is only taxed once for what is actually two units and a relationship. An assumption being made in this table is that students possess the ability to 'daisy-chain' composite units (i.e. they can make a connection between composite units by sharing one of the interiorized units and only counting it once). Therefore, a UC stage two student with m-capacity 5 is able to solve a task ranked 6. A student operating with UC stage three, can construct a composite of three units. Hence, a student with UC stage three and m-capacity 5 is able to solve a task ranked 11 at most (this can be done using a daisy-chain approach or using separate relationships).

UC\M	2	3	4	5
1	1	2	2	3
2	3	4	5	6
3	5	7	9	11

Figure 2. Units Coordination & M-Capacity

Defining Domains

The three different mathematical domains chosen for analysis are whole numbers, fractions, and algebraic reasoning. I have compiled a task list for each domain by finding tasks used in previous research and ranking them based on a student-independent task analysis. Task lists are to be used in future research endeavors. These domains were chosen based on the availability of reliable tasks and their presence in elementary and middle school curriculum.

Student-Independent Task Analysis

The scale used for task ranking is derived from the relationship between units coordination and mental attentional capacity. At the least, a student can solve a task ranked 2 and at the most, a task ranked 11. This creates a scale of tasks ranked 2-11. In analyzing tasks, both units and relationships are counted. Representation of units can be found in Figure 3, and relationships are represented by arrows. Additionally, no composite units were created in the making of these diagrams, as they are student-independent. In other words, the tasks are diagramed from the point of view of a stage one units coordination student, even though they would not be able to solve a task greater than rank three.

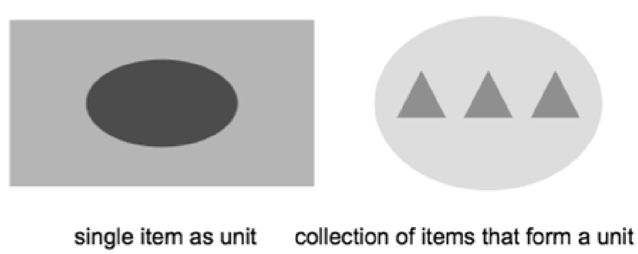

Figure 3. Unit Representation

Figure 2 can be referenced to see which UC stage and m-capacity a student would need to have in order to perform a given task. There will be obvious variation to how different students approach a task. However, students' mental actions can be represented by the same diagram, following the process from a different starting point.

Whole Numbers Task Analysis

Analyzing the problem diagramed in Figure 4, "Susan went on vacation for three weeks. How many days was she on vacation?" (Kamii, & Housman, 2000). The student needed to first recognize a week as a unit (1); a day as a unit (2); form the relationship that a week is made up of seven days (3); refer to the notation of three weeks, forming a new whole (4); finally representing three weeks in terms of 21 days (5). These units and relationships lead to a rank equal to 5. According to Figure 2, a student with UC stage 2 m-capacity 4-5, and a UC stage 3 m-capacity 2-5 could successfully solve this task.

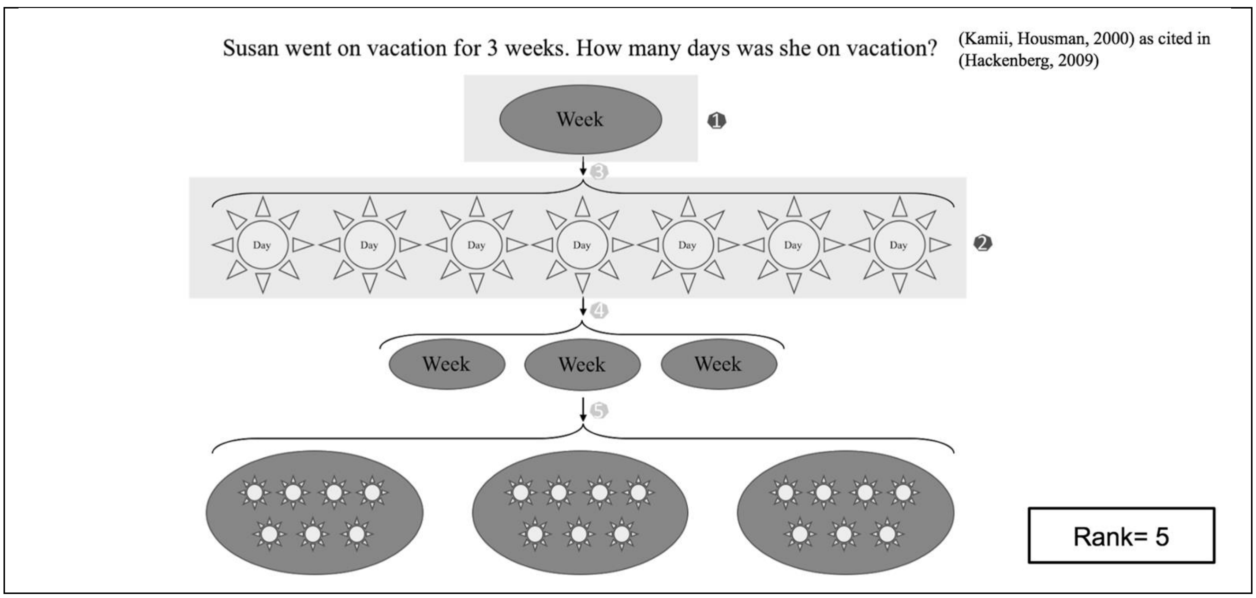

Figure 4. Whole Numbers Task

Fractions Task Analysis

Analyzing the problem diagrammed in Figure 5, "What is 2/3 of 1/5?". The student must first recognize the whole (1); partition into fifths (2); choose one fifth (3); recognize one fifth as part of the whole (4); take ⅔ as a unit within ⅕ (5); partition into thirds (6); choose 2 (7); recognize the ⅔ as a unit (8); the unit in relationship to the whole (9). These units and relationships lead to a rank equal to 9. According to Figure 2, only a student with UC stage 3 and m-capacity 4-5 would be able to arrive at an accurate answer to this task.

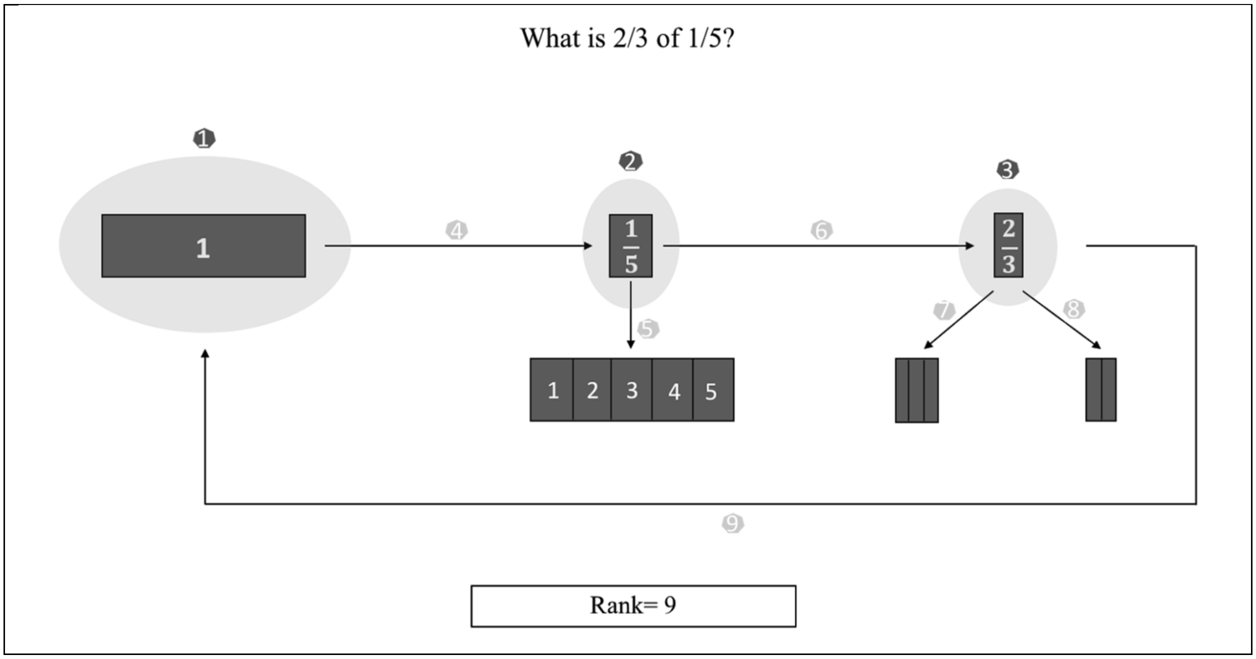

Figure 5. Fractions Task

Algebraic Reasoning Task Analysis

Analyzing the problem diagramed in Figure 6, "There are 5 identical candy bars (rectangles) and each candy bar weighs some number of ounces. Let's say that h= the weight of one bar. Can you write an expression for the weight of 1/7 of all the candy?" (Hackenberg, 2013). One candy bar represents the whole (1); the whole is partitioned into sevenths (2); one seventh is chosen to work with (3); keeping in mind the seventh's relationship to the whole (4); representing one seventh in terms of 'h' (5); recognizing 'h' as a variable (6); which is the weight of one candy bar (7); the relationship between 'h' and one candy bar (8); recognizes 5 candy bars as the new whole (9); forms relationship between 1/7h of one bar and the new whole (10). According to Figure 2, only a student with UC stage 3 and m-capacity 5 would be able to perform this task.

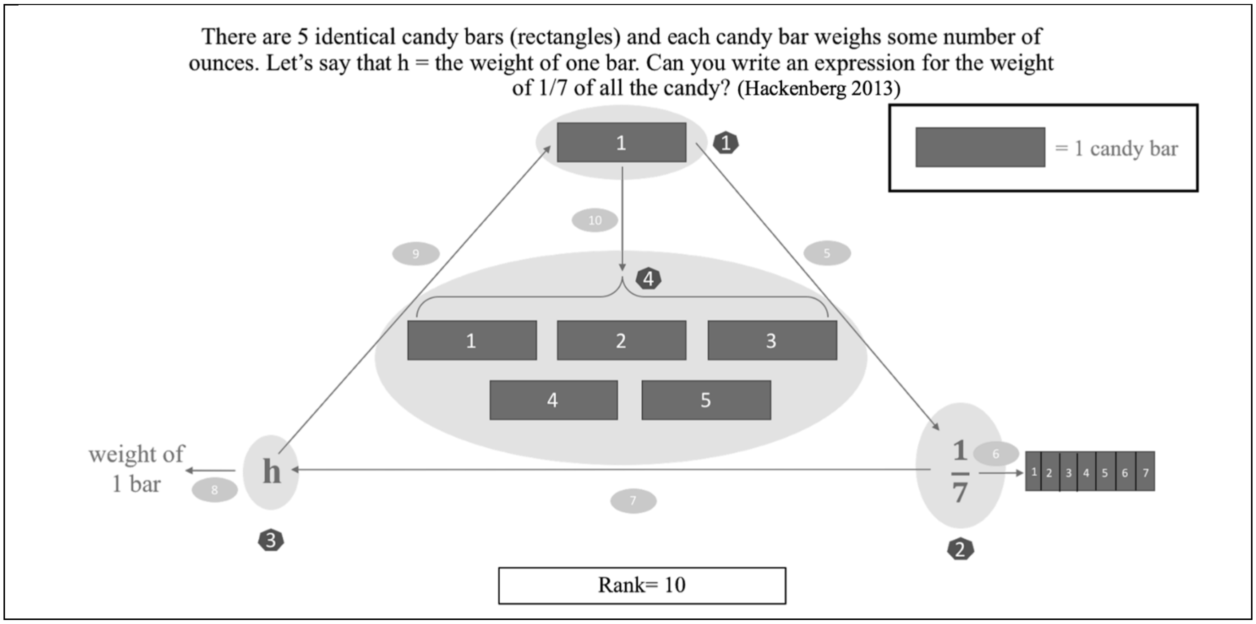

Figure 6. Algebraic Reasoning Task

Conclusion & Future Plans

There are many unanswered questions in regard to how students learn best and why certain tasks are more challenging than others. This student-independent task analysis in relation to units coordination stages one, two, and three and mental attentional capacity provides insight into the limitations of students ability. The diagramming is meant to help teachers better understand their students' thought processes and road-blocks. This connection between units coordination and m-capacity leads to a better comprehension of what conceptually challenges students ages 7-12 and why.

One of the things to be further discussed is the mental demand of a variable. Variables are a proven difficulty for students and it is unknown how many relationships can be unpacked from a single variable. It is possible that the weight of a variable is dependent on the context of the task. It is currently being hypothesized that a student 'unpacks' a variable in a solution strategy before they begin solving a task. Further classroom studies should answer the question of whether students are able to daisy-chain composite units to solve a task or if they need to account for a separate relationship. It is possible students may use one approach or the other, or a combination of the two. It is hypothesized that a student will be able to reach a similar rank on each of the three domains. Meaning a student who can solve tasks up to a rank of 5 on the whole numbers task list (based on their UC stage and m-capacity) should be able to solve tasks up to rank 5 on the fractions and algebraic reasoning lists as well.

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CONCEPT MAPS: PROFESSIONAL DEVELOPMENT AND ASSESSMENT

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The authors initially studied the use of concept maps to help understand the organization of elementary teacher content knowledge. We found that teachers had very little background in constructing concept maps or in thinking about the organization of the mathematics they taught. The initial maps suggested a typical elementary school teacher understood mathematical content as closely aligned to a table of contents in a typical elementary textbook. As the research shifted to include instruction in making concept maps, teachers engaged in building and comparing maps became better at seeing and organizing the mathematics they taught in a more interconnected way.

Introduction and Theoretical Perspective

We have been working with K-6 teachers for the past 3 years with a goal of strengthening their knowledge of the mathematics they teach. One measure of teacher content knowledge we used in this research was the Learning for Mathematics Teaching (LMT) test (Hill & Ball, 2004). We found this assessment somewhat limiting because each year we focused on a small area of mathematics topics and thus only some subtests were appropriate. The second issue with this summative assessment was it told us what areas our teachers were stronger/weaker in, but not how these teachers conceptualized this mathematics.

We reviewed the research regarding using concept maps to identify how they were used to facilitate learning various concepts and how they were used to study student thinking. Williams (1998) compared concept maps created by students studying Calculus with those created by mathematicians on the concept of function. She concluded that there was promise in distinguishing concepts maps of novices from those of experts. Also, Novak and Cañas (2007) posit that using concept maps can uncover knowledge that is tacit. Thus, creating concept maps may function as a shared metacognitive activity. There have been many researchers in mathematics education who have described meaningful learning and its importance to students (and in our case, teachers) studying mathematics. Piaget described the processes of assimilation and accommodation whereby an individual relates new information to existing information or makes changes in existing information or the organization of this information in light of a perturbation or some cognitive dissonance. Others have referred to this type of learning as meaningful learning (Ausubel, 1968), and Skemp referred to those engaged in this type of learning as building a relational understanding (Skemp, 2006). These authors were all describing the importance of making connections among mathematical ideas to develop a personal sense of the mathematical ideas they are learning. We believed that creating a concept map of specific mathematical ideas held by these teachers would give us some insight into how they were organizing and making sense of them. Because of this we're interested in answering two research questions. The first was what can teachers' concept maps tell us about how they organize the mathematical ideas they teach? The second question is how do teachers' concept maps of the mathematics they teach change as a result of their engagement in professional development? The purpose of this article was to communicate what we learned, how the focus of the research evolved, and what new questions we have formed. From this research we believed that concept map data seemed like a productive supplement to our LMT data.

Background

The professional development began each year with an intensive week-long summer course which combined mathematics content and pedagogy. This was followed by a week-long summer camp in teachers' home district built around Lesson Study. Grades K-6 students engaged in summer classes where a teacher skilled in experienced-based (Moses, 2001) pedagogy taught the lesson while the participants observed and took notes. The participants then discussed what they observed with the teacher of the lesson and others who facilitated the discussions. At the end of the camp week these teachers were tasked with integrating a few lessons based on the experienced-based pedagogy within their first month of teaching in the fall. Starting in September, teachers participated in monthly 1-day workshops throughout the 9-month academic year, culminating in their own Lesson Study events in December and May. There was a second-year summer course, and enhanced leadership training for continuing teachers. In total there were about 105 contact hours per year, with a typical teacher spending three years in the program.

Concept Maps as Data

We used concept maps as a pre, near-post (after the summer sessions), and delayed-post (after the academic year) assessments to collect concept map data. Because this was not a quantitative research project, we modified our use of the concept maps during the 3-year implementation. What follows is a description of how they were used to gather data.

First Iteration

There was a content focus each year of the PD. For this first iteration, the content area was measurement. Teachers were given a short (10 minute) introduction to the idea of a concept map and shown different samples of generic concept maps: the linear map; the tree map; the circular map; and the hub and spoke map (from Meagher, 2009). They were also given a page with short written instructions, and some prompting questions to think about before making their individual maps. The teachers were given ample time to complete their maps.

Teachers created this initial version at the beginning of the summer, then they were given red pens to modify their initial map at the end of the summer, and finally at the end of the next academic year they were asked to construct another map, all focused on the concept of measurement.

This map contains one participant's initial map with her modifications in red after the summer session. In her modification, she uses red ink to emphasize the sequence of the nodes with numbers, and the outer arrow to illustrate a change in order. The arrangement is: (1) Basic Addition, (2) Repeated addition, (3) Linear measurement experience (standard and non-standard), *(4) Area/Perimeter concepts, *(5) Measuring shapes, (6) Arrays and multiplication. Central to all the nodes is the node "Concepts of area and relation to addition and multiplication". There are no linking phrases. (* = modified item)

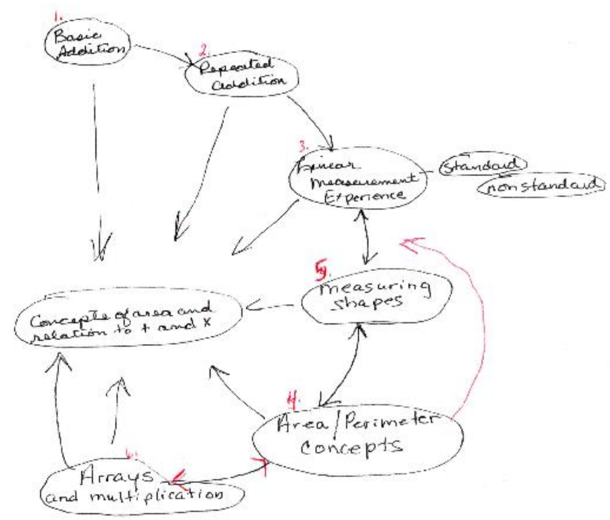

*Figure 1***.** Example of a concept map (on measurement) completed and modified during academic year 1.

This concept map (CM) was one of the better rated (score = .68, where low = .40, high = .80) by both researchers after the class and camp using a specific rubric constructed by the researchers for this concept. The maps were analyzed in terms of five essential items for the concept of *measurement*: (1) things that can be measured, (2) feature to be measured, (3) units, (4) comparison, and (5) counting. Maps were scored based on the presence of these features and given a separate score on the logical relationships among these items.

The rubric had each researcher look for the 5 essential items and give it a score of 1 to 5. There was very little written on connecting lines among concepts in their maps, so adjacency relative to one another and direction of arrows were the only way for the researchers to evaluate connectedness from the maps. We also had some notes that teachers made on the back of their maps regarding key terms/features of measurement to read to see if there was any thinking about relationships among the nodes of their maps. Each researcher again rated the connectedness of each map with a score of 0 to 1. We then "leveled" these two scores and came out with an average overall score. As mentioned above the participant scores ranged from .40 to .80. The participant CM in Figure 1 above was scored at .68.

Results of the initial analysis

From the map in Figure 1 one can see that she created a list of important features related to measurement and used arrows to connect these ideas. Some of the arrows were one-directional and pointed from one node to another while others were bi-directional. We weren't sure how she saw these connections as there was nothing written on the connectors. This limited our analysis of her organization of this topic. Several of the other teacher maps that were created and adjusted (red pen changes) reflected pedagogical issues instead of the intended target of conceptual analysis on the topic of measurement. This made it very difficult for researchers to use the first set of concept maps as an assessment of changes to their understanding of measurement. We attributed at least part of how the teachers initially drew and then modified their concept maps on how the task was defined (not enough grounding orientation and training in using concept maps) and the strong emphasis on pedagogy used in the one-week summer course. In addition, we noted that five of the participants were K-1 teachers whose curriculum had very little specific mention of measurement (i.e., not much teaching experience with this topic). In order for us to get a better idea of how these teachers conceptualized the mathematics they were teaching, we needed to provide more direct support for creating concept maps. We also wanted them to see that concept maps were one way to view how the mathematics they teach on a day-to-day basis fit into a comprehensive whole.

The Second Iteration

Trying to learn from our mistakes, we set out to limit the scope of the content (measurement was too large) and provided direct support for what a concept map is and how they could be created. At the beginning of the second year summer class the facilitator discussed the concept of number. Key ideas regarding number were brainstormed by the class and written on the board. There was a discussion of how these ideas were connected and then the whole class discussed how these ideas could be arranged. Using sticky notes with these terms on them teachers began the process of creating a collective concept map for (whole) number. They used yarn to connect related sub-ideas and used white strips of paper to write down why the "yarn" was used to connect two (or more) ideas. After this was done together in class, they were assigned to work on another related topic: Addition and subtraction of whole numbers and to create their own maps for this new topic. Before they began individually, they had another whole group discussion to create a new list of key features for this concept. They were told they could add to this list or ignore anything on the list for their own concept map. They were also told that they could use the same materials as used during the whole class (sticky notes, strips of white paper, and yarn). Most used sticky notes, but a few of them used either the yarn or strips of paper.

Analysis of Second Iteration

We were happy with the initial results with the new process for creating the CMs. Clearly, we were getting more of an idea of how teachers saw the larger picture of the mathematics they were being asked to map. You can see that there appears to be a hierarchical order for the subtopics below in Figure 2. The connections and grouping of addition and multiplication and the word "opposite" indicates relationships among these features of operations with whole numbers. We also learned that in the large and small group work, we were starting to see the potential for pedagogical uses of concept maps. Some teachers remarked they had never thought about how topics within mathematics were related beyond the idea that you needed students to learn chapter 1 material before they could learn chapter 2 material. Thus, questioning why certain material was introduced prior to other material and what it was building for later development led to powerful discussions by participants. While we completed a similar scoring of these 2nd year maps, we didn't compare pre and post assessments. Rather, we focused more on honing the pedagogical use of creating concept maps for the 3rd year group.

Figure 2. Second iteration: a concept map (on operations for whole numbers) completed and modified during academic year 2.

This map used an array to organize the concepts. The author used hierarchy between the top row ("Higher Level") and the bottom row ("Basic Understanding"). The inner array is vertically connected in categories. The original map had linking phrases on pink sticky notes ("repetition"), and the revision added two more linking phrases ("opposites"), written with a red felt pen.

Third Iteration or Using Concept Maps as an Organizational Learning Tool

During our third iteration we used what we learned from the first two implementations of CMs and decided that we needed to take an even more guided approach to introducing the idea of creating concept maps with teachers. Thus, we took a more structured approach to teaching CM construction, using a model described by Salmon and Kelly (2015). In addition to group brainstorming and keeping the focus of the CM narrow, we added a number of deliberate strategies. Each session began with a focus question (e.g., "What do students need to know to understand fractions?"). Teachers were tasked with generating facts about the focus topic, written in complete sentences ("feature sentences"). A typical teacher-generated example was "A fraction has a numerator and a denominator." The facilitator freely probed the sentences after they were reported out ("What does denominator mean? Can you explain with an example or a model?"), leading to new feature sentences. From their list of sentences, the teachers generated a list of concepts. The facilitators then asked the teachers to rank the concepts in a hierarchy, using prompting questions like "Which of these concepts are the most important to fractions, and which of these would be subordinate to the others?" Once concepts were in a rough grid (concepts at equal level of hierarchy were listed horizontally), the feature sentences could be added to the CM as links between concepts.

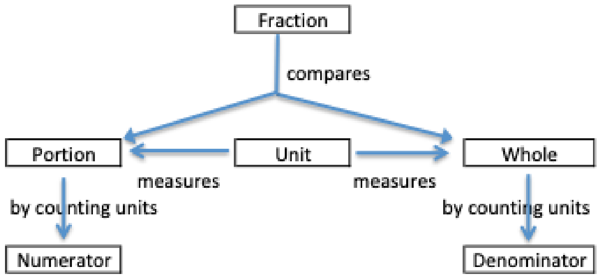

Figure 3. Third iteration: a facilitated concept map (on fractions) completed during academic year 3. (Neatly typed but generated from teacher work.)

The facilitator conducted this process as well, by using dialog and questioning to sharpen teachers' thinking ("How is 'numerator' related to 'portion'?") Through this process, teachers would realize (for example) that the numerator is a number while the portion is an object, that a unit measures both the portion and the whole, and that numerators and denominators are obtained by counting. The final map would have links reflecting their understanding. The result of this process is predictably close to an "expert model" designed in advance.

Conclusion

Many educators in various disciplines have documented the utility of concept mapping to increase metacognition, develop flexibility in asking questions, and serve as a powerful tool for instructional planning. Science education has developed concept mapping as a way to teach and learn science with a focus on "big ideas". Concept mapping has been employed in mathematics education as well, but very little research has been carried out to document its effectiveness as a systematically employed tool for mathematics teaching and learning. In terms of APOS theory, our research may help to clarify how concept mapping can help teachers and students develop a *schema* for understanding a particular mathematics topic, and eventually to build larger, connected schemata for the mathematics that they know (Arnon et al., 2014).

Next Steps

Ultimately, the responsibility for constructing CMs should shift to the teachers themselves. It is interesting to note that at least two teachers have begun using CMs to help students organize their own mathematical problem solving. Many agree there is an overwhelming need to move toward a conceptual understanding of mathematics for all people. We would like to establish CM as a tool for teachers and K-6 students to approach mathematics learning as a conceptual domain in which meaning is derived from linkage between ideas. We would like to see if teachers could organize different concept areas into larger CMs, to address questions such as "How do operations of whole numbers help students understand operations with other number systems?"

Finally, we hope that as teachers have a better understanding of how to create a concept map for mathematical ideas, we can study how these maps provide insight into their level of organization of these topics. We also would like to see how their scores on CMs may be related to their scores on a standardized test like the LMT.

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CONNECTING OBSERVATION PROTOCOLS AND POST-OBSERVATION FEEDBACK

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In this study, two universities created and implemented a student-centered graduate student instructor observation protocol (GSIOP) and a post-observational Red-Yellow-Green feedback structure (RYG feedback). The GSIOP and RYG feedback was used with novice mathematics graduate student instructors (GSIs) by experienced GSIs through a peer-mentorship program. Ten trained mentor GSIs observed novice GSIs, completed a GSIOP, and provided RYG feedback as part of an observation-feedback cycle. This generated 50 semester-long data sets of three observation-feedback cycles of novice GSIs. Analyzing these data sets helped identify how certain feedback influenced GSIOP scores.

Introduction

Mathematics graduate student instruction significantly impacts undergraduate courses and students (Belnap & Allred, 2009). Graduate student instructors (GSIs) have been identified as a key component of success for collegiate mathematics departments (Bressoud, Mesa, & Rassmussen, 2015, p. 117). As a result, mathematics departments and research in undergraduate mathematics education continue to focus on supporting and improving GSIs' student-centered instruction (Rogers & Yee, 2018; Speer & Murphy, 2009; Yee & Rogers, 2017). There are multiple methods of student-centered pedagogical support for GSIs (e.g. professional development, mentoring, pedagogically-focused courses; Speer, Gutmann, & Murphy, 2005; Yee & Rogers, 2017), but there is currently limited research on GSI teaching observation protocols and even less research on post-observation feedback (Reinholz, 2017). Multiple observation protocols exist to assess undergraduate mathematics instructors' classrooms (e.g. MCOP^2, RTOP, C-LASS, etc.), often with scalar metrics such as point values 1-4, but few discuss how to connect that assessment with observer feedback.

To this end, we created a GSI observation protocol (GSIOP) and a post-observation feedback structure at two universities to provide ongoing support for novice GSIs. Together, the GSIOP and feedback were implemented for two years as part of a peer-mentorship model where novice GSIs were mentored by experienced (two or more years of experience) GSIs who had completed a mentor professional development (PD) seminar. This mentor PD included training with the GSIOP and post-observation feedback (See Rogers & Yee, 2018 and Yee & Rogers, 2017 for more information on peer-mentorship). The purpose of this paper was to help bridge the research gap between observations and post-observation feedback by identifying how feedback within this peer-mentoring model informed and influenced future observations. Our research question for this study was in what ways (if any) did the feedback structure lead to changes in teaching observations throughout a semester?

Related Literature

Feedback

Although K-12 mathematics education research has extensively studied feedback within practicum courses (e.g. student teachers are observed regularly by their master teacher and university supervisor as a critical means of ongoing teacher development) our review of the literature has found few studies focusing on mathematics GSI peer feedback (Reinholz, 2017; Rogers & Yee, 2018). One exception was a recent study by Reinholz (2017) that explores peer feedback with mathematics graduate students as equal peers. Reinholz had six GSIs provide peer-feedback to one another and found that feedback not only helped the novice, but enhanced teacher noticing and reflection in the observer, aligning with Reinholz's previous work (2016) where peer assessment led to improved self-assessment. Rogers and Steele (2016) concluded that novice instructors struggle to discuss teaching methods, which Reinholz (2017) argues could be aided by peer feedback. Thus, Reinholz's (2017) and Rogers and Steele's (2016) research supported post-observation feedback as a means of improving GSIs' teaching through discourse and reflection.

Complexities of Observations and Feedback

Reinholz (2017) reminded us that "how instructors engage with peer feedback is complicated" (p. 7) due to GSIs' beliefs about mathematics and its often-assumed relationship to innate intelligence. Kluger and DeNisi's (1998) meta-analysis of 607 studies on feedback interventions (i.e. providing people with some information regarding their task performance) showed that while overall feedback improves performance, it can also sometimes reduce performance, depending on the type of feedback and means by which it is delivered. In light of the complexity that links observations and feedback, we questioned what type of feedback is most effective for GSIs.

Framework of Study

Our peer-mentorship research (Rogers & Yee, 2018) and current literature (Reinholz, 2017) has found observational protocols need to have complementary feedback structure where novices are able to reflect more openly about how they can modify their teaching to achieve their goals. Hence, our design emphasized post-observation feedback as reflective to complement the more evaluative observation protocol.

GSIOP

The initial goal of our peer-mentorship model was to provide feedback and facilitate discussions among novice GSIs around student-centered teaching strategies to improve undergraduate mathematics instruction (Yee & Rogers, 2017). We modified the MCOP^2 (Gleason, Livers & Zelkowski, 2017) to observe GSIs to develop the GSIOP which focuses on both student and instructor actions. The GSIOP contained questions on an ordinal scale from 0 to 3 for four sections: classroom management, student engagement, teacher facilitation, and lesson design.

RYG Feedback

Mentors were educated through the mentor PD to use the GSIOP and facilitate post-observation conversations using a Red-Yellow-Green feedback structure. Using this structure, mentors identified key points from the GSIOP that they could summarize for the novice in three categories: methods the novice is doing well (green), methods the novice could work on (yellow), and methods the novice needs to address (red). The mentor would summarize points of discussion from the GSIOP and keep the feedback manageable by discussing at most two concerns within the yellow and red categories.

Methods

In this mixed-methods study, we quantitatively analyzed changes to GSIOP scores. We then qualitatively coded the RYG feedback for types of actionable feedback and compared the types of feedback with the changes in GSIOP scores to answer our research question.

Participants & Observations

This study included 10 mentor GSIs and 32 novice GSIs from two universities in the United States over two semesters. New novices were added between semesters while other novices completed their training after one semester. For this reason, we focused on sets of semester-long observations, which consisted of three observations with feedback for each novice on average.

This generated 50 data sets of semester-long observations with feedback (totaling 151 individual observations with feedback). Mentors submitted novice teaching notes, videos of the novice's class, observation summaries, completed GSIOPs, and RYG feedback for analysis.

Data Analysis

As our research study emphasized student-centered instruction and RYG feedback, we focused only on the two sections of the GSIOP that emphasized student-centered instruction, the *student-focused* (student engagement) and *teacher-focused* (teacher facilitation) sections. One research assistant at each university longitudinally analyzed the GSIOP scores from both the student- and teacher-focused sections for each novice over an entire semester. Similarly, each research assistant analyzed the RYG feedback and observation summaries for student-focused feedback and teacher-focused feedback that aligned with the questions from appropriate sections of the GSIOP. This created 100 longitudinal data sets of semester-long observations and 100 data sets of semester-long feedback (50 student-focused and 50 teacher-focused).

To answer our research question, we summed the questions on the GSIOP student-focused section (4 questions) and the GSIOP teacher-focused section (5 questions) separately. Thus, for each observation of each novice each semester, there was a teacher-focused GSIOP score and a student-focused GSIOP score. We looked at change in GSIOP scores over a single semester by looking for trends and subtracting novices' final GSIOP score from their initial GSIOP score for both the student- and teacher-focused sections. Additionally, we looked at the data collected by the mentor during each observation and the feedback each novice received from the mentor. We analyzed feedback through an *advice and improvement* framework. We looked at RYG feedback, GSIOP comments, and mentor observation summaries for suggestions that provided the novice with *advice* on teaching that focused on student learning or teacher facilitation. We then looked through the data sets at each novice to see if the mentor noted any observed *improvements* related to advice given previously in the semester.

Next, we coded each piece of advice and each noted improvement as *broad* or *specific*. To frame broad versus specific objectively, we used Nilsson and Ryve's (2010) definition of contextualization where the context of an event must be given to make a situation specific and not referencing a context or event (often referred to as decontextualized) would be considered broad. Looking at feedback as advice or improvement concomitantly as broad or specific provides a categorization demonstrated on Table 1 with prototypical examples.

The last two categories, Advice Without Improvement (AWI) and No Advice Nor Improvement (NANI) took into account if advice and improvement were not given. AWI implied advice (broad or specific) was given, but improvement was not noted in subsequent observations. NANI lacked advice and therefore no improvement could be noted in subsequent observations.

To triangulate the qualitative coding of advice and improvement as broad or specific, after each research assistant qualitatively coded the results according to Table 1, two additional researchers went back and verified their work by comparing 75 of the 151 observations and post-observation feedback artifacts for both teacher-focused feedback and student-focused feedback.

Table 1

Qualitative Coding Scheme for Feedback across an Entire Semester

Code	Description	Example
SASI	**Specific Advice Specific Improvement**: Feedback included at least one contextualized suggestion the novice could take to improve their teaching. In subsequent observations, the mentor noted that the novice had addressed the issues through particular contexts, actions, and/or strategies.	"Elaborate with the material and explain the importance of the concept. For example, one instance in which you could give a little more insight and explanation was when the student used P(A U B) = P(A)+P(B) - P(A cap B)"...(*later observation*) "You elaborated more than last time.. I felt that this was the perfect amount of elaboration. Also, you asked well thought out questions, and you rarely missed good opportunities to ask further questions."
BASI	**Broad Advice Specific Improvement**: Feedback included suggestions without context on when or how to improve the novice's teaching. In subsequent observations, the mentor noted that the novice had addressed the issues through particular contexts, actions, and/or strategies.	"Have tiny bits of student involvement through to keep students engaged" … *(later observation)* "Student questioning chosen was very effective in engaging students [with 2^x and log_2(x)]"
SABI	**Specific Advice Broad Improvement**: Feedback included at least one contextualized suggestion the novice could take to improve their teaching. In subsequent observations, the mentor noted that the novice had improved upon previous issues, but without referencing specific contexts.	"I encourage you to give more wait time before answering the questions yourself, this can have them participate more" … (*later observation*) "I saw great improvement since last time with student engagement….(*later observation*) "Great student interaction".
BABI	**Broad Advice Broad Improvement**: Feedback included suggestions without context on when or how to improve the novice's teaching. In subsequent observations, the mentor noted that the novice had improved upon previous issues, but without referencing specific contexts.	"Student engagement should be addressed" … (*later observation*) "Even though she ask[ed] many questions, students are not really active in this particular class"…(*later observation*). "She did not just answer but encourage[d] students to respond".
AWI	**Advice Without Improvement**: Feedback included suggestions, but the suggestions did not appear to be noted throughout the subsequent observations.	"For the next time, I hope that he can get more active participation during his lecture portions" *No follow up.*
NANI	**Neither Advice Nor Improvement**: Feedback was either statements extolling the novice's instruction or platitudes on teaching. Mentor did not provide advice nor improvements.	"He did a great job in his lesson of engaging the students, explaining material adequately and also giving his students problems to work on at the end of class". *No advice.*

Interrater agreement was initially 94% and after discussion of the coding discrepancies, researchers agreed on the appropriate coding for the remaining 6%.

Results

Due to limited space, we will briefly summarize the longitudinal trends. Each novice's three GSIOP scores from both the student-focused and teacher-focused sections determined how each set of three scores varied. Results show that for both the student- and teacher-focused sections, on a 0-3 point scale, there was an average positive change of 1.01 points per section. Although a majority of the GSIOP scores had less than a one point change from previous GISOPs (33 out of 100), there were significantly more novices whose score increased by more than one point (44) than those that decreased by more than one point (15) over a semester. Thus, our results indicated there was an observed change in teaching throughout a semester via the GSIOP score showing an overall increase in point value.

We tallied the total change in score for all novices during a semester by taking the final GSIOP score for each section and subtracting it from the initial GSIOP score for that section. We then divided the total change by the number of novices to get the average change per novice.

Table 2

Inductive Analysis of Feedback Types Cross-Referenced with Change in GSIOP score

Feedback Types	SASI	BASI	SABI	BABI	NANI	AWI	Grand Total
Student-Focused Feedback	4	2	7	12	11	14	50
Average GSIOP Change Per Student-Focused Section	4.50	3.50	3.57	0.58	-0.73	-0.93	0.72
Teacher-Focused Feedback	10	4	4	8	5	19	50
Average GSIOP Change Per Teacher-Focused Section	3.40	3.00	-0.25	2.38	0.80	-0.16	1.3
Student and Teacher Feedback	14	6	11	20	16	33	100
Average GSIOP Change Per Student- and Teacher-Focused Feedback	3.71	3.17	2.18	1.30	-0.25	-0.48	1.01

Table 2 shows that of all 100 data sets of semester-long feedback, the one with the highest average change in GSIOP score was when mentors provided and noticed Specific Advice and Specific Improvement (SASI, *M=3.71)*. SASI feedback also resulted in the highest change in GSIOP scores for both student and teacher sections. Both Advice Without Improvement (AWI, *M=-0.48*) feedback and No Advice and No Improvement feedback (NANI, *M=-0.25*) had the least change in GSIOP scores.

We provide a small excerpt demonstrating SASI semester-long feedback that generated a substantial increase in his novice's student- and teacher-focused GSIOP scores. Consider Roberto's yellow feedback and following green feedback which had a substantial increase in his novice's student- and teacher-focused GSIOP scores.

> *(Yellow Feedback)* Engage more with the students. Particularly, ask more questions. I see that you are using the PowerPoints…I will do a demonstration for you in the one-on-one for a slide that was in your lecture. The main thing is to actively think if this is a moment I can ask a constructive question to engage with the learning… *(Following Green Feedback)* You are asking more questions to your students and you are getting more participation! This is great. Keep it up but remember that you can also… (*Coded SASI*)

The specific advice to engage through questioning, followed by specific improvement promoting growth demonstrates actionable feedback that can positively frame post-observation feedback.

Discussion

In answering our research question, we found that the RYG feedback in our study there were more increases than decreases in GSIOP scores over semester-long observation-feedback iterations, illustrating novices were attending to mentor feedback. Additionally, our coding of feedback (advice/improvement and broad/specific) illustrated how GSIOP scores on the teacher and student sections would change relative to the type of feedback. Feedback that included specific advice and specific improvements had the largest positive change in GSIOP observation score indicating that contextualizing feedback leads to more actionable feedback.

Limitations and Implications for Research and Practice

The structure of the post-observation feedback and the overall design of the peer-mentorship model could have influenced the results of this study. Specifically, the training of mentors and the use of the peer-mentorship model may be critical factors in the results of this study. This in no way voids the results but is a limitation of implementing RYG feedback with another observation protocol or using the GSIOP with a non-RYG feedback structure.

Table 2 verifies Kluger and DeNisi's (1998) argument that change depends on the type of feedback. When mentors provided specific advice and noted specific improvement, or provided broad advice and noted specific improvement, novice GSIOP scores improved on observation questions focusing on student engagement and teacher facilitation of student-centered learning. However, if the mentor's feedback provides no advice nor improvements, or advice without improvements, there was a minor positive or negative change in GSIOP score for both student engagement and teacher facilitation of student-centered learning. Our research provides undergraduate mathematics education with a framework for looking at post-observation feedback using a tested observation protocol and a post-observation feedback structure. Our results (Table 2) indicate providing specific improvements had the most actionable (Cannon & Witherspoon, 2005) results with respect to the observation protocol.

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EXPLORING STUDENTS' STATISTICAL REASONING

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Statistical reasoning, defined as making sense of statistical investigations, is essential for developing college and career ready students for the world in which data and statistics are omnipresent. This study aimed at assessing the statistical reasoning of 84 high school students and college students before taking a college statistics course. The findings reveal strong skills in computing and interpreting probabilities, and lack of significant differences between statistical skills for middle school, high school, and college students. Implications for further research are discussed.

Objective

As Ramirez, Schau, and Emmioglu (2012) asserted, "The ultimate goal of statistics education is to produce statistically literate adults who appropriately use statistical thinking" (p. 57). Statistical literacy involves understanding basic statistical skills, concepts, and vocabulary including probability concepts. Relatedly, statistical thinking involves understanding key ideas, the whys and hows of statistical investigations. This study focuses on statistical reasoning. While there is no unified way to define statistical reasoning, the conceptual approach of this definition focuses on making sense of statistical information (Garfield & Chance, 2000), applying statistical ideas to summarize and make conclusions on data (Lovette, 2001) from observations, experiments and surveys. Statistical reasoning also involves making connections between different concepts, and fluency in interpreting or explaining the results of statistical investigations (Zvi & Garfield, 2004). The key difference between statistical reasoning and mathematical reasoning, as Van De Walle, Karp, and Bay-Williams (2013) argue, is that essential focus in statistical reasoning is the interplay of variability and context of data. Thus, the aim of assessing statistical reasoning is not to explore how students perform or read the shallow statistical procedures but rather to assess their knowledge of big ideas underlying statistical investigations.

The call for statistical reasoning has continued to grow stronger in the past decade as the world is surrounded by quantitative information requiring its citizens to properly evaluate claims and reactions based on data (Zvi & Garfield, 2004). In the frameworks and initiatives for college and career ready K-12 education, it is natural to focus on how students make sense of statistical investigations and reports that are omnipresent in their world. Such a focus is evident in the increase of statistics content in the curriculum frameworks and instructional materials. The objective of this study is to assess the statistical reasoning of middle school through early college students prior to taking their college statistics course. This study is as necessary and timely as the need to prepare college and career ready students who can make sense of their statistics-dominated world. The study contributes to evaluating progress and informing educational practices in developing statistical reasoning prior to college. Furthermore, it addresses a concern expressed by Sotos, Vanhoof, Van den Noortgate, and Onghena who after surveying literature reported that "the literature on statistics education, and particularly publications providing empirical evidence of misconceptions in statistics, is sparse" (2007, p.100). More recent studies have also urged further research in statistical reasoning (e.g. Kawakami, 2018; Sabbag, Garfield, Zieffler, et al., 2018).

Framework: Enculturation into Statistical Reasoning

Statistics is a discipline that has independent intellectual method and calls for unique investigative and questioning habits of mind (Pfannkuch & Ben-Zvi, 2011). After a review of the research on expert-novice thinking and statistical thinking, Garfield, Zieffler, and Ben-Zvi (2015) strongly asserted that statistical reasoning is a type of expert thinking. Such assertions are supported by scholarship on hierarchies or trajectories in statistical literacy, thinking and reasoning such as Garfield (2003) and those who analyzed different models of statistical reasoning such as Pfannkuch and Wild (2002). Enculturation into statistical processes is a process in which novice statisticians become experts. It is rooted in socio-cultural perspectives and perceives the goals of education as initiating the novice statisticians into becoming experts or full statistics participants with growing identities as learners and knowers (Greeno, 2003; Claxton, 2002). With this theoretical view, tools are an integral part of learning and may include physical tools, level of mastery, and mental tools that are accessible to participants of learning. The teacher or other experts who are enculturators use such tools to scaffold learners' processes of becoming. Informed by this theoretical framework, this study explored statistical reasoning of students who are in middle school through college prior to taking a college statistics course. The guiding research questions were 1) How do novice statisticians reason about key statistical concepts such as sampling, distributions, spread, and center? and 2) Are there differences in the reasoning skills across grade bands?

Methods

This study had a total of 84 participants who were either in high school (junior and senior high school) or in college. There were 39 participants in college and data were collected prior to taking any college statistics course. All college participants attended the same college and 37 of them did their high school education in the same state. For this subgroup of participants in college, a Statistical Reasoning Assessment was administered electronically in low stakes supervised classroom settings. The validity of the Statistical Reasoning Assessment, an instrument developed as "part of the project to evaluate the effectiveness of a new statistics curriculum in US high schools" (Tempelaar, 2004, p.4), is discussed by Garfield (2003) and others who have used this instrument with college and high school students. Participants took as much time as they needed to answer questions, and it took them an average of 53 minutes to complete the assessment. An example of the assessment items is in Figure 1. A second subgroup consisted of 28 senior high school students who had just completed grades 9-12, and 17 students who had just completed grades 7 or 8. Their high schools were also located in the same state as the college students. These high school students took the same Statistical Reasoning Assessment (Garfield, 2003) via paper and pencil over two days. The high school students were conveniently sampled from a STEM summer camp and attended 23 different high schools.

Several skills, chosen based on state curriculum benchmarks, were assessed. Using Classical Test Theory (Bechger, Maris, Verstralen et al., 2003), item difficulty for each skill was assessed by finding the proportions for the number of the actual correct responses to the number of possible correct responses. Proportion values are reported on a scale of 0 to 1. Consistent with Classical Test Theory, if 90 out of 100 students responded correctly on an item, the proportion value for that item will be 0.9 and is considered an easy item because most of the students got the item correct. Alternatively, if only 10 out of 100 students got the item correct, the proportion value will be 0.1. This item will be considered difficult. In general low proportion values (closer to 0) are associated with difficult items while high p-values (close to 1) are associated with easy items. Items with average difficulty have proportion values around 0.5. We anticipated higher proportion values for correct reasoning skills. These proportion values are typically reported as p-values but in this paper are simply referred to as proportion values. We use p-values in the results section to report probability values for ANOVA.

Data were disaggregated into grade bands, 7-8, 9-10, 11-12, and college. Disaggregating data this way was necessary because of the curriculum opportunities and the order in which junior and senior secondary students take mathematics classes, which potentially affect opportunities for students to be enculturated into statistical reasoning. Higher proportion values of correct reasoning skills are interpreted in this study as higher reason skills, relatedly, low proportions of misconceptions indicate high reasoning skills, and vice versa. One-way ANOVA tests were used to test whether the mean item difficulties (p-values in Classical Test theory terms) were significantly different for both correct reasoning skills and misconceptions across grade level bands.

A marketing research company was asked to determine how much money teenagers (ages 13 - 19) spend on recorded music (cassette tapes, CDs and records). The company randomly selected 80 malls located around the country. A field researcher stood in a central location in the mall and asked passers-by who appeared to be the appropriate age to fill out a questionnaire. A total of 2,050 questionnaires were completed by teenagers. On the basis of this survey, the research company reported that the average teenager in this country spends $155 each year on recorded music.

Listed below are several statements concerning this survey. Place a check by every statement that you agree with.

a. The average is based on teenagers' estimates of what they spend and therefore could be quite different from what teenagers actually spend.
b. They should have done the survey at more than 80 malls if they wanted an average based on teenagers throughout the country.
c. The sample of 2,050 teenagers is too small to permit drawing conclusions about the entire country.
d. They should have asked teenagers coming out of music stores.
e. The average could be a poor estimate of the spending of all teenagers given that teenagers were not randomly chosen to fill out the questionnaire.
f. The average could be a poor estimate of the spending of all teenagers given that only teenagers in malls were sampled.
g. Calculating an average in this case is inappropriate since there is a lot of variation in how much teenagers spend.
h. I don't agree with any of these statements.

Figure 1: Assessment Item [Source: Garfield, Joan B. (2003). Assessing statistical reasoning. *Statistics Education Research Journal* 2(1), 22–38.]

Results

What are students' statistical reasoning skills across grade-bands?

In Table 1, high proportion values for correct reasoning skills indicate that students had most of the required skills. Low proportion values for correct reasoning skills indicate lack of required skills for correct statistical reasoning. Our data showed the strongest skills in computing and interpreting probabilities, and the weakest in understanding independence. It is notable that there are only three proportional values (probabilities and understanding large numbers) over all that are greater than 0.5. What is also notable is the general trend for the college students and the 7th-8th grade students to have proportion values that are higher than the other grade bands.

Table 1

Proportion Values (Item Difficulty) for Correct Reasoning Skills

Correct Reasoning Skills	Whole Group (n=84)	College (n=39)	Grade 11-12 (n=14)	Grades 9-10 (n=14)	Grades 7-8 (n=17)
1. Correctly interprets probabilities	0.64	0.73	0.57	0.54	0.56
2. Understand how to select appropriate average	0.34	0.35	0.34	0.30	0.35
3. Correctly computes probabilities	0.79	0.90	0.64	0.93	0.53
4. Understands independence	0.25	0.23	0.36	0.14	0.29
5. Understands sampling	0.38	0.42	0.45	0.29	0.31
6. Distinguishes between causation and correlation	0.42	0.41	0.48	0.36	0.43
7. Correctly interprets two-way-tables	0.39	0.44	0.50	0.21	0.35
8. Understands importance of large numbers	0.51	0.54	0.71	0.36	0.41

When all the correct reasoning skills are treated as one unit by finding the means for each grade band, one-way ANOVA revealed that there are no significant differences between the skills of different grade-bands ($F=2.187$, $p>0.05$) after running homogeneity tests where equal variance assumption was met. However, comparing means of the bands for the different skills yielded slightly different results. Only one skill – computing probabilities – showed differences in the means across grade bands for both Brown Forsythe test and the more conservative Welch test ($p= 0.012$ for Brown Forsythe and $p=0.025$ for Welch test). These tests were used because homogeneity of variance was not met ($p<0.05$) and post hoc analysis of the results using Games-Howell test indicate that skill means for grade 7-8 band on computing probabilities was significantly lower than the 8-9 grade band ($p= 0.004$).

What are students' misconceptions across grade bands?

In Table 2, high proportion values for misconceptions scale indicate that students had very limited understanding of concepts for statistical reasoning. Low proportion values for the misconception scale indicate that students have higher statistical reasoning skills. Misconceptions assessed included outcome orientation (overlooking a series of events when making probability decisions and focusing on one) and representative misconception (believing a sample is representative only if it resembles the parent population). Proportion values for all misconceptions are lower than 0.5, indicating good reasoning skills. None of the students in grades 9-12 showed any confusion between the mean and the median. The most prevalent misconception is students' failure to consider outliers when interpreting data. Similar to correct reasoning skills, a comparison of means showed no significant difference for each skill and aggregated means for the whole set of misconceptions.

Table 2

Proportion values (Item Difficulty) For Misconceptions

Correct Reasoning Skills	Whole Group (n=84)	College (n=39)	Grade 11-12 (n=14)	Grades 9-10 (n=14)	Grades 7-8 (n=17)
1. Misconceptions involving averages					
a)Averages are the most common	0.25	0.32	0.14	0.21	0.21
b) Fails to take outliers into account	0.40	0.31	0.43	0.64	0..41
c) Compares groups based on their averages	0.26	0.27	0.36	0.18	0.21
d) Confuses mean with median	0.06	0.08	0.00	0.00	0.12
2. Outcome orientation misconception	0.14	0.14	0.13	0.17	0.12
3. Good samples have to represent a high percentage of the population	0.03	0.02	0.02	0.02	0.06
4. Law of small numbers	0.29	0.26	0.36	0.32	0.26
5. Representativeness misconception	0.08	0.04	0.05	0.11	0.15
6. Correlation implies causation	0.19	0.14	0.07	0.39	0.24
7. Equiprobability bias	0.44	0.41	0.29	0.50	0.59

Discussion

The purpose of this study was to explore students' statistical reasoning prior to taking a college statistics course and to see if there are significant differences in the reasoning across grade bands. A Statistical Reasoning Assessment was administered and data were analyzed using the Classical Test Theory approach of using proportional values to study item difficulty. ANOVA was used to compare reasoning across grade bands. The study revealed mostly very weak statistical reasoning skills that did not increase across grade bands.

Framing the development of statistical reasoning as a process of becoming anticipates students' statistical reasoning to become stronger the more students participate in statistical reasoning environments. Additionally, the cumulative inherent nature of mathematics and curriculum expectations positions one to expect a significant increase of reasoning skills across different grade bands even after taking into account that growth of understanding and the process of enculturation are often recursive. Since statistical reasoning "appears to be universally accepted as a goal in statistics classes" (Garfield, 2003, p.3), lack of differences across grade bands is a finding that calls us to question the enculturation of statistical reasoning by looking at affordances and constraints of different pedagogies and curriculum in different learning environments.

Consistent with existing literature (e.g., Garfield, 2003; Bechger, Maris, Verstralen et al, 2003), understanding independence, sampling variability, and selection of appropriate average were the correct reason skills with very low proportions indicating high difficulty and weak reasoning . The more prevalent misconception was the failure to take outliers into account when reasoning about data. These sampling concepts are fundamental to statistical reasoning; understanding appropriate averages for different samples is fundamental to reading data and inferential statistics. More research and professional development that focus on instructional materials and pedagogies are needed to foster reasoning about samples and averages.

As stated earlier in this paper, this study sampled all high school students who attended a free STEM camp. This kind of sampling resulted in having participants from more than 23 high schools. The participants were not randomly but conveniently sampled, a limitation most quantitative studies in the field of education faces. Although this study has generalization limitations because of the convenience sampling, it makes significant contributions to statistics education. This Statistical Reasoning Assessment instrument has been used widely to sum statistical skills for high school and college students, to compare skills of students in different countries, and compare reasoning between genders. To our knowledge, it has not been recently used to compare skills at different K-12 grade levels. The overall lack of significant differences across grade levels that emerged in this study with smaller sample size invites further research with bigger sample sizes to see if the results would be similar. Since curriculums guide teaching which may result in different curriculums creating affordances and constraints to statistical reasoning, comparing statistical skills outcomes with different curriculums and a focus on how teachers specifically teach students to use and apply different types of statistical reasoning need to be explored.

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Leading and Learning for Pre-Service and In-Service Teacher Support

EXAMINING FACTORS THAT INFLUENCE MATHEMATICS LEARNING: AN AREA UNITS LESSON EXPERIMENT WITH PROSPECTIVE TEACHERS

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A lesson experiment is an intentional process to examine factors that influence learning, here used to evaluate and enhance prospective teachers' (PSTs) understandings of area units. An important construct for area measurement is understanding how area units are constituted by linear units. After the lesson experiment, nearly all PSTs increased their understanding of area units as 1-unit by 1-unit squares while approximately half understood how area units are defined by linear units. Instructional recommendations for enhancing PSTs' understandings of the relationship between area and linear units are provided, potentially applicable for K-12 students as well.

Area measurement is a significant concept in school mathematics and is mandated in the curriculum guidelines for nearly all countries. As such, it is an important content area for teachers to understand. Yet, area measurement is more abstract than other measures such as length (Murphy, 2011). The purpose of this study is to use a lesson experiment to evaluate and enhance prospective elementary teachers' (PSTs) understandings of area measurement and units.

Related Literature: Area Measurement and Lesson Experiments

Area measurement is a challenging concept for students and teachers. One factor is our cultural practice of using units of length to determine area measurements, rather than counting area units (Nunes, Light, & Mason, 1993). Multiplying such linear measures is conceptually far removed from the idea of coverage with square units, yet students have to master this historically developed practice. With a rectangle, students structure it as an array of rows and columns, join the individual units within a row into a composite unit, and then iterate that composite unit a number of times corresponding with the width (Battista, 2003). Furthermore, even once students internalize this process, they still need to understand how linear units constitute individual area units. Consider the following problem:

> Two people work together to measure the size of a rectangular region. One measures the length and the other the width. Each uses a stick to measure with; the sticks, however, are of different lengths. Louisa says, "The length is four of my sticks." Ruiz says, "The width is five of my sticks." What have they found out about the area of the rectangular region? (Simon & Blume, 1994, p. 487)

Although we do not conventionally measure rectangles with different units for the length and width, Louisa and Ruiz may say the area measurement is 20 sub-rectangles where each is one of Louisa's sticks by one of Ruiz's sticks. In other words, the two sticks generate each rectangular area unit. Thus, a full understanding of area extends beyond just counting individual square units and requires coordinating two dimensions (Lehrer, Jaslow, & Curtis, 2003).

Researchers have found that K-12 students as well as PSTs struggle with understanding how linear units constitute an area unit (e.g., Baturo & Nason, 1996; Lehrer, 2003; Simon & Blume, 1994). In a previous study, we too found that PSTs experience challenges with understanding area units and their constitution by linear units (Chamberlin & Candelaria, 2018). Observing such struggles as a teacher educator led me to want to address such difficulties through further instruction. Thus, I undertook the present lesson experiment to examine elementary PSTs' understandings of the relationship between length and area units.

For a lesson experiment, a teacher-researcher engages in cycles of testing hypotheses about cause and effect relationships between teaching and learning. The intent is to *examine factors that influence learning* by asking, "What did students learn during the lesson, and how and why did instruction impact such learning?" (Hiebert, Morris, Berk, & Jansen, 2007, p. 48). The experiment is composed of four steps. The first step consists of explicating the learning goals and instructional hypotheses, both of which inform the lesson planning. The second step entails assessing to what extent students achieve the learning goals by gathering during the lesson and analyzing afterwards evidence of students' thinking from videos, transcripts, written work, or verbal statements. The third step consists of evaluating the hypotheses for why the lesson did or did not achieve the learning goals. The fourth step entails revising the lesson based on evidence from the previous steps. The intent of the revised lesson is to help students achieve the learning goals even more so than in the first lesson. Lesson experiments shift an instructor's focus from teaching in the moment to including preparation and reflection outside of the classroom.

Method

I conducted the lesson experiment in an undergraduate geometry and measurement class for elementary PSTs with 31 enrollees. My learning goals were for the PSTs to understand how area units are constituted by length units. The lesson experiment included three instructional activities, aligning with three instructional hypotheses. My first hypothesis was creating standard area units would enable the PSTs to recognize the corresponding length units, visualize a standard area unit as a 1-unit by 1-unit square, and understand why we square standard area units. Accordingly, the first activity involved the PSTs in using rulers and masking tape to physically generate a square millimeter, a square centimeter, a square decimeter, and a square meter. My second hypothesis was presenting the PSTs with an area problem in which only the length units for the area unit are given would encourage them to understand how length units constitute an area unit. As such, the PSTs worked on the Stick Problem above. My third hypothesis was working with a rectangle in which two different area measurements are feasible would open up the PSTs to considering the size and shape of two different area units. Thus, the PSTs worked on the Jack and Jill Area Problem for the third class activity (see Figure 1).

Jack and Jill Area Problem

Suppose Jack and Jill were given a rectangle to measure. The only measuring tool they had was a smaller rectangle. (You have been provided with copies of the large rectangle and the smaller rectangle.) Jack measured the width of the rectangle using the smaller rectangle, rotated the smaller rectangle 90 degrees, and then measured the length of the rectangle with the smaller rectangle. He determined the rectangle has an area of $2 \times 4 = 8$. Jill did a similar thing but oriented the smaller rectangle the other way, as shown below. She determined the rectangle has an area of $5 \times 10 = 50$. How can it be that they both have found some useful information about the area of the large rectangle? What *area unit* did each of them use?

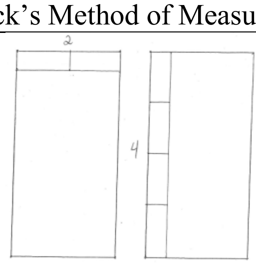
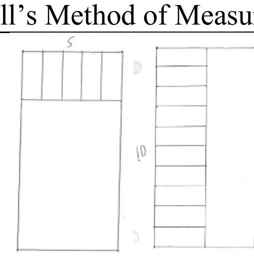
Jack's Method of Measuring	Jill's Method of Measuring
	

Figure 1. Jack and Jill Area Problem completed by PSTs as part of the instructional activities.

For evidence of the PSTs' thinking during and after the lesson, I video-taped all whole class discussions and collected from each PST their work on a pre-assessment item, an in-class formative assessment (completed immediately following the whole-class discussion of the Jack and Jill Area Problem), and two homework questions (turned in 1-2 class periods after the lesson activities) (see Figure 2). My data analysis consisted of two phases, determining the extent to which the learning goals were achieved and then evaluating the hypotheses and instruction for whether and how they supported the learning goals. For Phase I, I used open coding (Strauss & Corbin, 1998) to analyze the PSTs' responses on the assessment items, identifying themes within the PSTs' responses and the frequency of those themes. For the video-tape analysis, I transcribed each discussion. For Phase II, I began by noting instances in which the learning goals were attained by the PSTs and returned to the transcripts to identify which lesson activities and hypotheses engendered such understandings. Next, I noted instances in which the learning goals were not attained, or misconceptions remained. I again returned to the transcripts to identify how the lesson activities or hypotheses may have failed to address such issues. These issues pointed to possible instructional improvements.

Assessment Items

Pre-Assessment: Why do we square the units for an area measurement, for example cm^2?

Formative Assessment: Suppose Anthony tessellated a rectangular piece of paper with 36 1-inch by 2-inch rectangles. Can Anthony say the area of the paper is each of the following area measurements? If so, what are the associated area units and their size? If not, why not?

a. 36 b. 72 c. 18

Homework Items:

1. What does in^2 represent? What does cm^2 represent? What does ft^2 represent? What does any standard area unit represent (look like)? Why? What are their dimensions? In other words, why do we raise all standard area units to the second power (square them)?
2. Sally and Joe measured a standard sheet of paper and CORRECTLY found its area to be 598 square centimeters. Next, they are supposed to measure the front of a standard door and decide to do so in terms of sheets of paper. They find that it takes approximately 7 sheets of paper oriented the long way to match the height of a standard door and approximately 3 sheets of paper also oriented the long way to match the width. They then take 7 x 3 x 598 = 12,558 and state that the area of the front of a standard door is approximately 12,558 square centimeters.
   a. Explain the error in their measuring process for the area of the front of a standard door.
   b. What was the size of the area unit that they (unintentionally) used?

Figure 2. Assessment items completed by PSTs after the instructional activities.

Extent to which the Learning Goals Were Achieved

On the pre-assessment, 37% of the PSTs associated squaring standard units with the fact that area is measured with *square* units. Thirty-three percent realized that standard area units are squared because two linear units are multiplied. As one PST explained, "We square units when reporting area because we have to take the heights times the lengths so we are also multiplying the inches by inches." However, these PSTs did so in terms of the overall lengths of a presumed rectangle rather than the dimensions of a single area unit, revealing the potential misconception that all areas are found by taking $length \times width$. Forty percent explained area units are squared because area is a two-dimensional attribute. Only 17% revealed that a standard area unit represents a 1-unit by 1-unit square.

The first class activity began when I asked the PSTs to use a ruler to draw a square millimeter, a square centimeter, and a square decimeter. Upon confirming that all of the PSTs' drawings were squares, I asked, "How did you decide how big to make these standard area units?" One PST said she took one centimeter by one centimeter. I then asked, "Why do we say a square centimeter? Why do we square that?" Selene (a pseudonym, as are all names) explained, "We talked about how it was enclosing that space so that you are accounting for everything in the middle." I reiterated her comment and continued, "So, we square the centimeter here. Why? Where did that exponent of 2 come from?" Jody commented, "Because you're multiplying the two sides together for both of those." I summarized, "How do you find the area of a square? We multiply the lengths of the two sides so for example a square centimeter will be $1\, centimeter \times 1\, centimeter = 1\, centimeter^2$." I then directed the PSTs to use masking tape to create a square meter on the floor. Finally, we talked about how to visualize a square kilometer (1-kilometer by 1-kilometer) and compared the faces of the base ten blocks to a square centimeter and a square decimeter.

For the second class activity, the PSTs worked individually and then in groups on the Stick Problem. Our whole class discussion began with acknowledging that if Louisa's and Ruiz's sticks had been the same length, then they could have said the area was "20". I then pushed the PSTs, "Even though their sticks were of different length, can we say that the area is 20 of something? Try drawing some pictures perhaps." After some time to discuss in their groups, Lola shared, "I was thinking about rather than using square units, they would be like rectangular units." I asked her, "Did you draw a picture of it or did you just kind of talk about it?" She said they just talked about it, so I proceeded to draw a picture of the scenario with two different geostrips, using one to draw four length segments across the top and the other to draw five length segments along the side. I then asked, "So, do you see these rectangles that Lola was talking about? What would be the size of those smaller rectangles? Does 20 relate to those in any way?" Kaitlyn explained, "We said that there are 20 rectangles on the inside." I drew in the corresponding lines for the 20 rectangles inside the large rectangle. I commented, "This makes sense right because the dimension along here was 4 and this was 5, so 4 rows of 5 or 5 rows of 4. So, what's the size of each of those rectangular area units?" One PST shared, "One stick by one stick." I reiterated, "Could we say the area of their rectangle is 20 sub-rectangles where each sub-rectangle is the size of Louisa's stick by Ruiz's stick? . . . We completed this activity because I wanted to highlight the fact that what determines your area unit is the lengths of the two sides of your unit. … We're using those linear dimensions to define what the area unit is." The PSTs then worked in groups on the Jack and Jill Area Problem.

Our whole class discussion began by acknowledging that if Jack had not changed the orientation of the smaller rectangle, his area unit would have been the smaller rectangle. I then asked, “Can anyone help us when they did change the orientation, what area unit were they using?” Debra asked, “Were they like just making a square?” On the document camera, I used the length of the small rectangle to mark corresponding lengths along the top and side of the large rectangle. I commented, “Talk at your tables, do you see the square that Debra was talking about?” After time to discuss in their groups, I commented, “Jack said the area was 8. So the area could be 8 if you take what as your area unit?” The PSTs collectively responded, “squares”. I asked, “With what size of squares?” One PST answered, “length by length”. I concluded, “The area units are squares with a side dimension equal to the length of the small rectangle. . . . Talk at your tables about what Jill was doing. Can one also argue the area is 50? If so, 50 what?” After the PSTs discussed in their groups and appeared to understand Jill’s procedure, I summarized, “Did it make sense that Jill used the squares that were the width by the width. … Okay, so the area unit is generated by its two length dimensions.” This concluded the lesson.

On the formative assessment, 53% of the PSTs were able to identify the area unit on all three parts, 20% on two parts, and 17% on one part. Seventy-seven percent of the PSTs acknowledged throughout that different area measurements are possible given different area units, with five more PSTs acknowledging this fact on two of the three parts. On the first homework item, 90% of the PSTs acknowledged that standard area units are squares, with 80% also explaining that each area unit is a 1-unit by 1-unit square. Fifty-percent of the PSTs explained that we square all standard area units because when finding the area of a 1-unit by 1-unit square we multiply the unit times itself. Six more PSTs explained that one has to square the unit in order to form a square; otherwise, one would just have a length. On the second homework item, 48% of the PSTs realized that Sally and Joe produced the unintentional area unit of a square with dimensions equal to the long side of the paper. Five PSTs realized Sally and Joe’s error in using the long side of the sheet of paper to measure both height and width but were not able to determine the size of the area unit. Finally, nineteen percent of the PSTs failed to realize that a new area unit had been generated.

Evaluating the Instruction and Lesson Revisions

The instructional activities supported the PSTs in attaining some of the learning goals. First, the recognition that standard area units were squares improved from 37% on the pre-assessment to 90% on the first homework item, supported by the first activity of creating standard area units. Second, many of the PSTs understood different area measurements are possible for the same region given different area units, addressed by the formative assessment and the Jack and Jill Area Problem. Finally, the number of PSTs that visualized a standard area unit as a 1-unit by 1-unit square improved from 17% on the pre-assessment to 80% on the first homework item, also likely a result of the first activity.

Other learning goals were attained by only approximately half of the PSTs, indicating areas of improvement to the lesson. First, only 50% of the PSTs explained standard area units are squared because their areas are $1\ unit \times 1\ unit = 1 unit^2$. Upon analyzing the transcript, I realized this point was quickly addressed by me with little PST input. Thus, I recommended projecting and discussing this question after the first activity: How does the size of an area unit relate to the fact that we square standard area units? Second, the percentage of PSTs that understood how the area units are constituted by the linear units on the various assessments ranged from 50-65%. Upon reviewing the transcripts for the Stick Problem and the Jack and Jill Area Problem, I found the PSTs did not generate drawings representing the scenarios as much as I intended, reducing the opportunity for me to call upon them to share their drawings during the whole-class discussions (see comments from Lola and Debra above). Thus, before completing the Stick Problem, I would ask the PSTs to complete the formative assessment as a group activity. Being more concrete in terms of area measurements and dimensions, the PSTs may be more likely to generate drawings representing the problem. Next, I would have the PSTs again complete the Stick Problem and expand the Jack and Jill Area Problem to include Jack, John, and Jill. Jack and John would measure the rectangle as Jack did but use two rectangular units with the same length and different widths. The PSTs would consider, “How could Jack and John get the same area measurement when they used different rectangular units? What area unit did they each use?” Then, I would ask the PSTs to compare the results for Jack and John to those of Jill.

Discussion

In this study, I used a lesson experiment to evaluate and enhance elementary PSTs’ understandings of how area units are constituted by length units. Nearly all PSTs increased their understanding that standard area units represent 1-unit by 1-unit squares with approximately half understanding the relationship between area and linear units. Instructional recommendations included re-ordering and adapting the classroom activities, intended to increase the likelihood that PSTs will generate their own models of the area and length units. The lesson experiment process revealed factors influencing the PSTs' mathematics learning, in particular affordances of working with physical models of the length and area. I look forward to implementing the revised lesson and undertaking a second cycle of the lesson experiment, examining how the proposed changes may increase PSTs' attainment of the learning goals.

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TEACHING MOVES AND RATIONALES OF PRESERVICE ELEMENTARY SCHOOL TEACHERS

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This study explored teaching moves used by preservice teachers during one-on-one problem-solving interviews with children. Teaching moves used fell into six categories: (a) exploring details of children's strategies, (b) investigating multiple strategies and representations, (c) ensuring comprehension of the problem, (d) telling information, (e) re-voicing children's language, and (f) asking for a final answer. The preservice teachers' rationales for making teaching moves were also examined through stimulated recall interviews. Rationales were either geared to support children's understandings or to support the preservice teachers' understandings. Recommendations are made for teacher educators.

Conceptual Framework

The current vision of mathematics teaching and learning is supported by policy documents that emphasize children's sense making, which presents a changing narrative of what it means to engage in mathematics (National Council of Teachers of Mathematics, [NCTM], 2014). In the traditional approach, children solve problems using prescribed steps to arrive at correct answers with limited opportunities for reasoning. In contrast, the current vision positions teachers as facilitators who elicit and build on children's mathematical thinking in order to promote sense-making. Overall, shifts in how we view mathematics teaching and learning support an approach where teachers listen closely to children in order to make instructional decisions.

Responsive Teaching and Children's Mathematical Thinking

One approach to instruction that promotes reasoning and foregrounds children's thinking is responsive teaching. *Responsive teaching* is centered around teachers listening carefully to children's thinking and not only provides space for children to share ideas but also emphasizes teachers' own understanding of children's thinking to support their instructional decision-making (Robertson, Atkins, Levin, & Richards, 2016). Further, use of children's mathematical thinking during instruction has resulted in improved achievement (Carpenter, Fennema, Peterson, Chiang & Loef, 1989). In short, responsive teaching is an approach that has the capacity to support the current vision of mathematics teaching and learning because as children engage in sense-making, teachers learn about their thinking and decide what to do next.

Teaching Moves

Within the questioning and facilitating discussion in mathematics literature, a variety of terminology have been used to describe ways in-service and preservice teachers (PSTs) elicit and build on children's ideas. For the purpose of this paper, teaching moves will be used. *Teaching moves* are defined as any intentional teacher action, often in the form of questions, series of questions, statements or actions (Jacobs & Empson, 2016). Across the literature, five major categories of teaching moves were recognized, and teaching moves were grouped based on the similarities in the primary goals of the moves. Teaching moves can be used to build on children's thinking, but whether or not this goal is achieved may depend how the moves are enacted.

The first category is one that requires teachers to attend to the mathematically important aspects of what children say and do. The most common forms include pressing and probing, which consists of asking children to further explain their thinking or invite children to share their strategy (Franke et al., 2015, Moyer & Milewicz, 2002). Other forms may include linking children's strategies to story problem contexts or expanding children's understanding of quantities (Jacobs & Empson, 2016). Second, investigating multiple strategies and representations is a category of teaching moves that allows children to see, hear, or work with ideas in different ways. For example, teachers may ask children to solve a problem using an alternate strategy or look for relationships within or across strategies (Cengiz, Kline, & Grant, 2011; Franke et al., 2015). The third category of teaching moves aims to make certain that children understand the context of a problem they are solving, which can support children's access to the mathematics. For instance, teachers may help children unpack a problem by providing background knowledge or orient them toward details of the problem (Jacobs & Empson, 2016). Fourth, telling is a category of teaching moves that provides children with pieces of information teachers believe to be important for problem-solving. It occurs when teachers tell children about concepts, show particular strategies, or label terminology (Chazan & Ball, 1999; Moyer & Milewicz, 2002). Lastly, revoicing is a category of teaching moves that elevates children's ideas because teachers use the language of children when communicating. For example, a teacher may repeat word for word or rephrase what a child has explained (O'Connor & Michaels, 1993). In sum, there are a range of teaching moves expressed within the literature. However, teaching moves have primarily been studied with in-service teachers, and fewer studies included PSTs (Moyer & Milewicz, 2002).

Current Study

The purpose of this exploratory study was to get a better sense of what teaching moves PSTs make when engaging children with story problems as well as their rationales for making the moves they do. As teacher educators strive to support PSTs to enact the current vision of mathematics teaching and learning, it is important to consider what strengths PSTs have in eliciting and building on children's thinking as they begin their program. Similar to how research has shown the importance of teachers being responsive to children's thinking, it is important for teacher educators to be responsive to the needs of PSTs in development of methods courses.

Methods

Participants were in the second semester of a teacher preparation program at a university located in the southeastern region of the United States. Five PSTs, one male and four females were recruited at the start of their mathematics methods course. The study aimed to answer two research questions: (1) What teaching moves do PSTs make when engaging in math conversations with children around story problems? and (2) What rationales do PSTs give as to why they make the moves they do?

Data Collection

Each PST participated in two problem-solving interviews (PSIs) and two stimulated-recall interviews (SRIs). The PSIs were 15-minute one-on-one conversations between one PST and a first or second grader. PSTs were asked to pose a list of seven story problems as time permitted. Place value was chosen as a topic of the problems because it spans a large part of the elementary mathematics curriculum. The story problems included all four operations with whole numbers to provide PSTs with opportunities to respond to a range of children's strategies (Carpenter et al., 1989). The PSIs were both audio and video-recorded.

Immediately following the PSIs, PSTs engaged in an audio-recorded SRI in which the video from the PSI was viewed to retrospectively elicit their reasoning about each of the teaching moves they made (Gass & Mackey, 2000). This elicitation was accomplished in two ways. First, PSTs were asked to pause the video any time they wanted to share their reasoning. They were allowed to initially control pausing of the video, so they would feel empowered in their thoughts. Second, after the PSTs had shared the rationales they chose to discuss, other segments of the video were revisited to elicit rationales for any teaching moves not yet discussed. Rationales were elicited based on the context, but phrasing was similar in all cases.

Data Analysis

To analyze the data, two different coding methods were employed. First, PSI data were analyzed with provisional coding that began with the five categories of teaching moves identified in the literature. In iterative cycles, categories were either eliminated, collapsed, or expanded based on what was seen in the data. The unit of analysis was a single teaching move—not a talk turn—in order to capture the complex nature of teaching moves. Then, patterns within each category were further explored. At this stage of the analysis, the goal was to gain a general sense of what teaching moves were made across all PSIs, thus focusing on total number of teaching moves. Later analysis further examined frequencies of teaching moves per each PST. Second, SRI data were transcribed and "in vivo" coding was used to honor the voices of the PSTs as short words or phrases were taken from the PSTs' own language and grouped into categories of rationales that held similar themes. Last, SRI transcripts were coded using these categories with a single rationale serving as the unit of analysis. At this stage of analysis, describing the rationale categories were the focus and future analysis will consider frequency of each rationale.

Results

Teaching Moves

In this study, the goal was to describe the overall range and frequency of teaching moves made, not how well the moves were executed. Six categories of teaching moves were identified—the five found in the literature and one new category, asking for the final answer. Table 1 identifies the relative frequency of the six categories across all PSIs, and the following sections describe and illustrate the two most commonly used categories.

Table 1

Categories of Teaching Moves and Their Relative Frequency

Category of Teaching Moves	**Frequency** ***(% of total number of moves)***
Exploring details of children's strategies	52%
Ensuring comprehension of the problem	29%
Telling information	7%
Revoicing children's language	6%
Investigating multiple strategies and representations	3.5%
Asking for a final answer	2.5%

Teaching moves to explore details of the child's strategy. The most prevalent category of teaching moves were moves that explored details of children's strategies, and these moves appeared in three distinct forms: inviting children to share, pressing for reasoning, and linking strategies to the story-problem context. Within this category of teaching moves, 47% emerged as the first form, 41% as the second form, and 12% as the third form. Each of the three forms were further described below with representative examples that emerged from the data. Then, an example is provided to explicitly illustrate the three forms PSTs most commonly used to explore details of children's thinking.

Inviting children to share. PSTs asked children to share their strategy by posing general questions to begin or continue conversations. Children were typically invited to share their strategy soon after solving the problem by asking, "*How did you get [the answer]?*" This move may be easier to employ but PSTs did not invite children to share their thinking consistently across PSIs. However, this finding does show promising ways PSTs supported children in providing space to make their ideas visible and could be a starting place for teacher educators.

Pressing for reasoning. PSTs explored details of children's strategies by pressing for reasoning, which involved focusing on part or all of a strategy and asking children to further explain. This finding demonstrated the potential for PSTs to attend to some detail in children's strategies. Further analysis revealed patterns in how PSTs focused their pressing. Almost half of the pressing moves were directed on the tools selected by the child to solve the problem. For example, when a child was unsure of how to solve the problem, one PST asked, *"How are you grouping [the base ten blocks]?"* The emphasis on tools seemed interesting to PSTs and easier for them to gravitate toward when children were unsure of how to solve a problem.

Linking strategies to problem context. PSTs explored the details of children's strategies by asking them to link part of their strategy to the context of the problem. It can be helpful for children to use familiar contexts to solve problems in mathematics, and PSTs showed beginning abilities to recognize the importance of context for children's understanding (see Figure 1).

Example of exploring details of children's thinking. One PST posed the problem: *Aaliyah had 15 toy cars. Her mom gave her some more toy cars for her birthday. Then she had 25 toy cars. How many toy cars did her mom give her for her birthday*? The child used the hundreds chart as a tool. She circled 15 and used the marker to indicate small jumps as she counted to 25. However, as she was counting by ones, she paused on 21 and shaded in that number before continuing to count to 25.

Child: (Shows strategy on hundreds chart) 10.
PST: How did you get 10?
Child: I started with 15, then I jumped so (pause) 1, 2, 3, 4, 5, 6…11 (pointing to jumps on the hundreds chart).

Explore details: Invite child to share

PST: So, you started counting how many jumps you need?
Child: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (counting jumps). I got 10.
PST: Ok, and how come you marked out your 21?
Child: Because I (pause) so when we cross the line, we get 21. That is what my teacher always does.

Explore details: Press for reasoning

PST: Okay. So, what is 10?
Child: 10 is a whole row (pause) 10 is the whole row (pause) the whole row.
PST: Is 10 how many cars (pause) how many she started out with, or how many cars she got or how many cars she ended up with?

Explore details: Link strategy to

Figure 1. Three teaching moves working together to explore details of children's strategies.

Teaching moves to ensure the child comprehended the story problem. The second category of teaching moves used the most by PSTs were moves that ensured the child comprehended the story problem, and these moves occurred in two distinct forms: re-reading part or all of the problem and orienting children to details in the problem. Within this category of teaching moves, 87% emerged as the first form and 13% as the second form. PSTs re-read the problem, or part of the problem often without prompting. This re-reading usually occurred simultaneously as children were solving and appeared to interrupt their thinking. The extensive use of this move suggests that PSTs need to learn more ways to support children's understanding of context.

Rationales for Enacting Teaching Moves

PSTs rationales for the moves they made were analyzed in addition to the teaching moves. During analysis, two themes emerged including making a teaching move to support the child's understanding or making a teaching move to further support the PST's understanding. Each theme is further described and illustrated below, with examples representative of the data.

Rationales linked to supporting the child's understanding. The first theme that emerged from the data were rationales focused in supporting the child's understanding. PSTs shared making teaching moves to help children accomplish different goals such as understanding the story problem context, arriving at the correct answer, or children articulating their thought processes. For example, PSTs expressed using a teaching move because they wanted children to understand the story-problem context, "*If I break [the story] down for her, that will help her process it.*" Other examples illustrated how PSTs made teaching moves to help children arrive at the correct answer because children would get *"stuck"* or in their opinion, the child's strategy became *"messy"*. In one instance, a PST said, "*I wanted her to focus on the drawing because I thought that would probably best help her solve the problem.*" PSTs seemed aware that they should not give away too much information and expressed wanting children to articulate their own thoughts and identify their own errors. One PST described, *"I didn't want to be like—well—no, you had the right answer the first time. So, I figured that she would show me when I asked her that and then she would catch herself afterwards and realize it.*" Collectively, the evidence implied PSTs recognize the importance of children taking on the mathematical work in problem-solving, but PSTs may also have an end goal of correct answers.

Rationales linked to supporting the teacher's understanding. The second theme that emerged from the data were rationales focused in supporting their own understanding. PSTs conveyed making teaching moves because they were unfamiliar with children's strategies. In particular, PSTs expressed that they "*felt lost,*" or said, "*it was hard to follow what she was thinking".* For example, a child used a number line to solve a problem. The PST expressed her confusion of the child's strategy saying, *"Ok, that threw me for a loop because I've never seen that before, putting on a number line like that….to hop from random numbers until you get to a number, just threw me for a loop."* Additionally, PSTs compared their own thinking with that of children, expressing curiosity in how children solved. For instance, one PST observed a child solve a multiplication problem using unifix cubes and he asked the child about her use of the cubes because, "*She never counted them. Her thinking was different than mine.*" Collectively, these examples suggested that PSTs may have an incomplete understanding of children's strategies.

Discussion and Implications

In responsive teaching, enacting particular teaching moves is challenging in-the-moment. In preparing responsive teachers, it is worthy for teacher educators to recognize beginning skills PSTs may already possess and find ways to build on them. PSTs invite children to share their thinking which is the first step in PSTs understanding children's ideas. Findings show PSTs are also able to attend to some level of detail in children's thinking as they often pressed children for reasoning. The findings suggest PSTs need additional exposure to other teaching moves. For example, PSTs need more specific ways to help children comprehend story problem contexts aside from repeating all or part of the problem to help children gain access to the mathematics.

It was important to ask PSTs to explain their rationales in making the moves they do. In the first theme, PSTs expressed the rationale of making teaching moves because they were unfamiliar with or unable to follow children's strategies. Similar to findings by Carpenter et al. (1989) in their work with teachers, PSTs also need access to research-based frameworks of children's thinking to develop a better understanding of their ideas. In the second theme, PSTs voiced the rationale of making teaching moves to guide children toward a correct answer. It can be implied PSTs need to learn and experience both the benefits of enacting teaching moves after the correct answer and other end goals of problem-solving (Jacobs & Empson, 2016).

Looking forward, future studies need to include PSTs in work that explores teaching moves in order for teacher educators to build on strengths of PSTs and offer continued support through methods. Although not a focus of this study, it would be important to examine implementation of teaching moves because although PSTs enacted a range of moves, some elicited and built on children's thinking, while others did not. Lastly, although this analysis did not specifically explore the relationship between teaching moves and the rationale provided, rationales may be connected with particular moves and could potentially be a direction for this line of research. Overall, findings provide promising knowledge teacher educators need to be more responsive to the needs of PSTs as they develop and teach methods courses.

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PRESERVICE TEACHERS' BELIEFS ABOUT THE USE OF THE NATIVE LANGUAGE BY ENGLISH LEARNERS IN MATH

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This paper reports on the beliefs that preservice teachers have about English learners using their native language in the mathematics class. A beliefs survey was administered to 620 preservice teachers and qualitative interviews were conducted with 14 participants. Overall, preservice teachers believe that the English learners will develop a better understanding of mathematics if they have opportunities to use their native language in the classroom. However, the preservice teachers are also concerned that allowing the use of the native language in class will interfere with the English learners eventually learning English.

English learners (Els) are a rapidly growing population in schools across the United States. ELs speak a language other than English at home and are deemed to require language services at school. In 2014-2015 there were 4.6 million English learners (ELs) in schools, 9.4% of the school population (NCES, n.d.). The overall increase in numbers over the years, combined with the *No Child Left Behind* act, has seen an increase of ELs in mainstream mathematics classes.

Currently, mainstream teachers are underprepared to work with ELs (Lucas & Gringberg, 2008; NCES, n.d.). Along with knowledge, skills, and dispositions, teacher beliefs are a key factor that impact the decisions they make during teaching (Lucas & Grinberg, 2008; Philipp, 2007). For example, Thompson (1984) report that a teacher who viewed mathematics as a collection of facts promoted the memorization of rules and procedures in teaching, instead of problem-solving. In addition to beliefs about mathematics, the teachers' beliefs about the students' language and culture are important (Sztajn, 2003). In the case of culturally and linguistically diverse students, language is central to their identity. The interactions between the ELs and the teacher can be influenced by the way the teacher positions the student's language and culture in the classroom. Many experiences of ELs outside the classroom remain encoded in their native language (Domínguez, 2011). Thus, native language is an important resource that teachers can tap into during their teaching. Given that PSTs will likely work with ELs in their future classrooms, beliefs they have about ELs using their native language in the classroom can inform teacher preparation.

The research question guiding the study was: What beliefs do PSTs have about ELs using their native language in the mathematics class? The study discussed here is part of a larger study that examines the beliefs that PSTs have about the teaching of mathematics to ELs.

Theoretical Framework and Literature

According to Philipp (2007) beliefs are "Psychologically held understandings, premises, or propositions about the world that are thought to be true" (p. 259). Further, "beliefs might be thought of as lenses that affect one's view of some aspect of the world or as dispositions toward action" (p. 259). Beliefs do not exist in isolation and are part of an overall beliefs system that share certain characteristics (Green, 1971). Some beliefs are primary, and others derived. Beliefs have a quasi-logical structure, which implies that one belief may follow from another if the person perceives this to be the case. Additionally, beliefs also have a psychological property of being central or peripheral in the system. If the belief system is viewed as concentric circles, then the central beliefs form the core, are held more dearly, and are harder to debate and change (e.g. beliefs about faith and religion). The peripheral beliefs on the other hand are not as psychologically strong for the person, are open to debate and easier to change (e.g., beliefs about the benefits of social media). Teachers' beliefs influence the actions they take in the class (Philipp, 2007). In addition to their beliefs about mathematics, teachers' beliefs about their students also influence the way they teach students. Sztajn (2003) illustrate how teachers with the same beliefs about mathematics reform implemented the curriculum differently based on the students' backgrounds. The teacher engages the students of middle-income parents with problem solving and projects. On the other hand, students of low-income parents engage in basic facts, drill, and practice.

A non-deficit view regarding ELs and their communities guided the design of the survey and the analysis of the interviews in this study (Civil, 2007; Moschkovich, 2010). Moschkovich (2010) states that "deficit models stem from assumptions about learners and their communities based on race, ethnicity, SES (socio-economic status), and other characteristics assumed to be related in simple and typically negative ways to cognition and learning in general" (p. 11). Non-deficit models, on the other hand, view the ELs and their communities as holding valuable resources, which can facilitate learning within the classroom. In this study, the students' native language was a key resource in their mathematics learning.

In the case of ELs, research shows that teachers make judgments about students' academic performance in content areas based on their English proficiency. Teachers believe that students who speak Standard English will do better in school (Marx, 2000; Walker, Shafer & Liams, 2004). Teachers also believe that the ELs are the responsibility of the English as a Second

Language (ESL) teachers and should come to the mathematics classroom after they are proficient in English (Reeves, 2006).

Prior research examines teachers' beliefs about language within the context of bilingual education. Even though teachers believe that the knowledge can transfer from one language to another, they are less supportive of the use of native language in the classroom (Karabenick & Noda, 2004; Karanthos, 2009; Reeves, 2006; Shin & Krashen, 1996). According to the research in bilingualism, forcing students to only use English can hamper EL students learning the content and English (Gandára & Contreras, 2009). Reeves (2006) and Walker, Shafer, and Liams (2004) found that a significant number of teachers in their studies questioned the use of the native language in the classroom. Teachers with strong ideological beliefs, like those that believe classes should be taught in English only, find it challenging to see the benefits of using native language in the classroom (Gandára & Contreras, 2009; Gutiérrez, 2002; Lucas & Katz, 1994). Despite the pervasive beliefs of teachers about the interference of native language in learning English, research shows that there is no difference in the rate of learning English in bilingual or English-only programs (Gandára & Contreras, 2009).

Research also showed that ELs use their native language in discussions to understand mathematics problems, share strategies, build on the strategies of other students, and manage the social interactions (Domínguez, 2011; Moschkovich, 2007; Turner, Domínguez, Maldonado, & Epson, 2007). Domínguez (2011) found that Spanish-English bilingual students were more likely to engage in mathematical discussions when interacting in Spanish than in English. In English, the students reported on their individual work and were less likely to push each other on their mathematical explanations. The students' native language was also tied to common experiences they had outside school, and these were resources that were brought into mathematics discussions to make meaning. Clarkson (2006) also demonstrated how students who were bilingual in Vietnamese and English at a certain threshold of proficiency were able to perform better in mathematics compared to students who were monolingual. The bilingual students were able to make strategic use of their two languages to make sense and solve problems. In sum, EL students' native language can be a resource in the mathematics class when drawn on by the teacher.

Methods

Quantitative and qualitative data were collected as part of the larger study that examined

PSTs beliefs about teaching mathematics to ELs. In the larger study, a 40-item survey was administered through an online platform (SurveyShare) to 620 PSTs across the U.S. (see Fernandes & McLeman, 2012) for the validity and reliability of the survey conducted with a similar group of 330 PSTs). Thirty Likert scale items in the survey were about PSTs' beliefs about teaching mathematics to ELs (1- Strongly Disagree, 2 -Disagree, 3 – Undecided, 4 – Agree, 5 – Strongly Agree), and 10 items related to demographics and prior experiences of the PSTs. Additionally, qualitative interviews were conducted with 14 PSTs from a university in the southeast. In the interview, the PSTs were asked to respond to 18 survey items that related to the language demands in mathematics and the resources that ELs brought to the classroom (e.g. Math is not language intensive; The different ways that ELs learned math in their home country are a valuable resource in the math class). PSTs were interviewed on 18 out of the 30 belief items to prevent fatigue. After the PST responded to an item in the interview, they were probed in detail about their responses. The PSTs were pushed to think about alternate perspectives to examine the strength of their beliefs. For example, if the PSTs mentioned that they would allow students to use their native language in small groups, they were asked to respond to another teacher who comments that the students should be speaking in English since they were in the U.S. Inferences about the strengths of their beliefs were made based on their responses.

Given the focus of this study on native language as a resource for ELs, data analysis examines quantitative and qualitative data for three items that relate to the PSTs beliefs about the ELs using their native language in the mathematics class (see Table 1 for items). Table 1 shows the descriptive statistics related to each item consisting of the mean, standard deviation, and percent of agreement. All 14 interviews were transcribed, and the PSTs' responses related to the three items were analyzed for this study. The qualitative interviews provided insight into the PSTs' responses to survey items.

Results

Most of the 620 respondents to the survey were in North Carolina (60%), followed by Michigan (14%), Texas (7%), and Washington (7%). Most of PSTs were female (90%), interested in teaching K-5 (70%) and White (74%). About 11% of the PSTs were Hispanic and 5% Black. Most of the PSTs (80%) did not have classroom teaching experience.

Table 1

Items related to the use of the ELs' native language in math class

	Item	M	SD	D/SD%	U%	A/SA%
1	ELs will not learn English quickly if I allow them to speak their native language in my math class.	2.87	1.06	39.03	31.45	29.52
2	All math classrooms should allow ELs to discuss ideas with each other in their native language.	3.53	0.93	13.87	30	56.13
3	Whenever possible, new ELs should be taught math in their native language and English.	3.54	0.91	13.55	27.9	58.55

In item 1, almost a third of the PSTs believed that the ELs would not learn English fast enough if they used their native language in mathematics class. Looking at the interview responses for this item, one PST agreed with the statement, five disagreed and eight were undecided. The eight undecided PSTs believed that the native language would contribute to the ELs' understanding of the material, however, like the one PST who agreed with the statement, they believed that too much use of native language in the class would hamper the ELs learning English. One of the elementary PSTs, Hilda commented,

> If they take five minutes or ten minutes out of a class time to get their thoughts together and speak their native language, I don't see that as a hindrance at all because it's refocusing them. I'm undecided because I don't know if I would let them do it all the time. But if it is like let me take five minutes and regroup, I think I will let it slide, if it meant that they progressed (in mathematics).

Like Hilda, the other PSTs agreed to a limited use of the native language in the class, believing that allowing it for an extended period would not motivate ELs to learn English. The PSTs who disagreed with the item were more concerned about the ELs learning mathematics than the language(s) used in the classroom. The PSTs believed that ELs could use native language to build stronger connections to the mathematics learned in their native language. One PST, who disagreed, mentioned learning English should not come at the expense of the students' native language.

More than half the PSTs agreed with item 2, with almost a third remaining undecided (Table 1). In the interviews, eight PSTs agreed with the item. Like item 1, the PSTs reiterated the usefulness of native language in helping ELs. The PSTs believed that peers could translate the teachers' instruction for the ELs. Further, PSTs also pointed out that native language could foster discussion among students, which could promote mathematical understanding. Jason, an elementary PST said,

> Like I said it is all about the end result (of learning mathematics)….(if) they are learning it, then teach it to them how they understand it. And it works, so yeah, I would allow them to discuss ideas in their own language. I think that it makes it easy. Because you have to be able to discuss things if you learn it. … Eventually, they will understand how to translate that into English and discuss it in English. But until then let them discuss it in the way that they can.

In the interviews, three PSTs were undecided about allowing the students to discuss mathematics in their native language. Two PSTs expressed concern that ELs would get comfortable using their native language and not be motivated to learn English. The PSTs felt that there was a need to get the ELs out of their 'comfort zone'. The third undecided PST believed that allowing the ELs to speak their native language in the long term would lead to segregation with other students in class.

Item 3 had a similar number of PSTs who agreed, disagreed, and remain undecided, compared to item 2. In the interviews nine PSTs agreed, three disagreed and two were undecided. Though a large proportion of PSTs agreed with item 3 in the interviews compared to the survey, all the PSTs assumed that the ELs were new to the country and were at early levels of English proficiency. The PSTs believed that native language would allow the ELs to make connections to mathematics they learned in their home countries in their native language. Once students were in the U.S. for a while (about a year), instruction needs to be in English. Sandra, an elementary PST says,

> On the premise that English is really new to them and they do not have all the language that they need to understand what you are trying to get across. I would agree with the statement, but (only) in certain scenarios.

Two of the three PSTs, who disagreed with the statement, believed that ELs could use their native language at home, but should be taught in English in class. In line with the survey responses from item 1, the PSTs believed that the ELs would lack motivation to learn English. The third PST who disagreed, stated that mathematics was less language intensive (compared to other subjects) and would be comprehensible, even when taught in English to ELs. However, this PST agreed that native language would be better for ELs. Finally, two PSTs who were undecided on item 3 brought up practical issues that related to language use. One PST pointed out that the teacher may not know the students' native language, unless it was in Texas or California where it was possible that the teacher shared the same native language of Spanish with the students.

Discussion and Implications

Though the survey results show that there was some variability in the PSTs beliefs about the impact of using English in the mathematics class (item 1), about 60% (items 2 and 3) believe that ELs would benefit from the use of native language in class discussions and teaching. The PSTs who agree with the use of native language, in class discussions and teaching, believe that it will promote mathematical understanding as the ELs make connections with their prior knowledge. Across all three items, the PSTs express concerns about ELs not learning English if they use their native language in the classroom. Few PSTs express an interest in getting all students to be bi or multilingual. The beliefs of the PSTs in this study align with teachers' beliefs from other studies (Shin & Krashen, 1996) where they support the theoretical aspects of bilingualism but are reluctant to let the students use their native language in the classroom. The PSTs believe that restricting the time spent using their native language is an effective approach to ensure ELs learn English. Note that using three items from the survey and interviewing PSTs from one university limits the scope of this study, however, the results point to the need for further research to understand the PSTs' beliefs about the teaching and learning of mathematics to ELs.

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PRESERVICE ELEMENTARY TEACHERS DEVELOPMENT OF PROFESSIONAL VISIONS AND IMPLEMENTATION OF MATHEMATICAL TASKS

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This paper presents the results of a longitudinal study following three preservice elementary teachers (PSTs) throughout their teacher preparation program and into their first year of teaching. The focus of the study is centered around the teachers' use of mathematical tasks and how they developed in their professional visions and implementation of tasks over the course of three years. Results indicate that teacher preparation programs should examine field placements, continue to focus on increasing MKT, and provide PSTs opportunities to reflect on their memories, practice, and visions as they develop over time.

Throughout the past several decades, student achievement has been a central focus of educational reform (NCTM, 2000; NGA & CCSSO, 2010). This reform in mathematics education has emphasized the need to develop students' conceptual understanding of mathematics (Ma, 1999). However, some teachers, particularly novices, struggled with implementing this way of teaching due to their own previous experiences in mathematics (Gurbuzturk, Duruhan, & Sad, 2009). Despite these struggles, teacher preparation programs can help refine PSTs ideals, or visions, to which they strive (Gurbuzturk et al., 2009; Hammerness, 2001, Swars, Smith, Smith, & Hart, 2009).

Although refining all visions of PSTs seems challenging, one particular area in which teacher educators can focus is mathematical tasks. Tasks are central to the instruction in a mathematics classroom and influence other components such as discourse (Hiebert & Wearne, 1993) and representations. The authors of the NCTM (2014) *Principles to Actions* document suggest that teachers need to "regularly select and implement tasks that promote reasoning and problem solving" in order for students to "have the opportunity to engage in high-level thinking" (p. 17). In order to better understand teachers' professional visions of mathematical tasks, Munter (2009) created a framework for describing teachers' visions of high-quality mathematics instruction (VHQMI). One rubric within the framework focuses specifically on *Mathematical Tasks*, which draws from Stein and Smith's (1998) work (i.e. memorization, procedures with and without connections, and doing mathematics) to create a five-point scale (0-4) in order to classify the teacher's vision of mathematical tasks.

While the importance of focusing on mathematical tasks is clear, little has been done to understand how PSTs develop over time in their visions and implementation of mathematical tasks. This deeper understanding can inform teacher educators as they work to prepare PSTs in teacher preparation programs. Therefore, two main questions guide this study: (1) how do preservice and novice elementary teachers, with varying MKT, develop in their visions and implementation of mathematical tasks? and (2) to what extent are their visions and implementation of mathematical tasks aligned?

Methods

Participants

To understand PSTs' authentic experiences, a case-study design (Creswell, 2013) was used to follow three PSTs over three years. All participants in the study took the LMT-MKT assessment focused on number and operations at four time points. Mathematical Knowledge for Teaching (MKT) was used as a selector because MKT has been shown to have a positive relationship with quality of instruction (Hill, Ball, & Schilling, 2008). Sixteen teachers were followed in depth, and from these 16, three were chosen for this study. The three PSTs chosen were either below (*Jamie*), above (*Jordan*), or close to the cohort's average (*Charlie*) MKT throughout their teacher preparation program.

Data Collection and Analysis

Throughout the three years, data was collected at six time points: beginning and end of their junior, senior, and first year of teaching. This data consisted of: benchmark interviews that asked about the PST's visions of mathematics (e.g. "how do good teachers choose the best tasks or activities?" or "describe what you think the teacher and students should be doing during math instruction") as well as video-recorded lessons to understand how they implemented a task during that same time point. Each case was coded chronologically using the VHQMI in order to understand their visions of mathematical tasks and the Implementation of High Quality Mathematics Instruction (IHQMI; adapted from the VHQMI framework) to understand their implementation of mathematical tasks (see Table 1). Furthermore, participants were sometimes coded as falling "between levels" meaning they exhibited traits from two different levels and an average of the two levels was taken in order to best capture their visions and implementation.

Table 1

VHQMI and IHQMI Mathematical Tasks Rubric

Level	VHQMI	IHQMI (adapted from Munter, 2009)
4	Emphasizes tasks that have the potential to engage students in "doing mathematics".	The chosen task has students "doing mathematics".
3	Emphasizes tasks that have the potential to engage students in complex thinking, including tasks that allow multiple solution paths or provide opportunities for students to create meaning for mathematical concepts, procedures, and/or relationships.	The task chosen has the potential to engage students in multiple solution paths or higher-order thinking. However, the task does not have students generalizing or making connections between other mathematical topics or the world around them.
2	Promotes "reform"-oriented aspects of tasks without specifying the nature of tasks beyond broad characterizations, and without elaborating on their function in terms of providing opportunities for "doing mathematics".	The task is "hands on" or "real world". However, it doesn't provide students with the opportunity for "doing mathematics"
1	Emphasizes tasks that provide students with opportunity to practice a procedure before then applying it conceptually to a problem.	The task provides students with the opportunity to practice a procedure before applying it conceptually to a problem.
0	Either (a) does not view tasks as inherently higher or lower quality or (b) does not view tasks as a manipulable feature of classroom instruction.	The task is focused on memorization or learning a procedure without any connection.

Results

Jamie

Jamie was chosen because she was consistently below average in her MKT as compared to her cohort. In an interview during her junior year, she recalled struggling with mathematics during her time as a student of mathematics. Additionally, she recalled learning mathematics through memorization.

Visions. Throughout Jamie's junior year she envisioned the teacher as someone who modeled a task before allowing students to work independently on the task (VHQMI between Levels 0 and 1). Once in her senior year she continued with these beliefs, but also mentioned that the teacher should "create a deeper understanding of math so [students] can solve a lot of different types of problems" (VHQMI Level 1). Finally, during her first year of teaching, Jamie felt that a teacher's job was to make tasks engaging for students in order to keep their attention (VHQMI Level 0).

Implementation. Jamie began her junior year in a kindergarten classroom teaching a lesson on alike and different. She used a real-life situation to engage in higher-order thinking, but also had students completing a procedural worksheet (IHQMI Level 2). At the end of the year she taught a 4th grade lesson on comparing fractions and writing equations. She used a real- world context but solved the problems with the class procedurally (IHQMI Level 1). During her senior year, she was placed in a fifth-grade classroom. She taught a lesson on rounding during the beginning of the year and focused mostly on the procedure for rounding (IHQMI Level 0). At the end of the year she taught a lesson on fraction word problems by engaging students in practicing a procedure and focusing on 'key words' (IHQMI Level 1). Finally, during her first year of teaching, Jamie taught in a second-grade class. She taught her first lesson on addition of two-digit numbers in which students used manipulatives but did not have a context for the problems. She also used a practice-and-apply approach to her lesson (IHQMI Level 1). By the end of her first year, she taught a lesson on repeated addition. She began the lesson by practicing as a class, and then students solving problems on their own using the learned procedure (IHQMI Level 1).

Jordan

Jordan was chosen because she was consistently above average in her MKT as compared to her cohort. In an interview during her junior year, she recalled loving mathematics as a student. She felt that she was good at memorizing procedures and that made her excel.

Visions. Throughout Jordan's junior year she felt that tasks should be engaging and allow students to "have a conceptual understanding and not just take everything at face value". She envisioned students to "actively explore mathematical concepts and use appropriate representations" (VHQMI between Levels 1 and 2). Once in her senior year, Jordan envisioned lessons to "have some kind of hands-on practice or higher-order thinking questions that they're grappling with... not just standing in front of the room and teaching a lesson" (VHQMI Level 2). Finally, during her first year of teaching, Jordan envisioned good tasks to have students "collaborating to solve problems and using different methods to solve them and show each other their methods." She encouraged real-life scenarios in order to make personal connections with her students (VHQMI between Levels 2 and 3).

Implementation. Jordan began her junior year teaching a second-grade lesson on addends that make 10. She used a real-world context, but the majority of her lesson focused on the students' memorization of the partners of 10 (IHQMI between Levels 0 and 1). At the end of the year she was placed in a fifth-grade classroom where she taught a lesson on volume. The students used multi-link cubes in order to build prisms to explore volume (IHQMI Level 3). During her senior year, she was in a fourth-grade classroom. Her first lesson of the year was focused on addition and subtraction. She used word problems and focused on sense-making throughout. However, towards the end she encouraged the use of the standard algorithm (IHQMI Level 2). At the end of the year she taught a lesson on comparing fractions in which she required the use of higher order thinking by showing her students a shortcut by examining the denominator of the fraction (IHQMI Level 2). Finally, during her first year of teaching, Jordan began in a fifth-grade classroom with a lesson reviewing whole-number division. She had centers with a variety of tasks (computers, real-world problems, manipulatives for solving naked number problems; IHQMI between Levels 2 and 3). However, by the end of the school year she had moved to a new school in a fourth-grade classroom where she taught a lesson on reviewing fractions. Again, she used centers, but this time they were mostly lower in cognitive demand or procedural in nature (IHQMI Level 1).

Charlie

Charlie was chosen because she was consistently close to average in her MKT as compared to her cohort. In an interview during her junior year, she recalled struggling with mathematics during her time as a student. She remembered being frustrated with timed tests and only knowing one way to solve a problem.

Visions. Throughout Charlie's junior year, she was of the practice-and-apply mentality as to the purpose of mathematical tasks (VHQMI Level 1). However, by the end of the year she valued multiple ways to solve a problem and the importance of multiple representations (VHQMI Level 2). Once in her senior year, Charlie expressed visions of reform-oriented mathematics (i.e. high cognitive demand tasks, use of manipulatives, student-to-student discourse; VHQMI Level 2). Finally, during her first-year of teaching she valued practice-and-apply tasks again as well as hands-on tasks (VHQMI between Levels 1 and 2).

Implementation. Charlie began her junior year in a kindergarten classroom teaching a lesson on alike and different. Despite the lesson being disjointed, she used multiple tasks such as subitizing cards, number lines, the hundreds chart, etc. (IHQMI between Levels 1 and 2). By the end of the year, she taught a fifth-grade lesson on volume in which her students used multi-link cubes to build prisms to explore volume (IHQMI Level 3). During her senior year, she was placed in a third-grade classroom where she taught her first lesson on rounding. She used manipulatives, but then focused on the procedure for rounding (IHQMI between Levels 0 and 1). At the end of the year she taught a lesson on area and perimeter where she used a real-life scenario, but guided the students to draw the models (IHQMI level 2). Finally, during her first year of teaching, Charlie was placed in a fourth-grade classroom where she taught her first lesson on multiplication. She used a timed test to begin the lesson and then used a practice-and-apply strategy during her lesson to solve 3-digit multiplication problems (IHQMI Level 1). By the end of the year she taught a lesson on converting units of measurement with a variety of centers which had real-life contexts and asked students to solve any way they wish. However, she brought down the cognitive demand of some tasks by guiding them and not allowing for time to share strategies (IHQMI Level 2).

Cross Case Comparison & Discussion

When comparing all three participants over time, it was noteworthy to mention that all three began their junior year with similar visions (ranging from a level 0.5 to a level 1.5) and ended their first year of teaching with a wide variety of visions (ranging from a level 0, to a level 1.5, to a level 2.5; see left Figure 1). Additionally, both Jordan and Charlie were influenced by their cooperating teachers as they used strategies they observed in their classrooms when describing how they envision quality tasks.

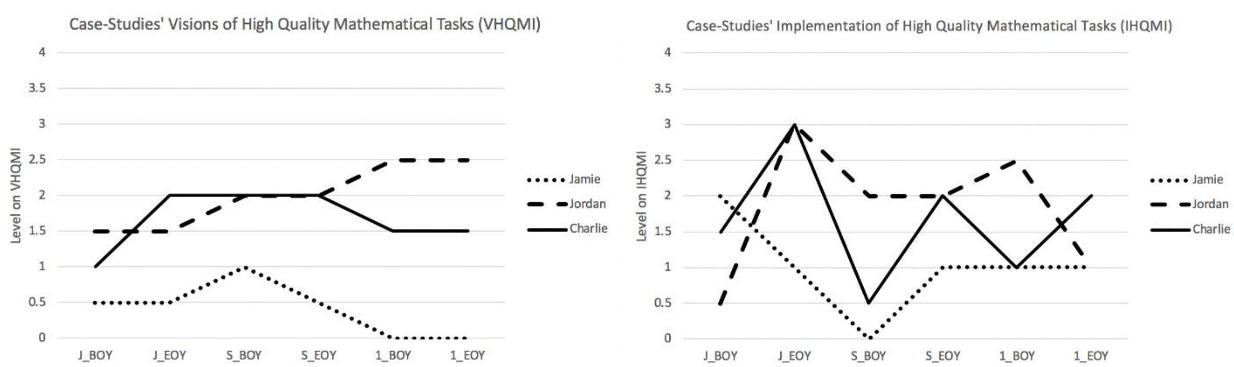

Figure 1. VHQMI (left) and IHQMI (right) for all three participants

When comparing all three participants' implementation of tasks, it was noteworthy to mention that all three struggled with behavioral issues which brought down the level of their tasks (see right Figure 1). Additionally, implementation varied widely during the first three time points (during their junior year and beginning of senior year), while their implementation levels seemed to 'stabilize' during the last three time points (end of senior year and first year of teaching). It was also important to note that all three participants used tasks given to them by their cooperating teachers during their junior and senior years, which may or may not have aligned with the PSTs' visions of quality tasks. However, during their first year of teaching they gave reasons for task choices such as needing to review, using the limited resources provided, and giving tasks based on what they felt their students needed to know. The influence of cooperating teachers on the PSTs' visions and implementation spoke to the importance of field placements during teacher preparation programs and placing PSTs with cooperating teachers that share visions with the teacher preparation program.

Impact of Memories on Visions and Implementation

As a student, Jamie recalled struggling with mathematics and learned through memorization. She consistently envisioned good tasks to be those in which the teacher models a procedure followed by students practicing it on their own. These visions directly aligned with her implementation of tasks, as she mostly used practice and apply tasks in her classroom. In contrast, Jordan recalled loving mathematics in school. She envisioned good tasks as those where students are engaged and learning conceptually. Similarly, she mostly implemented tasks focused on conceptual understanding, despite recalling memorizing procedures during her own K-12 schooling. Finally, Charlie envisioned good tasks as those that are reform-oriented and allowed for multiple ways to solve a problem. She mostly implemented real-world scenario tasks but used a "practice and apply" approach to them. She also gave timed tests to her students even though she recalled personal frustrations with them in her own experiences as a K-12 student. This study and its findings showed the importance of PSTs' memories and experiences in their K-12 classrooms. As noted in the examples above, the findings of this study showed how memories transfer to their visions and teaching practices once in their own classrooms. If PSTs experience tasks that do not align with reform-oriented practices, they oftentimes reverted back to these practices once they become a teacher themselves (i.e. Jamie and Charlie). However, strong teacher preparation programs can help to reform these beliefs and visions (Gurbuzturk et al., 2009; Hammerness, 2001, Swars et al., 2009), as shown by Jordan.

Mathematical Knowledge for Teaching

Similar to Hill and colleagues' (2008) study, MKT and quality of instruction were found to be positively related in this study. As shown in Figure 1, Jordan and Charlie both had higher visions and implementation on the VHQMI and IHQMI, respectively. While MKT was not intended to be a focus of this study beyond case selection, those with higher MKT demonstrated visions and task implementation that are more closely aligned to the "reform" advocated by mathematics educators (e.g. NCTM, 2000). Therefore, mathematics teacher educators should continue to find ways to increase MKT in preservice and novice teachers.

Conclusion

The findings of this study provide implications for mathematics teacher educators. These implications include: the need to examine field placements, a continued focus on increasing MKT, and providing PSTs opportunities to reflect on their memories, practice, and visions as they develop over time. These opportunities, whether through journaling, video-recorded lessons, or discussions, can help PSTs better align their visions with what they actually do when implementing mathematical tasks in the classroom.

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SECONDARY PROSPECTIVE TEACHERS' UNDERSTANDINGS OF THE COGNITIVE DEMAND OF MATHEMATICS TASKS

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In this qualitative study, we examined 14 secondary prospective teachers' (PSTs) understandings of the cognitive demand of mathematics tasks during a mathematics class taken the semester before student teaching. The PSTs' understandings were investigated both before and after a class lesson on cognitive demand. Results showed the PSTs gained a positive understanding of cognitive demand after the lesson as well as reported beneficial intentions for their future mathematics teaching.

While developing as a mathematics teacher is a life-long process, many teachers' pedagogical strategies and understandings are formed during their teacher education programs. The purpose of this research is to examine secondary prospective teachers' (PSTs) understandings of the cognitive demand of mathematics tasks during a mathematics class taken the semester before student teaching. We examine the PSTs' classifications of the cognitive demand of such tasks both before and after an associated lesson as well as how their future mathematics teaching may be impacted.

Related Literature: Cognitive Demand and the Task Analysis Guide

Cognitive demand refers to the kind and level of thinking needed to solve a mathematics task. The Task Analysis Guide (TAG) may be used to classify mathematics tasks according to cognitive demand (Stein, Smith, Henningsen, & Silver, 2000). The TAG distinguishes between lower-level (*memorization* and *procedures without connections*) and higher-level (*procedures with connections* and *doing mathematics*) cognitive demand mathematics tasks, similar in nature to the progression of cognitive processes in the revised Bloom's Taxonomy (Anderson & Krathwohl, 2001). *Memorization* tasks involve reproducing previously learned details or committing mathematical ideas to memory; no connections are made amongst mathematical ideas nor are any procedures used. *Procedures without connections* tasks are algorithmic. The procedure to be used is specified in the task or apparent from previous instruction so students experience little cognitive demand. No connections are made between the procedure and any concepts or underlying meanings. *Procedures with connections* tasks focus students on a particular procedure with the intent of helping them develop deeper levels of understandings of the concepts and meanings associated with the procedure. Thus, the procedure is suggested but students cannot follow it mindlessly. Finally, *doing mathematics* tasks require considerable cognitive demand because students explore and make sense of relationships among mathematical concepts and processes. Students have to utilize complex and non-algorithmic thinking as no predictable solution processes are provided in the task or from previous instruction.

One intent of the TAG is for teachers to select a task with a level of cognitive demand that aligns with the goals of a lesson (Stein et al., 2000). For example, if teachers expect students to develop an understanding of a new mathematical concept, they may want to select a *doing mathematics* task. If they want students to practice and gain efficiency with a particular mathematical procedure, they may want to select *procedures without connections* task. As such, all levels of the TAG are fruitful for use in the classroom and may have positive impacts on students' mathematics learning. Additionally, student performance gains are greatest for students in classrooms that appropriately and consistently incorporate higher-level cognitive demand mathematics tasks (Boaler & Staples, 2008; Hiebert & Wearne, 1993; Stein & Lane, 1996; Tarr et al., 2008). This potential of the TAG inspires additional teacher educators to develop a corresponding framework for science tasks (Tekkumru-Kisa, Stein, & Schunn, 2015).

Unfortunately, challenges exist for effective classroom implementation of the TAG. Mathematics textbooks often lack higher-level cognitive demand mathematics tasks, leaving the identification or development of such tasks to the teacher (Wang, 2016). Furthermore, even when teachers select higher-level cognitive demand tasks, they may be implemented in less than ideal ways (Stein et al., 2000). The teacher may reduce the cognitive demand by completing challenging components for the students, by emphasizing the correctness of answers, or by failing to hold students accountable for explanations. To address these challenges, many are using professional development to inform practicing teachers about the cognitive demand of mathematics tasks and associated pedagogical implications (e.g., Arbaugh & Brown, 2006; Boston, 2013; Boston & Smith, 2009). As a result, practicing teachers find it better to evaluate the cognitive demand of mathematics tasks, to implement more higher-level cognitive demand tasks in their instruction, to maintain the high-level cognitive demand of such tasks, and to understand the influence of mathematics tasks on students' learning. Although nearly all teacher educators have used the TAG with practicing teachers, Osana, Lacroix, Tucker, and Desrosiers (2006) investigate prospective teachers' classifications of elementary mathematics tasks according to the TAG following a 45-minute lecture on cognitive demand. The PSTs had difficulty classifying tasks at a higher-level of cognitive demand, while those with stronger mathematics content knowledge were better able to accurately classify tasks. The authors present their study as an initial step towards addressing the literature gap on PSTs' understandings of cognitive demand but acknowledge that their study did not investigate the thinking processes used by the PSTs to classify the tasks. Due to the benefits of addressing cognitive demand with practicing teachers and the scarcity of research investigating task classification by PSTs, we decided to address the TAG with PSTs and examine the impacts on their understandings of cognitive demand.

Method

The purpose of our qualitative study was to address two research questions: (1) What understandings of the cognitive demand of mathematics tasks do secondary PSTs have before and after an associated lesson in a mathematics class taken the semester before student teaching and (2) How do secondary PSTs report their enhanced knowledge about the cognitive demand of mathematics tasks may influence their future teaching? The participants included all of 14 secondary mathematics PSTs (9 females; 5 males) enrolled in an undergraduate mathematics class (15-week duration) the semester before student teaching. The participants were selected as a convenience sample, and all consented to participate in the research. The purpose of the undergraduate class was to examine the mathematics that the PSTs would address in their future teaching assignments, including the underlying concepts and meanings as well as the thinking of middle and high school students. The second author, a mathematics teacher educator, served as the instructor for the course, while both authors served as researchers for the study. To examine the PSTs' understandings of cognitive demand before the associated lesson in the class, the PSTs completed a homework pre-assessment. The pre-assessment asked them to locate or develop a mathematics task that they felt would support middle or high school students in learning about and understanding mathematics. The task could be for any 6-12 grade level and for any mathematical topic in algebra or probability (the two mathematical foci for the class).

The lesson on cognitive demand began with the PSTs working on two *doing mathematics* tasks to experience a higher-level cognitive demand task. Next the PSTs sorted ten mathematics tasks (see Stein et al., 2000, p. 19 plus the two *doing mathematics* tasks) in at least three different ways that made sense to them. After discussing their initial sortings, the PSTs completed a reading about the TAG (Stein et al., 2000, p. 11-16) and then classified each of the ten tasks accordingly. They worked on this individually and then discussed their classifications in small groups and as a whole class. Ideas addressed during the discussion included:

- The TAG does not imply that one level is best. Rather, the goal is to match the level of a task with the learning goals for the lesson.
- The students completing the mathematics task, e.g., their age, grade level, and prior experiences, will impact the level of a task and deserve consideration.
- In classifying mathematics tasks, one should watch out for superficial features of tasks. Low-level cognitive demand tasks may look high-level if they involve real world contexts, manipulatives, multiple steps, diagrams, or explanations. High-level cognitive demand tasks might look low-level if they appear like standard textbook problems or seem to lack reform features (such as requiring an explanation). Features of the task are not per se indicators of the cognitive complexity, rather the quality of thinking required by the task is the main criterion for classifying the task (Stein et al., 2000).

After the lesson on cognitive demand (encompassing three 75-minute class periods during weeks 2-3 of the semester), the PSTs completed a related homework assignment one week later and an in-class exam 5 weeks later. Finally, at the end of the semester the students wrote a journal entry, describing what they learned about the TAG and how it may impact their teaching.

The study included four data sources: the pre-assessment, selected questions from the homework assignment, selected questions from the in-class exam, and the journal entries. For the homework questions, the PSTs used the TAG to categorize the mathematics tasks that they submitted for the pre-assessment, including explanations of why they categorized each task as they did. For the exam questions, the PSTs were given additional mathematics tasks to classify according to the TAG. For the journal entries, the PSTs were asked to write at least three pages reflecting on a large project completed throughout the course, which included many components related to cognitive demand. The two questions relevant for this study included "How has our work with the TAG impacted your perception of mathematics tasks for supporting student learning?" and "How do you feel this experience might impact your teaching of mathematics?"

To analyze the pre-assessment, we classified each of the PSTs' submitted tasks according to the TAG. For the homework and exam questions, we also classified the associated tasks according to the TAG. We then compared each PST's classifications against our classifications. If the PST's classification matched with our classification, it was highlighted green. If the PST's classification did not match with our classification but was in the same level of cognitive demand (lower-level or higher-level), then it was highlighted yellow. If the classification did not match with our classification nor with our level of cognitive demand, then it was highlighted red. Because the PSTs explained their classifications on the homework, we used open coding (Strauss & Corbin, 1998) on the red responses to identify why the PSTs may have been incorrectly classifying the mathematics tasks. To analyze the journal entries, we went through each PST's journal entry and highlighted any text related to the previous two questions. We then organized the answers into similar groups and again used open coding to find consistencies in the PSTs' responses about how their future teaching may be impacted by learning about cognitive demand.

Results

The pre-assessment was used to examine the PSTs' understandings of cognitive demand before the lesson. For the pre-assessment, the PSTs submitted one m*emorization* task (7%), six *procedures without connections* tasks (43%), two *procedures with connections* tasks (14%), three *procedures with* or *without connections* tasks depending on the previous experiences of the students (21%), and two *doing mathematics* tasks (14%) (see Table 1).

Table 1

Three Examples of PSTs' Submitted Tasks along with Our Classifications and Rationales

Task	TAG and Explanation
For 8th grade: 1. Given two dice, how many different possible outcomes are there? Express each combination as an ordered pair (a, b) where a represents the first die and b represents the second. 2. How many different ways are we able to combine the dice such that they sum to 5? 3. Determine the probability of rolling a sum of 5.	*Procedures without connections:* Task is algorithmic. Use of the procedures are specifically called for in the directions for the task. Little cognitive demand required because little ambiguity about what to do or how.
For 9th grade algebra: 1. Graph the two equations. For each be sure to plot 8 points. a. $y = (x – 4)^2 – 3$ b. $y = -(x + 1)^2 + 6$ 2. (Students are then given the graph of a parabola that models the graph of a punt in a football game. They are directed to find the quadratic function that represents the given graph.)	*Procedures without* or *with connections:* If students have previously studied quadratic equations and parabolas, including vertex form, then this problem is algorithmic. If students have not, then this task helps to develop connections between the symbolic and graphical forms of quadratic equations.
For 7th grade: Some family friends have asked you to plan a rafting expedition. A rafting company has agreed to take your group down the Babbling River. The rafting company has given you specific details on how much weight a raft can hold. A raft can safely carry the weight of 24 babies. As everyone knows, the weight of 12 babies is exactly equal to the weight of 4 teenagers; the weight of 6 teenagers is exactly equal to the weight of 3 adults. Figure out the least number of rafts necessary that is needed for a trip with 11 adults, 5 teenagers, and 21 babies.	*Doing mathematics:* Because no pathway for solution is given, this task requires complex thinking about ratios for 7th graders. They have to make sense of relationships between various mathematical quantities, while analyzing associated constraints.

The homework and exam questions were used to examine the PSTs' understandings of cognitive demand after the lesson. The homework included 14 tasks for each PST to classify, resulting in 196 total responses. Of these, 54% of the responses were classified accurately, 76% within the correct level of cognitive demand (either lower-level or higher-level), and 24% incorrectly. The in-class exam asked the 14 PSTs to classify eight tasks, resulting in 112 responses. Fifty-seven percent of the responses were classified accurately, 92% within the correct level of cognitive demand, and 8% incorrectly.

Four themes emerged for why the PSTs appeared to incorrectly classify some of the homework tasks: (1) over- or under-estimating the mathematical complexity, (2) over-estimating the connections between procedures and meanings, (3) missing pathways given in the task statement, and (4) being misled by real-world contexts or the use of manipulatives. First, the PSTs over- or under-estimated the complexity of the mathematics in the task for the given students (over-estimating occurring 17/196 times at least once by 10 of the PSTs; under-estimating occurring 4 times at least once by 3 of the PSTs). For example, some of the PSTs overclassified the following computational task submitted for ninth grade algebra students:

> An emergency plumber charges \$60 as a call-out fee plus an additional \$65 per hour. He arrives at your house at 9:30 and begins to work. If the total bill is \$196.25, how long did it take the plumber to do the repairs at your house?

Second, the PSTs sometimes over-estimated the connections that the students would make between the associated concepts and the procedures used (occurring 12/196 times at least once by 9 of the PSTs). For example, consider the following task for seventh grade algebra students:

> Let x represent the cost of a notebook. A pencil costs two dollars less than a notebook. A pen costs three times as much as a pencil. The pen costs nine dollars. Which equation describes the story? A. $3x – 6 = 9$, B. $x – 6 = 9$, C. $3x – 2 = 9$, D. $3(x – 2) = 9$

Some PSTs felt this problem would help students develop connections between symbolic and narrative representations; however, the task is limited due to its emphasis on only selecting a correct answer. Third, some of the PSTs missed when suggested pathways or guiding procedures were given as part of a task, thereby over-rating its cognitive demand (see the first task in Table 1) (occurring 13/196 times at least once by 10 of the PSTs). Finally, some PSTs overestimated the cognitive demand of tasks when a real-world context or some type of hands-on activity was involved (occurring 12/196 times at least once by 12 of the PSTs). For example, one of the submitted tasks involved ninth grade students in measuring the rise and run of stairs to investigate the slope of a line. Many of the PSTs classified this task as higher-level cognitive demand, when working with slope should be fairly algorithmic for ninth grade students.

The journal entries were used to assess how learning about cognitive demand may impact the PSTs' future mathematics teaching. Thirty-three percent of the PSTs reported realizing that all four levels of tasks are necessary for student learning and therefore plan to use all four types in their future classrooms. One PST remarked, "I think all four tasks, *memorization*, *procedures without connections*, *procedures with connections*, and *doing mathematics* are needed for student learning; however, I feel our schools do not do enough *doing mathematics* problems." Another 33% of the PSTs believed that students gain a better understanding of mathematical concepts when teachers effectively use the TAG, especially when higher-level cognitive demand tasks are implemented. As one PST wrote, "A *doing mathematics* problem … forces the students to learn through self-discovery. Having students learn in this way, I believe, is one of the greatest ways for students to learn." Finally, 25% of the PSTs felt that teacher and student involvement was improved with the TAG as using appropriate tasks provides an effective way for teachers to see where students may be struggling with a concept. In conclusion, the PSTs reported many advantages of using the TAG and planned to use it in their future classrooms.

Discussion

We examined secondary mathematics PSTs' understandings of the cognitive demand of mathematics tasks during an undergraduate mathematics class. After the lesson on cognitive demand, the PSTs accurately classified tasks as either lower-level or higher-level cognitive demand a large majority of the time. However, the PSTs occasionally over- or under-estimated the mathematics or connections involved in a task, missed the guiding nature of some tasks, or gave too much emphasis to hands-on or realistic contexts. Finally, the end of class journals indicated that the PSTs valued the TAG and intended to incorporate it into their future mathematics teaching. Our results were similar to those found with practicing teachers participating in professional development focused on cognitive demand. Further research is warranted to examine the degree to which the PSTs actually incorporate the TAG into their future mathematics teaching. However, the encouraging results here suggested that introducing PSTs to cognitive demand during their teacher education programs may serve as one important experience for PSTs during this formation time.

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WHERE'D THEY GO? SUSTAINING AND GROWING INTEREST IN MATHEMATICS TEACHING

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Recruiting and retaining qualified mathematics teachers are well-documented challenges. Our research objective was to examine factors that influence mathematics teaching majors to change academic direction. We utilized a sequential explanatory mixed methods protocol to identify key attrition points for mathematics teaching majors and better understand the differences in motivations between those who leave and those who persist. Particular attention is given to mentoring and support networks, barriers to persistence, and the timeline by which students matriculate and exit the program. One group within the population had a drastically higher success rate, and their motives are examined.

Related Literature

While the demand for professionals in mathematics and the sciences continues to rise, the supply of quality mathematics and science teachers has languished (National Academy, 2010; Watt, Richardson, & Pietsch, 2007). This puts significant pressure on K-12 administrators to recruit and retain mathematics and science teachers with content knowledge and efficacy to instill both expertise and interest into the next generation (National Research Council, 2011; National Academy, 2010). In 2000, the Texas Center for Educational Research released a study quantifying the costs of teacher turnover to the state at somewhere between $300 million and $2.1 billion. Other results corroborate the tremendous costs of teacher attrition and call for in depth study of teacher attrition and its causes (Watlington, Guglielmino, & Flesher, 2010).

A nationwide decrease in undergraduate students who earn educator certification or major in education since 1970 (Marketwatch, 2018) has occurred and mirrors the consistent annual increase in teachers who leave education (Sutcher, Darling-Hammond, & Carver-Thomas, 2016). They attribute this to financial factors, but Betancourt (2018) adds concerns about hyper-accountability and federal and state political rhetoric among other causes. Hong, Greene, Roberson, Francis, and Keenan (2018) state that pre-service teachers need opportunities to explore career choices and receive feedback from people they trust in order to be able to persist to graduation. Research findings from Darling-Hammond (2010) identify key components of successful educator preparation programs including induction and retention. There is limited research focused specifically on how all of these factors impact pre-service STEM teacher attrition at the undergraduate level. Our research is an effort to fill that gap. This study examines the research question, what factors influence undergraduates pursuing mathematics teacher certification to change majors, not certify, or choose other certification pathways?

Context

Stephen F. Austin State University (SFA) is a rural comprehensive university with an annual enrollment of roughly 13,000, 50% reporting being the first in their family to graduate from college. Undergraduate mathematics teacher certification at SFA entails a major in mathematics taught exclusively by the Department of Mathematics and Statistics, then eight additional courses in an Education Preparation Program (EPP) taught exclusively by the College of Education. The EPP consists of six courses leading up to student teaching and a student teaching experience with a university assigned field supervisor and a secondary cooperating teacher.

The T4 program, formally Talented Teachers in Training for Texas, is a National Science Foundation program sponsored (NSF 1136416, NSF 1556983) through the Robert Noyce Scholarship initiative. Its purposes are: first, to create experiences through which university STEM majors can examine careers in high school teaching through early intensive field experience (Hubbard, Embry-Jenlink, & Beverly, 2015). Second, to target aspiring STEM teachers for authentic engagement in a community of practice with a structured mentoring network (including experienced classroom teachers, aspiring STEM teachers, STEM and education university faculty, and public school administrators) for two years before graduation and three years after entry into the teaching profession. Third, to longitudinally examine prospective STEM teachers for the purpose of identifying most effective practices in long-term STEM teacher training and retention. T4 Scholars are chosen based on GPA, professor recommendations, and essay and interview responses. They receive scholarships over the duration of the program and commit to regular participation in the mentoring network community. They participate in bi-weekly meetings with mentors and other T4 scholars to discuss a variety of STEM specific instructional, curricular, and classroom management issues. They also attend discipline specific conferences, meet regularly with education faculty and

STEM faculty mentors, and participate in other team building events with fellow T4 scholars and graduates who are current STEM teachers in public schools.

Methodology

In order to answer our research question, we chose a sequential explanatory mixed methods design. This allowed us to begin by examining the quantitative data available for the mathematics majors at SFA, and then move on to a deeper, qualitative analysis of data to "explore the participants views in more depth" (Ivankova, Creswell, & Stick, 2006, p.9).

In the initial analysis, we examined transcripts for all mathematics majors since 2007 who had at some point identified secondary education as a minor or emphasis. For these 216 students, we tracked graduation rates, secondary education courses taken, and majors and minors declared or completed. These data were analyzed to identify descriptive statistics for graduation, certification, and attrition points from mathematics teaching.

To more deeply understand why mathematics majors persist to certification or change course, we then conducted semi-structured interviews with a purposive sample of eleven mathematics majors based upon attrition and graduation categories in the quantitative data analysis. Each person was chosen to represent a group identified in the quantitative analysis. Three interviewees had continued on track toward graduation in mathematics with a certification in secondary education. Three interviewees had changed their major away from mathematics. Five interviewees had continued in mathematics while moving away from teaching. Within this group of five, one had already certified alternatively, one plans to certify alternatively, two are considering, while one has no plans to certify. Interviewees were selected in order of most recent attendance at the university, employing the assumptions that these students might be most representative of current curriculum and perspectives, and also would be most likely to respond. If a student or former student declined to be interviewed or did not reply to three attempted contacts, another student was sought using the same process.

Interviews were recorded, transcribed, and independently coded by the researchers to identify common themes through open coding and the constant comparative method of data analysis (Glaser & Strauss, 1967). Interviewers also took notes during the interview, which were digitized and compared to the open coding to improve fidelity of analysis.

Data Analysis

Since 2007, SFA has had 216 mathematics majors formally declare an intention to pursue secondary teaching certification through an undergraduate certification program. Of those, 52 were still pursuing an undergraduate degree at the university. We restricted our statistical analysis to the remaining 164 students no longer enrolled as undergraduates.

Since 2012, T4 has selected 29 mathematics students for targeted support. Of these, 12 were still enrolled in undergraduate coursework, meaning that the 164 former students may be disaggregated as 17 T4 students and 147 non-T4 math teaching majors who have graduated or left the university.

Table 1

Attrition and Graduation Categories in Quantitative Data

	Math Grad, Certified	**Math Grad, No Certification**	**Graduate, Not in Math**	**Left, No Degree**	**Total Students**
T4 students	100%	0%	0%	0%	17
Non-T4 students	17%	15%	22%	46%	147

Of the 147 non-T4 students, 67 left without graduating (46%), 33 graduated but not in mathematics (22%), 22 graduated in mathematics but without secondary education (15%), and only 25 graduated with a mathematics major and secondary education minor (17%). Of the 17 T4 students, 100% graduated with a mathematics major and secondary education minor.

Table 2

Number of Students and EPP Courses taken Before Discontinuance

# of Students	**# of Courses**
6	7 courses
2	6 courses
0	5 courses
4	4 courses
6	3 courses
43	2 courses
30	1 course
33	0 courses

Attrition from the secondary education program might appear most readily addressable from a STEM teacher pipeline perspective. While mathematics courses make up over half of the most failed courses at our university, the education courses required for certification all have success rates above 90%. Hence, we set out to pinpoint precisely where attrition from the secondary education program occurred. The certification program requires eight courses, with the last two courses constituting the student teaching experience. As Table 2 indicates, 85% of those who discontinue the secondary education program do so within the first two courses of the EPP program. Our quantitative analysis led us to examine the differences in experiences between students who persisted to certification, students who persisted only in mathematics, and students who left mathematics as shown in Table 1.

All three of the interviewees who had originally chosen mathematics as a major and switched to a different degree mentioned that someone in their ultimate degree field encouraged or recruited them to the program in which they are currently on track to graduate. One specifically said of the faculty and peers in her new program, “these are my people.”

The interests of mathematics majors who moved away from a teaching minor were diverse. The first changed his minor to animal science and ended up pursuing a master’s degree in pure mathematics. Although his parents (both teachers) actively discouraged him from becoming a teacher, it was actually his experience with substitute teaching that led him away from teaching. The second simply decided to graduate then certify alternatively to teach. This student indicated receiving no active encouragement or support regarding teaching from faculty or mentors and reported that her father had actively discouraged the career. The third switched her minor to Accounting. She indicated concern about the ability to procure a job in mathematics teaching. While she pointed to several faculty encouraging her in mathematics, she reported no such encouragement toward teaching. On the contrary, she had several accountants actively encouraging her into a career in accounting. Teaching was now her “backup plan.” The fourth interviewee indicated that her GPA precluded her from entrance into the EPP. The fifth interviewee mentioned timely graduation as an impediment to continuing with teacher certification. Listing no mathematics or education mentors, he indicated that he had a dance teacher who had encouraged him to pursue alternative certification if he ever decided to teach. He considers mathematics teaching “still an option.”

Interviewing those students who had persisted in both mathematics and EPP, the first encountered many hurdles from failing coursework to issues with being allowed to student teach. He was a T4 Scholar, however, and indicated that faculty mentorship was “huge,” The second interviewee, also a T4 Scholar, indicated that failing Calculus was a major inhibitor toward continuing. However, he indicated that although he received no direction from his parents, being a first-generation college student, his peers were very supportive; T4 professional mentors were supportive; and "faculty had the largest effect." The final interviewee was a successful mathematics teaching student who was not involved with T4. He reported encouragement but no academic direction from his parents, but an excellent group of peers who were also interested in mathematics teaching. He also listed former mathematics teachers and faculty with whom he has continuing relationships. He said, "I love the entire math department … it made everything worthwhile."

Findings

A number of themes were illuminated through the participant interviews. The most commonly identified theme from these interviews, as evident in Table 3, is that students respond to mentorship and encouragement, both positively and negatively. This is true of faculty, family, peers and professionals outside the profession. The research findings of Hong, et. al. (2018) also highlight the influence of mentorship upon persistence and choice to teach, stating that pre-service teachers "are continuously going through resynthesizing and reconfirming process" (p. 418). Thus, a mentoring network such as T4 provides a safety net for pre-service teachers within that process as they move through their undergraduate experiences.

Table 3

Summary of participant responses about reasons for leaving mathematics teaching

Reasons for leaving mathematics teaching major	**Students affected**
Lack of encouragement toward mathematics teaching	7 of 8
Encouraged by someone in a different field	5 of 8
Extending time to graduation/ cost	5 of 8
Parent actively discouraging teaching	3 of 8
Early teaching exposure was negative	2 of 8
Alternative certification seemed more efficient	2 of 8
Concerns about being able to find a job in mathematics teaching	1 of 8
Not meeting GPA requirements of EPP	1 of 8
Reasons for persistence in mathematics teaching major	**Students affected**
Faculty mentorship and encouragement	3 of 3
Supportive peer network	3 of 3
Faculty support during academic or system hurdles	2 of 3

A second identified theme, evident in five of the interviewees, was that additional coursework, postponed graduation, and overall cost were barriers. Several students viewed the EPP content as having limited "quantifiable value," which they indicated discouraged them from pursuing undergraduate certification.

A third identified theme is that six of the eleven indicated that they did not start college with mathematics teaching as the primary career path - they switched to it (including two of the three successful mathematics teaching graduates). This provides a limited “window” during which a potential mathematics teaching major would be most assisted by mentorship into mathematics teaching. This is supported by quantitative findings that 85% of those leaving the eight teacher certification courses, did so within the first two courses. It mirrors the research of Hong, et. al. (2018), whose research also indicates that pre-service teachers’ university experiences encourage or discourage persistence to certify and graduate.

Conclusion

While examining the training and retention of mathematics teaching majors is multifaceted, it is vital that programs internally examine their majors’ persistence to certification and the motivations for staying or leaving. Studies such as this allow programs to determine how their rates of attrition compare to other institutions and provide insight into how to go about supporting students most effectively.

Outside the T4 program, the traditional mathematics certification rate is 17%, while within the program it is 100%. While this program’s support is financial, experiential, and interpersonal, interviews make clear the dramatic and causal effect of mentorship in academic major and career choice. Those who left the mathematics major, to a person, indicate mentorship drew them to the new discipline. Though alternative certification was frequently mentioned among those who left secondary education, the one teacher in the study who pursued this course left teaching after having a very negative experience. This is consistent with national research on teacher attrition (Ingersoll, Merrill, & May, 2014). The costs, additional time, and perceived deficiencies in the secondary education coursework, real or imagined, form a substantial barrier to certification.

The emergent themes not only provided insight into our research question, but also aligned with the research of Darling-Hammond (2012) and Hong et al. (2018), who stated that quality EPP programs emphasize positive relationships between faculty and pre-service teachers, and that EPP programs must be identifying barriers to and improving retention for pre-service teachers. Based upon our findings, we believe that universities can create and support such an environment, as evidenced by the success of the T4 program at SFA.

It is noteworthy that successful mathematics teaching graduates indicated multiple mentoring and support sources, pointing to a mentoring network rather than a single mentor that might be most effective. More research and longer time horizons are needed to more effectively model and positively affect mathematics teacher training and retention but building mentoring networks for students appears vital to ensuring sufficient quantities of committed teachers capable of engendering an appreciation for mathematics in the generation to come.

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EXAMINING NOVICE SECONDARY MATHEMATICS TEACHERS' USE OF SUPPORT NETWORKS

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Teacher attrition has been a chronic problem for schools that serve low-income and minority students. Previous research has shown that low perceived levels of support can lead to teacher attrition, while high-quality relationships can support teacher retention. This multiple case study examines the challenges faced by four Teach for America second-year mathematics teachers and how they leveraged their support networks to overcome these challenges. Findings indicate that teachers have mixed success in overcoming their challenges, with quality of support network members and access to these networks both playing an important role.

Equity in mathematics education has been described as one of the grand challenges for the field (NCTM Research Committee, 2015). An area in which inequities have been identified and studied extensively is teacher attrition (Carver-Thomas & Darling-Hammond, 2017; Simon & Johnson, 2015). Although mathematics is generally a hard-to-staff subject area (Hamdan, 2010), the shortage of mathematics teachers is particularly problematic for schools serving populations with high rates of low-income and minority students. The teaching profession has an attrition rate of about 8% annually. In contrast, attrition rates are about 70% higher for mathematics and science teachers in Title I schools (schools whose population has a minimum of 40% of students coming from low-income households) compared to their non-Title I counterparts (Carver-Thomas, & Darling-Hammond, 2017). This problem is further exacerbated because teachers who remain in the profession but choose to switch schools tend to leave ones that serve low-income students for ones that serve higher income communities (Hanushek, Kain, & Rivkin, 2004). Teachers entering the profession through alternative certification pathways (ACP) also have high turnover, leaving the profession at rates that are 150% higher than those of their traditionally certified counterparts.

These statistics indicate that attrition is an especially critical issue for mathematics teachers who enter the profession through ACP. To address this issue, this paper describes the findings of a study that focused on mathematics teachers from Teach for America (TFA), an ACP that specifically places teachers in communities serving low-income populations (Zahner, Chapin, Levine, He, & Afonso, 2018). Research on TFA teacher attrition has found that 16.9% of TFA teachers leave their initial schools after they complete this commitment in order to teach in other schools, while 39.5% leave the profession entirely (Donaldson & Johnson, 2011), and attrition rates for mathematics teachers who are not math majors are higher than those who are (Donaldson & Johnson, 2010). Using a sample of 2,029 teachers, Donaldson and Johnson found that the most common reason for teachers deciding to leave was to pursue careers outside of K-12 teaching. This result was not surprising, as other research on TFA has found that many TFA teachers who enter the program do not intend to pursue teaching as a long-term career (Donaldson et al., 2011; Heineke, Mazza, & Tichnor-Wagner, 2014). The next most common reason TFA teachers cited for leaving the profession was poor environmental conditions at work, including lackluster administrative leadership and an absence of collaboration (Donaldson et al., 2011). The level of support experienced by TFA teachers from their colleagues has been studied by several researchers (Chambers, 2017; Heineke et al., 2014). This research suggested that the quality of teachers' relationships and the extent to which they feel supported at work impacts the likelihood that they remain in teaching beyond their two-year commitments.

The research presented in this paper aims to further our understanding of the way that TFA teachers use their support networks during their first years in low-income, urban schools. Through a multiple case study design looking at four second-year TFA mathematics teachers, this paper addressed the following questions:

1. What types of support do the TFA teachers receive and what are the sources of these supports?
2. To what extent do the TFA teachers feel that their support needs are met and how does perceived effectiveness vary by support source?
3. What are the teachers' post-commitment plans and what factors influence their decision-making?

Theoretical Framework

Regardless of certification pathway, support is needed to lay the foundation for professional growth and to ensure that professional development is meaningful (Stanulis, Burrill, & Ames, 2009). Prior research has investigated the importance and roles of principals, mentors, and colleagues in supporting novice teachers. These studies have used a variety of definitions and constructs to measure and describe support. The framework for support used in this study will be based on House's (1981) theory of social support, which describes four types of support that teachers need - appraisal, emotional, informational, and instrumental. Table 1 presents definitions and examples of each support type.

Table 1

Definition of Support Types and Examples (adapted from Cihak, 2015)

Support Type	***Example***
Appraisal Support – supportive behaviors that involve providing guidelines regarding job responsibilities and ongoing personal appraisal about the teacher's performance	A principal observing a teacher's class and offering constructive criticism to the TFA teacher about their performance and their ability to meet the goals of the lesson.
Emotional Support – supportive behaviors that indicate that teachers are respected, trusted professionals, and worthy of concern	A colleague expressing empathy for the challenges faced by the TFA teacher.
Informational Support – supportive behaviors that involve providing teachers with information that they can use to improve classroom practices	A mentor suggesting a way to implement a lesson plan designed by the teacher prior to class.
Instrumental Support – supportive behaviors that involve directly assisting teachers with work-related tasks, such as providing necessary materials, space, and resources.	A peer from university classes providing the TFA with curricular materials for a class they both teach for later use.

Even though research has investigated both the importance of how teachers and principals view these supports, and the extent to which teachers feel that they receive these types of support from principals and administrators (Cordeau, 2003; Littrell, 1992; Schindewolf, 2008), previous research has not examined how the combination of supports influence TFA teachers' career decisions. The use of the combination of resources available to TFA teachers was particularly complex and included sources at their schools (administrators and other teachers), sources within the TFA program (TFA staff for their region), and sources at the university where they are seeking their degrees/licensure (university faculty and their TFA peers). Furthermore, TFA teachers' support needs might differ from other teachers because of the difference in their backgrounds and skill sets. For example, TFA teachers often do not possess a degree in the subject area that they are teaching (Donaldson et al., 2010; Donaldson et al., 2011).

This study aims to understand how TFA teachers' use of support networks allow them to meet their early career challenges, and their decisions around their career plans beyond their two-year commitment in their placement schools. The collection of individuals from which the TFA teachers receive support will be defined as their support network. Each individual within this network will be described as a social network member. The goals of this study are to uncover the characteristics of effective support network members, conditions under which support networks are more likely to be effective in aiding novice teachers in overcoming their challenges, and the extent to which these factors influence teachers' post-commitment career plans. Figure 1 presents the framework which describes the theorized relationship between support and post-commitment plans.

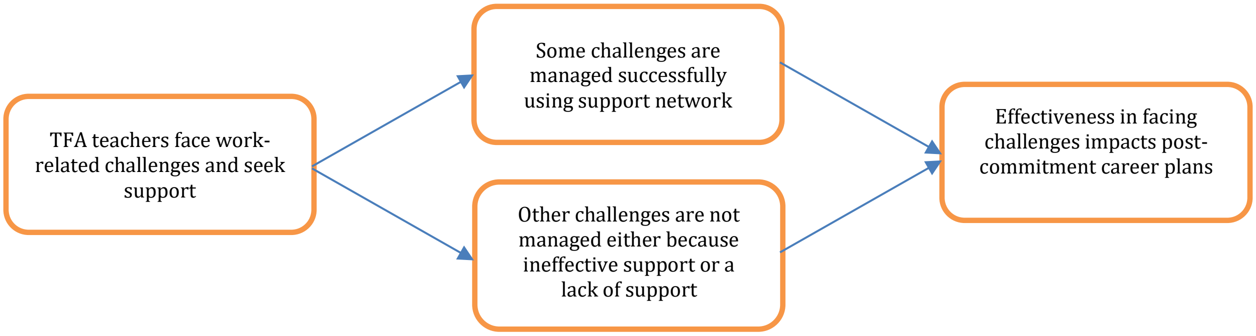

Figure 1. Theoretical Framework

It is worth noting that even though attrition may not always have entirely negative effects (e.g. when quality of departing teachers is low), teacher attrition has disruptive effects and can drive out teachers that might otherwise improve in teacher quality, which increases with years of experience (Henry, Fortner, & Bastian, 2012; Ronfeldt, Loeb & Wyckoff, 2013).

Methods

This study used a multiple case study design in order to explore how second-year mathematics teachers who have been placed in high-needs schools in the northeast United States leverage their support networks to undertake challenges faced by teachers. Second-year TFA teachers were asked to participate in interviews from a TFA-affiliated university as they completed their two-year commitment at their placement schools, and four of these teachers opted to interview with the researcher. Tables 2 provides some details on the teachers who participated in this study. Note that participant names have been replaced by pseudonyms.

Table 2

Participant Summary. Note: Family SES is listed as described by participant.

Name	School Level	Race	Family SES	Undergraduate Major(s)	Classes Taught
Harry	Middle (6-8)	Asian	Low/Middle	Sociology	Sheltered English Immersion Math I/II
Ron	High (9-12)	White	Upper-Middle	Human/Organizational Development	Algebra I, Geometry
Jimmy	High (9-12)	Asian	Lower	Computer Science and Economics	Computer Science, Algebra, Geometry
Karen	High (9-12)	Black	Upper-Middle	Public Health	Algebra II, Geometry

The teachers all taught secondary mathematics courses and came from a variety of SES and racial/ethnic backgrounds, which is characteristic of TFA teachers (Zahner et al., 2018). It is worth noting that none of the TFA teachers had a mathematics or education background despite being selected to teach mathematics classes.

Data Collection and Analysis. The teachers all agreed to meet the researcher for one semi-structured interview lasting between 45 to 60 minutes. The interviews took place in November or December of the teachers' second year in their placement school. Teachers were asked about what challenges they encountered and how they attempted to overcome them, including if and how they sought support for these challenges. They were also asked about their post-commitment plans and the factors that influenced their decisions. These interviews were audio-recorded, and transcripts were created for each participant. The types of support received by the teachers, support sources, and post-commitment plans were then identified using the transcripts. Each support source identified was then categorized as being from TFA, their school/district, or their university. The reasons for choosing the support and the perceived level of effectiveness of each support was also linked to the source when described by the participants, as well as any details about the interactions with each course. Each instance of support was coded as one of the four support types and then these categories were then analyzed together across cases in order to identify any themes that emerged when comparing across cases.

Findings

The following section describes how and why teachers sought and received each support type, and teachers' post-commitment plans.

Appraisal support. Appraisal support involves providing guidelines regarding job responsibilities and ongoing personal appraisal about the teacher's performance. For the most part, teachers described appraisal support as being received through classroom observations and through meetings with support network members. The amount of appraisal support varied tremendously by teacher and came from a wide range of individuals, including colleagues, TFA staff, instructional coaches within the schools, supervisors at schools, and principals. The data appeared to suggest that high quality appraisal support came both from support network members with and without mathematics expertise, and the preference of source varied by teacher. For example, Karen mentioned that she found it beneficial to get the perspective of an observer who was not an expert in mathematics.

Emotional support. Emotional support include supportive behaviors that indicate that teachers are respected, trusted professionals, and worthy of concern. This type of support was not mentioned much by any of the teachers, nor was the lack of this support mentioned as a significant challenge. This finding suggests either that other challenges are more important to these TFA teachers, or that these TFA teachers do not consider this type of support to be necessary to their professional success. Alternatively, this result also suggests that the interview questioning was not framed in a way that would elicit responses relating to this dimension of support. Further research may be needed to understand TFA teachers' emotional support needs.

Informational support. Informational support involved providing teachers with information that they can use to improve classroom practices. The TFA teachers received this support mainly in the form of learning about how to work effectively with English Language Learners and/or students of color, which are large segments of the populations that they taught. This type of support was provided by university classes, as well as the TFA Summer Institute through its Diversity, Race, and Inclusion seminars. Two teachers specifically mentioned going to TFA staff in order to receive this type of support, as they were helpful and readily available. These staff members were able to support teachers with information that allowed the teachers to meet challenges that they were facing. Across teachers, however, it appeared that results are mixed, with some teachers describing a need for more or different informational support than what is available to them. It was worthwhile to consider ways in which informational support can be provided on a more consistent basis, and in a way that is needed by teachers with different sets of needs.

Instrumental support. Inadequate instrumental support was the most commonly cited challenge for these teachers. The main concern for this group of teachers was the development of curricular materials. Although the teachers were able to get some support from TFA peers and colleagues at school, they often had to create their own materials from scratch or had to modify existing curricula to a great extent to meet the needs of their students. The teachers generally created materials using a combination of online resources and help from colleagues/supervisors at school or in TFA. This work appeared to be very time consuming, making up a significant portion of the TFA teachers' work outside of class. This type of support was most difficult to obtain in situations where the teacher was the only one at the school who was teaching that subject/grade level. This information highlights the importance of developing ways to bring resources to teachers who are not able to obtain them within their school settings.

Anticipated post-commitment plans. At the end of the interviews, the teachers were asked about their post-commitment plans. Table 3 below summarizes their tentative plans and explanations for why they are leaning in that direction.

Table 3

Anticipated post-commitment plans.

Name	Anticipated Post-Career Plans	Reason
Harry	Unsure on remaining in teaching.	Burnout / believing someone else could do better
Ron	Leaning towards staying in teaching, but at a different school	Desire for sustainable work-life balance
Jimmy	Leaving teaching for another field (potentially educational technology)	Workload is too much to handle
Karen	Staying in teaching, but at another school	Wanted to try a different school where the workload might be more manageable

All four teachers described the immense workload as being influential in their decision-making process, with burnout being a major factor in their decisions.

Implications and Future Directions

Work-life balance and burnout were named as the primary reasons that teachers would leave their schools, either for other schools or for other professions. It was noteworthy that none of the four teachers entered the program with pre-existing plans to leave after the two years, which is common among TFA teachers (Donaldson et al., 2011). However, each was overwhelmed by the tremendous workload, even with the support networks provided for them by TFA, their university, and their schools. The need to create curricular materials, often independently, appeared to be a challenge that was consistent across the cases investigated in this study.

Further research might investigate if these same challenges exist across a larger sample of TFA teachers, and if the patterns of support network use identified in these cases exist more generally for TFA teachers. Also, future work utilizing the House framework should create a set of interview questions that better capture teachers’ needs across all four support types effectively. A limitation of this study is that the questions used by the researcher did not effectively capture aspects of emotional support (as the House framework was applied to pre-existing data around teachers’ feelings of support), and care would need to be taken in the future in order to learn about all four types in more detail. It would also be important to see how these trends compare to teachers who enter through other certification pathways as well. Providing adequate supports to reduce the workload for TFA teachers may be an important step towards reducing teacher attrition in schools that are already struggling to recruit and retain teachers.

Creating environments where these teachers can thrive will provide benefits for both the schools and the students and communities that they serve. Finally, recognition of the needs of novice mathematics teachers and how these needs can be addressed by teacher educators and school staff and administration provides opportunities to improve attrition and success of these teachers, regardless of certification pathway.

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.FOLLOW-UP CONVERSATIONS: INSIDE OR OUTSIDE OF CHILDREN'S STRATEGY DETAILS?

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This study was contextualized in a vision of teaching that is responsive to children's mathematical thinking. We explored elementary school teachers' engagement with strategy details in children's written work, and our findings showcase the complexity of connecting these details to teaching moves during follow-up conversations. We introduce the lens of inside/outside moves as a parsimonious way for researchers to examine teacher-student conversations to understand and capture teachers' expertise in building on children's thinking. For teachers and professional developers, this lens can serve as both a heuristic and a reminder of the importance of attending to children's strategy details.

Current visions of high-quality mathematics instruction focus on teaching that emphasizes eliciting and building on the details of children's thinking, even in the midst of instruction. We talk about this type of teaching as *responsive teaching* and adopt three characterizing features identified by Richards and Robertson (2016): (a) attending to the substance of children's ideas, (b) recognizing important mathematical connections within those ideas, and (c) taking up and pursuing those ideas. In doing so, we connect with a large body of research and policy reports that document benefits for children and teachers when children's thinking plays a central role in instruction (National Council of Teachers of Mathematics, 2014).

Despite documented benefits of teaching that is responsive to children's thinking, developing the needed expertise has proven challenging. We addressed this challenge by examining one component of expertise—teachers' professional noticing—in the area of fractions at the elementary school level. Jacobs and colleagues (2010) defined *professional noticing of children's mathematical thinking* as a set of three interrelated skills: attending to details of children's strategies, interpreting children's understandings reflected in those strategy details, and deciding how to respond on the basis of those understandings. Their investigation of this expertise in four groups of prospective and practicing teachers provided evidence that professional noticing expertise was complex but learnable with time and support. In this paper, we focused on the component skill of deciding how to respond, which Jacobs and colleagues found to be the most difficult for teachers—teachers must integrate their interpretations of children's strategy details and understandings into responses, and there are always multiple productive responses possible. To explore this decision making in relation to individual children's thinking, we introduced a new lens of inside/outside moves. This lens foregrounds individual children's thinking given the important role that teachers' one-on-one conversations play in children's learning, especially when teachers circulate as children are solving problems (Jacobs & Empson, 2016; Stein, Engle, Smith, & Hughes, 2008).

Methods

The data for this paper are part of a larger project in which 92 grades 3–5 teachers engaged in multiyear professional development about children's fraction thinking and how to build on that thinking during instruction. The professional development consists of more than 150 formal workshop hours offered over three years, and also included school-based activities each year. Teachers collaboratively engage with children, video and written work artifacts, research-based frameworks, mathematics problems, and readings (see Jacobs et al., in press, for more information). In this paper, we explore the responses of a subset of the teachers (n = 54) who chose to focus on a particular child's strategy in an assessment of professional noticing. Teachers are drawn from three neighboring districts in the southeastern U.S., had a range of years of teaching experience (1–31 years, M = 8.5 years), and had completed one (n = 18), two (n = 16), or three (n = 20) years of the professional development.

Professional Noticing Assessment

This written assessment was designed to capture teachers' expertise in professional noticing of children's mathematical thinking in a variety of scenarios. For each scenario, teachers were asked to respond to prompts related to three component skills of professional noticing: attending, interpreting, and deciding how to respond. In this paper, we focused on teachers' decisions about how to respond to fourth graders' written work for this fraction story problem: *The teacher has 4 pancakes to share equally among 6 children. How much pancake does each child get?* Teachers were given the written work of three children then asked to select one child and anticipate a one-on-one follow-up conversation with that child, describing the proposed teaching moves and rationales for those moves. We shared our analysis of the anticipated conversations for the 54 teachers who chose to focus on one of the children—Joy.

Joy's strategy (Figure 1) included a valid problem-solving process with a correct answer presented in a non-traditional form. Joy began by drawing four pancakes and dividing them into fourths. She then distributed 1/4 to each of the six children twice, as indicated by the numerals

1–6 (representing children) in each of the fourths. At this point, each child had received 2/4 of a pancake (or “two courters” in Joy’s language), but there were not enough pieces in the last pancake for each of the six children to receive a piece. She divided the last pancake into eighths by cutting the fourths in half, and then distributed 1/8 to each child. At this point, each child had received 2/4 and 1/8 of a pancake, but there were again not enough pieces remaining to distribute them equally to the six children. She divided the two remaining pieces to create six pieces—one for each child—and distributed those pieces, which she determined were 1/24 of a pancake. She concluded that each child would receive 2/4, 1/8, and 1/24, and recorded her answer in a non-traditional form. Specifically, her final answer included multiple, different-sized pieces rather than a single total amount, and this answer was conveyed in words and pictures of fraction pieces rather than fraction symbols.

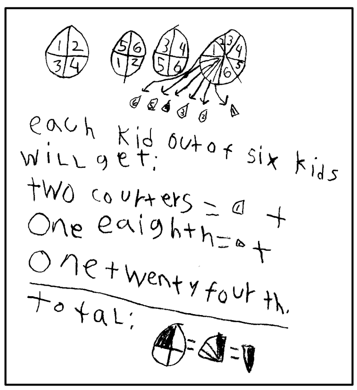

Figure 1. Joy’s strategy for 6 children sharing 4 pancakes.

Analysis

We coded teachers’ written responses holistically on a three-point scale indicating *robust*, *limited*, or *lack of evidence* of building on Joy’s mathematical thinking. These holistic codes were designed to capture the extent to which the moves teachers proposed in their anticipated conversations took into account Joy’s solution to the pancake problem, honored her existing understandings, left room for her future thinking, and were consistent with what the field knows about children’s thinking. We did not look for particular moves or a particular number of moves but instead considered moves in relation to the teachers’ goals as conveyed in their rationales. Written responses were blinded prior to analysis and each was coded by two researchers. Agreement was at least 80%, with disagreements resolved through discussion.

To further explore the nature of these anticipated conversations, we performed two additional analyses. First, we used the lens of inside/outside moves to make explicit connections between teachers' proposed moves and Joy's strategy. Second, we considered the relationship between the holistic code for teachers' anticipated conversations and a previously generated holistic code for those same teachers' descriptions of Joy's strategy. This comparison allowed us to connect the first and third component skills of professional noticing: attending to children's strategy details and deciding how to respond on the basis of children's understandings.

Findings and Discussion

Our holistic coding of teachers' anticipated conversations with Joy reflected a continuum of expertise in deciding how to respond. We found that many teachers were still learning how to build on children's thinking in follow-up conversations, which confirmed earlier findings about the complexity of developing this expertise (Jacobs et al., 2010). Specifically, 13% of the teachers' anticipated conversations were coded as *lack of evidence* of building on Joy's thinking, but most reflected teachers' efforts to build on her thinking—74% demonstrated *limited evidence* and 13% demonstrated *robust evidence*. In the following sections, we further unpack this expertise by describing our two additional analyses involving the lens of inside/outside moves and teachers' descriptions of Joy's strategy.

Lens of Inside/Outside Moves

To better understand how closely connected each teacher's anticipated conversation was to Joy's strategy, we classified each proposed teaching move as either an *inside move* (focused on Joy's existing strategy) or an *outside move* (focused on something Joy did *not* do). Inside moves took a variety of forms such as questions about specific parts of Joy's problem-solving process, fractional quantities connected with Joy's strategy, and links between Joy's representation and the story context (Jacobs & Empson, 2016). Outside moves also took a variety of forms but all introduced or targeted something Joy had *not* done. For example, teachers proposed introducing new fraction language or notation, new representations, or new ways for Joy to partition.

We want to be clear that we are not suggesting that teachers only use inside moves. Outside moves can be important tools to further children's learning. However, to be responsive to children's thinking, teachers need to elicit and extend children's thinking in ways that connect new ideas with existing ideas. This approach therefore requires teachers to create regular opportunities for children to engage with their own ideas, which is enabled by the use of inside moves. We illustrate the use of this lens with two contrasting cases (Figure 2).

Ms. Reed's anticipated conversation. Ms. Reed's response was holistically coded as *robust evidence* of building on Joy's thinking. In analyzing Ms. Reed's specific moves, we noted that she began by asking a general question (#1 in Figure 2) to elicit Joy's thinking before posing seven questions about mathematically interesting parts of Joy's strategy. Four questions (#2–5) asked Joy to describe or provide a justification for a part of her strategy, such as her partitioning and fraction labeling. Three questions (#6–8) asked Joy to articulate a final answer and compare it to the benchmarks of 1/2 and 1 to see how she was thinking about how much pancake each child would receive, even if she was unable to articulate a single fractional amount.

We viewed moves #1–8 as inside moves because they explored Joy's thinking around her existing strategy. Notably, with the exception of the first general question, Ms. Reed could not have crafted these questions in advance because they were based on details Joy generated in her strategy. Ms. Reed's final question (#9) was an outside move in which she inquired about the possibility of partitioning the pancakes differently. Her rationale suggested that she was not requiring new partitioning or advocating for a particular type of partitioning, but instead was wondering what Joy might have learned about partitioning from their earlier conversation.

Ms. Ward's anticipated conversation. Ms. Ward's response was holistically coded as *lack of evidence* of building on Joy's thinking. In analyzing Ms. Ward's specific moves, we noted that she began by asking Joy about partitioning differently—the same move that Ms. Reed used to conclude her conversation. Ms. Ward's second move also focused on partitioning differently. Her rationale for these moves revealed an emphasis on efficiency with the specific goal of helping Joy see her inefficiency and move to a new and more efficient partitioning (e.g., making thirds or halves). We considered all of Ms. Ward's moves to be outside moves because they did not focus on Joy's existing strategy and instead advocated for a new strategy.

Comparison of the two anticipated conversations. We found it significant that although both teachers' responses included the outside move of asking Joy to consider different partitioning, the surrounding moves varied. Ms. Reed began with an inside move, immediately communicating her curiosity about Joy's strategy. She made a total of nine moves, and eight of the moves (89%) were inside moves. Further, the outside move was strategically posed at the end of the conversation, after extensive exploration of Joy's existing strategy. In contrast, Ms. Ward began with an outside move and made a total of two moves, both of which were outside moves.

These findings were reflected throughout our data set. In the anticipated conversations holistically coded as *robust or limited evidence* of building on Joy's thinking, inside moves were dominant—all anticipated conversations but one began with an inside move and the vast majority (87%) had either the same percentage of inside and outside moves or a higher percentage of inside moves. In contrast, in the anticipated conversations holistically coded as *lack of evidence* of building on Joy's thinking, all began with an outside move and had a higher percentage of outside moves. Percentages of teachers' inside and outside moves (versus actual number of moves) were used to account for some teachers (like Ms. Reed) proposing multiple moves and others (like Ms. Ward) proposing only a few.

Mean percentage of inside moves also differed across the three groups of responses: 91%, 68%, and 0% for responses holistically coded as *robust, limited*, and *lack of evidence* of building on Joy's thinking, respectively. We found it noteworthy that the mean percentage of inside moves in the *robust evidence* group of responses was 91% and not 100%. Teachers who are building on children's thinking sometimes need to introduce new information or steer children in different directions. However, we argue these types of outside moves are most effective if conversations begin with children's ideas and focus predominantly on those ideas.

Ms. Reed's Anticipated Conversation	Ms. Ward's Anticipated Conversation
1. Can you tell me what you did? *(To understand the thinking behind the work)* 2. Why did you split the first 3 pancakes into 4 pieces? *(To understand the rationale, to see if she saw the relationship with the people.)* 3. Tell me about the last pancake. *(I want to see what she was thinking when she split this pancake.)* 4. You wrote here 1/24. Can you show me 1/24 in the picture? 5. How do you know that is 1/24? *(What thinking was behind this decision to split the pieces? What understanding does she have about it?)* 6. Do you know how much the kids will get altogether*? (Can she add her pieces?)* 7. Is it more than 1/2 or less? 8. More than 1 or less? 9. Is there another way to split the pancakes? *(Does she see the connection now?)*	1. I would ask Joy if there was another way she could share [4] pancakes with [6] children. *(She may have started with fourths without thinking about the outcome, so I would like to see if she could do thirds or halves.)* 2. I would ask her if 1/4 + 1/4 + 1/8 + 1/24 would be the most efficient way to share pancakes or if she could find a way to cut bigger servings. *(I know she understands equivalency so I would like to see what she [more efficiently] comes up with.)*

Figure 2. Teachers' written responses identifying proposed teaching moves and rationales (*in italics*) for a one-on-one follow-up conversation with Joy.

Teachers’ Descriptions of Joy’s Strategy

If teachers are going to build on children’s thinking by asking about specific strategy details, they must have already attended to those details. We return to Ms. Reed and Ms. Ward to consider the written responses they provided when asked to describe Joy’s strategy (Figure 3).

Ms. Reed’s Strategy Description	**Ms. Ward’s Strategy Description**
Joy drew her 4 pancakes and cut them into 1/4’s. I believe she did that because she is comfortable with 1/4’s. When she reached her last pancake she realized 1/4’s wouldn’t get each person a pancake piece. I think she then divided it into 1/8’s. Again, I think the 1/4’s and 1/8’s are comfortable for her. After she numbered 6 she realized she had 2 pieces left so she divided the last 2 1/8’s into 6 pieces. I believe she counted the pieces as if thirds were in each 1/8 to come up with twenty-fourths.	Joy drew 4 pancakes first, then automatically cut them into fourths. She may feel comfortable with fourths? She knew the last one could be cut into sixths except for 1 fourth would be cut into twenty-fourths. She understands equivalence but cannot notate her thinking yet!

Figure 3. Teachers’ written responses describing Joy’s strategy.

The teachers’ descriptions began similarly, noting the drawing of four pancakes and partitioning into fourths, and both speculated about Joy’s familiarity with fourths. However, the similarities end there. Ms. Reed walked sequentially through Joy’s strategy, highlighting the mathematically important details of the partitions, why Joy might have partitioned in particular ways, and her use of fraction terminology. Ms. Reed’s written description provided a clear account of Joy’s problem-solving process, the fractional quantities, and Joy’s sense making.

In contrast, Ms. Ward’s written description has many missing details, making it difficult to reconstruct how Joy solved the problem. The description also has a mathematical error (or misspeak)—the eighths in the last pancake are referred to as sixths—and the final sentence highlights what Joy *cannot* do (“notate her thinking”). This deficit approach was consistent with Ms. Ward’s anticipated conversation in which she focused on changing Joy’s strategy to one with a more efficient partitioning. Ms. Ward’s minimal attention to the details in Joy’s strategy may have prevented her from building on Joy’s thinking in the specific ways that Ms. Reed did.

These findings were reflected throughout our data set. In an earlier analysis, we coded teachers’ strategy descriptions holistically on a three-point scale indicating *robust, limited,* or *lack of evidence* of attending to Joy’s strategy details. For the responses coded as *lack of evidence* of attending to Joy’s strategy details, none were also coded as *robust evidence* of building on Joy’s thinking in their anticipated conversations. This finding supported the idea that teachers can only build on children’s thinking if they have seen and made sense of the strategy details. However, although attending to these strategy details is necessary, it does not guarantee building on children’s thinking. For the responses coded as *robust evidence* of attending to Joy’s strategy details, only 39% were also coded as *robust evidence* of building on Joy’s thinking in their anticipated conversations. Building on children’s thinking is challenging work!

Final Thoughts

Teaching that is responsive to children’s thinking is complex, and extensive time and support are required for this expertise to develop. To conclude, we highlight three ideas that inform efforts to study and support the development of this expertise. First, the lens of inside/outside moves provides a parsimonious way for researchers to examine teachers’ conversations with children to understand and capture the range of expertise in building on children’s thinking. Additional explorations of this lens are needed in particular for the construct of outside moves. Finer distinctions in how near or far outside moves are to a child’s thinking are likely to be important for understanding how teachers build on children’s thinking. Second, the lens of inside/outside moves can serve as a simple, yet powerful and memorable, heuristic for teachers. For instance, they might ask themselves “Am I starting with an inside move?” or “Are inside or outside moves dominating our conversation?” Third, we provide additional support for the idea that attending to details in children’s strategies is a necessary, but not sufficient, foundation for achieving a vision of teaching that is responsive to children’s mathematical thinking.

Acknowledgement

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DESIGNING AND EVALUATING OERS FOR EFFECTIVE TEACHING AND LEARNING

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This study describes the design and evaluation of open educational resources for mathematics modeling students. In a single academic year, we sought to use the NCTM's (2014) eight effective teaching and learning practices as a guide to develop course materials, pilot, modify and compare these materials to those that are currently in use at our institution. We use students' content growth, dispositions and mathematical views to make comparisons and conclusions. As we share our positive outcomes and highlight the affordances and challenges of the project, we focus on how open educational resources fit into instructional reform efforts.

Introduction

Over the course of a single academic year, we worked to develop and enact a set of open educational resources [OER] designed for a college-level introductory mathematics modeling course. Our work was motivated by two major factors. First, we had recently completed research into the implicit messages conveyed through mathematics modeling textbooks adopted by institutions in our state (Phipps & Wagner, 2017). The textbooks we examined conveyed that mathematics is best learned through a progression of algorithmic procedures, positioning the student as dependent on the instructor for access to mathematics and problem solving. This finding was dismaying given the field's current understandings about how students learn. In particular, the National Council of Teachers of Mathematics [NCTM] (2014) has identified eight effective teaching and learning practices, none of which was supported by the textbooks we analyzed. Focusing on our own institution's mathematics modeling course, we realized that the commercial textbook we used conveyed the same implicit messages as the textbooks in our study. For example, our institution's commercial textbook "portrayed an abstracted, purified form of mathematics which could only be used to solve problems after proficiency was achieved" (Phipps & Wagner, 2017, p.143). This was particularly troubling given that, in our experience, the mathematics modeling course is populated by non-STEM majors who desire a terminating course to meet core university requirements. Many of our students struggled with K-12 mathematics and exhibit poor self-efficacy and dispositions towards mathematics. For these students, the mathematics modeling course represented the last opportunity to affect change in their views towards mathematics. We concluded that pedagogy and the curriculum materials for this course must reflect teaching and learning practices supported by research in mathematics education.

The second major factor motivating our work was that our statewide university system, the University System of Georgia [USG], was simultaneously engaging in efforts to reduce the financial burden of course materials to college students. The *Affordable Learning Georgia* [ALG] initiative was designed "to promote student success by supporting the implementation of affordable alternatives to expensive commercial textbooks" (Affordable Learning Georgia, n.d., para. 1). As a part of this initiative, ALG was awarding *Textbook Transformation Grants* to support faculty efforts towards developing and/or implementing no- or low-cost resource materials for college courses. Upon application, we were awarded one of these grants to transform our mathematics modeling course by identifying existing OERs that reflect effective teaching and learning practices, developing OER materials when existing resources could not be found, and compiling all the materials into a cohesive course. As part of this process, we collected data to determine the effect of this transformation on students' dispositions and content growth.

The purpose of this paper is to describe the experience of creating no-cost course materials that support effective teaching and learning practices as outlined by NCTM (2014). Knowing that well developed textbooks go through a lengthy vetting process, we present our preliminary findings and invite response.

Literature Review

In 2014, NCTM outlined eight effective teaching and learning practices. Our previous work used these practices as a framework for analyzing the hidden curriculum in mathematics modeling textbooks (see Phipps & Wagner, 2017). The eight elements of effective teaching and learning, as characterized by NCTM, were: (1) establish mathematical goals to focus learning; (2) implement tasks that promote reasoning and problem solving; (3) use and connect mathematical representations; (4) facilitate meaningful mathematical discourse; (5) pose purposeful questions; (6) build procedural fluency from conceptual understanding; (7) support productive struggle in learning mathematics; and 8) elicit and use evidence of student thinking. In our study, we treated the text as agent capable of conveying messages—to both student and instructor—about student capabilities, what they should learn, and how they should learn it.

Research has established that the textbook plays an important role in the mathematics classroom (Fan, Zhu, & Miao, 2013). One of our goals was to develop materials that support effective teaching and learning practices (NCTM, 2014) with respect to the students *and* the course instructor. A number of studies have suggested that textbooks influence instructors' pedagogy (Fan & Kaeley, 2000; Porter, 2002; Tarr, Chavez, & Reys, 2006); however, the bulk of these studies concern pre-college mathematics instruction. The research base at the tertiary-level is thin but suggests that instructors view textbooks as a resource primarily for students (Mesa & Griffiths, 2012; Gonzalez-Martin, 2015). The course textbook, therefore, has little impact on instructors' pedagogy other than topic progression and determining the types of problems and examples to emphasize. Mesa and Griffiths observed that for curricular materials to impact tertiary-level instructional practices, textbooks "need to incorporate considerations for instructor use" (p. 101).

According to Mesa and Griffith (2012), although instructors viewed the textbook as a resource for students, they simultaneously assumed that students in lower-level mathematics courses did not read those textbooks beyond searching for example solutions to homework problems. A large study of college-level introductory mathematics students supported this assumption. Weinberg, Wiesner, Benesh, and Boester (2012) found that students' patterns of textbook use suggest "students are looking for algorithms and shortcuts" (p. 167) rather than building conceptual understanding of the mathematics. Interestingly, the students characterized their activity as efforts to *understand* the material. The authors posited this apparent disconnect to be the result of students' views of mathematics as a set of rules and procedures to be memorized and applied. In our experience, this view of mathematics is common among mathematics modeling students, raising questions about the efficacy of traditional style textbooks in such a course.

Methodology

Our goal was to develop or create OER materials that align with NCTM's (2014) effective teaching and learning practices. We considered ourselves representative of mathematics education faculty without any specialized technological skills. We decided to reflect on the process of designing a textbook, including affordances and challenges, in order to provide insight and encouragement to others who may engage in similar projects. As a part of the evaluation of our course transformation, we collected preliminary data to investigate students' affective domain and content growth. Particularly, we were interested in ensuring that student growth along the course objectives in our transformed course met or exceeded other sections at our institution that used the commercial textbook. Additionally, we were interested in whether the use of our materials resulted in changes to students' productive dispositions.

To measure the impact of the course materials on students' dispositions towards mathematics, we administered a pre- and post-Likert-scale survey composed of questions to assess their views of mathematics, self-efficacy, and dispositions. For example, students rated the extent to which they agreed or disagreed with the statements: *There are often many ways that a mathematics problem can be solved* and *I wasn't born with the math gene but if I work hard I will do well*. In addition, we conducted focus group interviews of a subset of each other's sections to understand how students interacted with the materials and to further explore their views of mathematics. Speculating that perseverance in the course may yield an indirect measure of student dispositions, we collected grade averages and withdraw/fail rates in our sections and compared them to sections not using the new textbook.

To ensure the new materials were meeting course objectives, we measured content knowledge growth using a comparative treatment-control design. We administered a 12-question assessment aligned with specific content objectives both at the beginning and end of the course. The pre-assessment controlled for significant differences of incoming ability level and afforded matched pairs analysis. Another math modeling instructor at our institution agreed to serve as a control so we could compare our students' performance with those using the commercial textbook. Participants in the control class completed the 12-question pre- and post-assessment.

Below, we share some of our reflections on the process of designing OERs and our preliminary findings as to their effectiveness.

Results

Reflecting on the unproductive ways students use traditional textbooks helped us develop a vision for a textbook. We did not want to explain concepts to students through our own writing but rather have students think about the concepts and make conclusions under the direction of a facilitator who guides and directs students' endeavors. We determined to "build procedural fluency from conceptual understanding" (NCTM, 2014, p. 42) and realized this depended on the actions of the instructor. Therefore, we determined that the textbook would position the teacher as a facilitator and the student as one who must engage in thinking and reasoning in order to learn. Because we wanted the textbook to reflect the effective teaching and learning practices identified by NCTM, we focused on how instructor actions could support student learning. Our final product reflected this focus.

The current iteration of our textbook consists of lesson plans designed to guide instructors in implementing high cognitive demand tasks (Smith & Stein, 1998) that promote reasoning, sense making and discussions. Simply, the textbook is an instructor's guide with accompanying student documents. We support instructors by including scaffolding questions, expected student responses and misconceptions, and extensions or remediation. Student resources include learning goals, student notes, homework problems, and websites for additional information and practice. The student notes contain classroom tasks with work space to record key ideas that are realized by the student, written in informal student language, shaped under the influence of peers, and confirmed or redirected by an instructor.

We encountered a number of affordances in developing this textbook. Firstly, we were situated in a way that provided specificity in our work. In particular, our previous work analyzing textbooks provided a framework for analyzing how textbooks support NCTM's (2014) effective teaching and learning practices. This enabled us to design a product that would optimize those practices. Our familiarity with the intended population of students allowed us to create a purposeful curriculum, thinking critically about what to include and how to ensure understanding of important ideas. Secondly, the Internet contains a wealth of ideas, applications, and open licensed materials. We relied extensively on prior OER works with licenses that allowed modifications and adaptations to suit specific purposes. The prevalence of free, powerful software programs that are accessible via laptop, tablet, and smartphone allowed us to incorporate multiple representations of mathematics into the process of solving problems. We chose GeoGebra (www.geogebra.com) as the primary software program to use in the course. This technology allows students to offload computational work and focus on the conceptual problem solving aspects of the course.

We also faced major challenges. Our project from start to finish was completed in one year. Creating the textbook, piloting its use, revisions prior to a second implementation, and evaluation of the product occurred at a tempo which did not relent. This timeline was uniquely tied to the external funding we received for this work; but our experience suggests that curriculum development was a necessarily time-consuming effort. Another challenge was that we only possessed the technological, designer, and programming skills of an average faculty member. We relied on familiar programs, such as Microsoft Word and PowerPoint, but regularly had to search the Internet for additional information. For example, the Americans with Disabilities Act [ADA] required us to learn such things as how to add closed captioning to videos, the functionality of different heading styles, and adding alternative text to PowerPoint images. We also had to educate ourselves in order to navigate the complicated structure of copyright law and open source licensing.

After designing and implementing the materials, we were interested in whether students using our textbook would show progress in productive dispositions. Statistical analyses of the surveys revealed no significant difference in overall dispositions of students from beginning to end of the semester. In fact, student optimism was greater at the beginning of the semester than at the end as evidenced by more agreement with the statements: *My success in this class is not related to hard work and effort* and *I think that it is important to complete all homework and assignments for this class,* and less agreement with the statement: *I frequently check my answers to see if they are reasonable.* We were intrigued by the seemingly contradictory implications of changes in beliefs about the necessity of homework and whether effort translates to success. It was unclear whether this is an anomaly or whether students do not consider the completion of homework as falling under the realm of effort.

Withdraw/fail rates may offer an indirect measure of student disposition. We analyzed the withdraw/fail rate for this course at our institution across all sections over the academic year of this study. The withdraw/fail rate for mathematics modeling averaged 18% across all sections compared with a rate of 9.3% in our sections. Grade point averages in the separate sections were similar, with a 2.6 average in our sections compared to a 2.7 average in the other sections. The difference in the withdraw/fail rates, however, may be attributed to a number of factors. It was possible that the lower withdraw/fail rate of our sections is due to our attempts to make the content more engaging and relevant. It was also possible that student self-efficacy was affected through the use of these materials. On the other hand, it was possible that the difference is simply coincidental or due to instructor enthusiasm. Additional study would be necessary to make a determination.

Perhaps most importantly, we evaluated to what extent our students exhibited content knowledge growth compared with the other section of the same course. The mathematics modeling class that served as a control had 12 students who agreed to participate in the study and the two experimental sections yielded 26 total participants. Statistical analysis of the pre-assessment revealed significantly greater incoming knowledge among students in the control group ($\alpha = 0.05$). Despite this, students in the experimental group performed significantly better on the post-assessment ($p = 0.0015$) than those in the control. Students in the experimental group scored significantly higher on 10 of the 12 assessment items ($\alpha = 0.05$) as compared to their performance on the pre-assessment. A 95% confidence interval suggests that students using our materials will increase their performance by 30 to 50 percentage points from beginning to end of the semester.

Given the small numbers of participants, these results must be considered preliminary; however, the evidence was strong that use of the developed materials is at least as effective as the commercial textbook in use at our institution. Importantly, these materials fall under a creative commons license and were free for all.

Discussion

The purpose of this manuscript is to share our process and to solicit critique and review of our materials from colleagues. The dissemination of our materials through Galileo Open Learning Materials Repository is in progress. Reaching out to other instructors of mathematics modeling is a critical next step in vetting the new textbook. We are interested in the usability of the materials from an instructor perspective, coupled with measuring student learning outcomes and dispositions. Additionally, we hope this article will encourage others to consider creating and using purposeful open educational resources as part of their ongoing professional endeavors. Doing so allows faculty to situate materials and classroom assignments for specific populations of learners. Our story demonstrates success from two faculties' perspectives in creating an OER for mathematics modeling students.

As open educational resources become readily available, adopters are encouraged to think about how their purpose aligns with timeliness, perspective, and their population of learners. Advantages of OERs may be greater access to materials for students and instructors, an alleviation of financial burdens, and content that is adaptable to a large population of students. On the other hand, disadvantages include quality control issues and sustainability for long term use. As popularity of OERs increases for situated contextual learning, future research should address how mathematics educators can evaluate effectiveness. Our study foregrounded

NCTM's (2014) effective teaching and learning principles as a theoretical framework. What other criteria could guide and inform resource design and evaluation?

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Leading and Learning for Professional Development

CHANGE IN DISCOURSE DIMENSIONS IN ELEMENTARY CLASSROOMS OF PROFESSIONAL DEVELOPMENT PARTICIPANTS

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We seek to understand change in the practice of orchestrating mathematical discourse among elementary teachers who participated in a professional development program designed to support teachers in promoting high-quality discourse during mathematics instruction. Results of analyzing discourse in classrooms of teachers show overall growth in teachers' orchestration of discourse, regardless of the teachers' initial knowledge and practice levels; i.e., initial knowledge/practice did not define teachers' growth. We also investigate teacher change in specific dimensions of discourse, including questioning, explaining, and communication patterns.

Introduction

The practice of orchestrating mathematical discourse is timely given its importance for mathematics education (National Council of Teachers of Mathematics, 2014). Rich mathematical discourse that involves both the teacher and students in the creation of mathematical ideas can be enhanced by teachers asking questions that encourage students to think conceptually and by allowing students to ask questions to each other as well as to the teacher (Boaler & Brodie, 2004). Furthermore, researchers recommend fostering multi-directional communications between the teacher and students and among students in order for the intended meaning to be delivered (Tofel-Grehl, Callahan, & Nadelson, 2017). However, communications observed in mathematics classrooms in the US are mostly unidirectional--teacher to student--inhibiting students' interest in mathematics (Herbel-Eisenmann, Steele, & Cirillo, 2013). Types of mathematical explanations discussed during lessons are also important for students' understanding. Kazemi and Stipek (2001) pointed out the importance of explanation that consists of mathematical argumentation, going beyond procedural explanation, to help students conceptualize mathematics.

While the practice of orchestrating high-quality mathematical discourse is important, it has been difficult for many teachers to implement (Kazemi & Stipek, 2001). Thus, professional development (PD) opportunities that effectively support all teachers in enhancing their abilities to promote high-quality mathematical discourse in their classrooms are critical. In this paper, we present an analysis of observation data to capture changes in the quality of mathematical discourse in classrooms of teachers who participated in one such PD program, thereby shedding light on the value of participating in the PD program.

This investigation is part of a larger design research study that involved several cycles of design, implementations, analysis, and revisions of the Project All Included in Mathematics (Project AIM) PD program. Project AIM is a 40-hour, year-long PD program that is designed to support elementary teachers in promoting high-quality discourse during mathematics instruction. A main feature of the program is the inclusion of ready-to-use strategies typically used to support discourse in literacy instruction that have been adapted by the AIM team to support discourse in mathematics instruction. Through AIM PD activities, participants learn about characteristics of high-quality discourse in mathematics and about using these strategies to achieve such discourse with their students.

Through analysis of multiple, previous implementations of Project AIM PD, the research team has consistently found value in teachers' participation in the PD. In every implementation, participants' mathematical knowledge for teaching is assessed pre- and post-PD using items selected from the Learning Mathematics for Teaching measure (Hill & Ball, 2004; Hill, Schilling, & Ball, 2004). Participants also complete a pre- and post-intervention practice questionnaire, which asks them to indicate the frequency with which they use, and students engage with, various discourse practices during mathematics instruction. Data from these instruments has consistently showed pre-post increases in mean scores for participants in their knowledge as well as in their planning for discourse, their preparedness to facilitate discourse-rich instruction, and their students' engagement in discourse. In this paper, we investigate Project AIM teacher change in the practice of orchestrating mathematical discourse by going beyond an examination of self-report measures.

Research Questions

Since observation data are particularly appropriate to capture the quality of discourse (Desimone, 2009), we conducted a retrospective analysis of observation data to capture changes in the quality of mathematical discourse in classrooms of participants of the 2012-13 implementation of the Project AIM PD, guided by the following research questions:

1. Does the richness of mathematical discourse change in classrooms of teachers who participate in a PD program focused on promoting mathematical discourse?

2. Does the richness of specific dimensions of mathematical discourse change in classrooms of teachers who participate in a PD program focused on promoting mathematical discourse?
3. Do teachers' pre-intervention levels of mathematical knowledge for teaching and/or pre-intervention frequencies of engaging in discourse-promoting practices matter for teacher change while participating in a PD program focused on promoting mathematical discourse? If so, how?

Methods

The 2012-13 Project AIM PD program consisted of a three-day summer institute followed by seven after-school sessions during the school year. This study reported on a sample of 15 of the 78 second-grade teachers who participated in the 2012-13 implementation of the PD. The classroom teaching of these 15 participants was observed two consecutive days at two time points--once early in the school year (Fall 2012) and again toward the end of the school year (Spring 2013). The observed teachers were selected using a stratified random sampling approach based on the levels of their responses to the pre-intervention content knowledge assessment (higher or lower) and practice questionnaire (higher or lower), resulting in four teachers in each of four strata. Observation data of one teacher (higher practice and lower knowledge) was dropped from the analysis reported here due to a delay in obtaining the early observation data, which resulted in a very short time-period between the early and late observations. Subsequently, the higher-practice, lower-knowledge stratum consisted of three teachers, while each of the other three strata consisted of four teachers. This reduced the sample investigated in this paper from 16 to 15 teachers.

In conducting the observations, members of the Project AIM research team, other than the authors, followed a classroom observation protocol, which included providing a written description of the discourse that occurred during each phase of the observed lesson--the launch, explore, and discuss. Guided by the protocol, the observers documented specific examples and verbatim quotes whenever possible. Observers were in each of the classrooms for two consecutive days for each of the observation time points. The length of a classroom observation at each time point was approximately two hours (one hour for each lesson observed). The 60 written observation protocols constituted the data source for this study.

The first two authors, who were not part of the project when the observations took place, coded the classroom observation records using the Mathematics Discourse Matrix (Sztajn, Heck, & Malzahn, 2013), a framework developed by the Project AIM team for use in the PD to analyze discourse in participants' classrooms. The Matrix characterizes four types of discourse--correcting, eliciting, probing, and responsive--across four dimensions--questioning, explaining, listening, and modes of communication. Informed by research on discourse (e.g., Hufferd-Ackles, Fuson, & Sherin, 2004; Willey, 2010) and the theoretical assumptions behind the development of the Matrix (Sztajn et al., 2013), we consider the four discourse types in the Matrix to represent levels on a continuum of discourse richness from left to right. In other words, correcting is the least rich level of discourse and responsive is the richest level of discourse. A correcting discourse is observed when the teacher initiates the communication and students respond with sole authority for the teacher. Higher in breadth, eliciting discourse involves more students' participation in discourse, describing their solutions including *what* and *how*. A greater depth of mathematical explanation is observed in the probing type, where the teacher presses for mathematical explanation and justification. Lastly, responsive discourse involves maintaining eliciting and probing in addition to evidence of making mathematical connections and students' agency and taking responsibility for their own learning (Sztajn et al., 2013). It is important to note that the different types of discourse can be used for different purposes during instruction. For example, correcting discourse can be useful and even needed in some instances. However, when the dominating discourse during a lesson is correcting, richness of discourse tends to decline, whereas richer discourse can lead to conceptual understanding.

The use of the Matrix to code the observation data for this investigation allowed the coders to characterize the type of discourse that took place based on what the teacher and students were doing during the lesson. Each pair of two consecutive lessons for a given teacher from the same time point (Fall/Spring) was analyzed together and a discourse type/level was determined for three dimensions of the Matrix--questioning, explaining, and communication patterns (a component of the modes of communication dimension) --for each of the Fall and Spring time points. Given limitations in the details included in field observations, the data did not allow for coding discourse types on the listening dimension and other components of the modes of communication dimension. As seen in Figure 1, a holistic discourse type was determined for each time point for each teacher based on the authors' interpretive judgment of the discourse types for each of the three coded dimensions, which were based on the field notes. After two rounds of practice using the Matrix as a coding tool, the two authors separately coded observation data for two randomly selected teachers (one teacher from the pool of 16 teachers in the 2012-13 AIM PD implementation and one from the 2013-14 AIM PD implementation). To determine the inter-coder reliability, a basic percent agreement was calculated at the item-level (discourse dimension per lesson phase). The coders achieved more than 80% agreement on both coding and their interpretive judgment on the overall type of discourse that took place during each of the two-day lessons.

Findings and Conclusion

Figure 1 shows the early and late discourse type classification each of the 15 teachers received for the richness of their classroom discourse—separated by their pre-intervention scores on the practice and knowledge assessments. When taking into account the dimensions of questioning, explaining, and communication patterns, 12 teachers (80% of the teachers) demonstrated an overall increase in the richness of the mathematical discourse in their classrooms. Discourse observed in the classrooms of two teachers stayed on the same level. Only one teacher decreased in the observed discourse level. This teacher, however, started with a responsive discourse in the Fall observation and maintained discourse above probing in the Spring. This result indicated that despite their initial scores, most teachers improved in their practice as the PD unfolded.

We further investigated the observed discourse by analyzing change in specific discourse dimensions: questioning, explaining, and communication pattern. Our results showed that discourse for the questioning and explaining dimensions had fewer teachers who grew than the communication pattern dimension. The questioning discourse levels declined from Fall to Spring in three classrooms (20%) and remained the same in three other classrooms (20%). When comparing change in the questioning dimension with change in the overall discourse (Figure 1), with the exception of one teacher, the improvement in discourse in the questioning dimension seemed less than the overall improvement in the lessons. Similarly, there were two cases of decline in the explaining dimension (13%) and three cases of no change (20%).

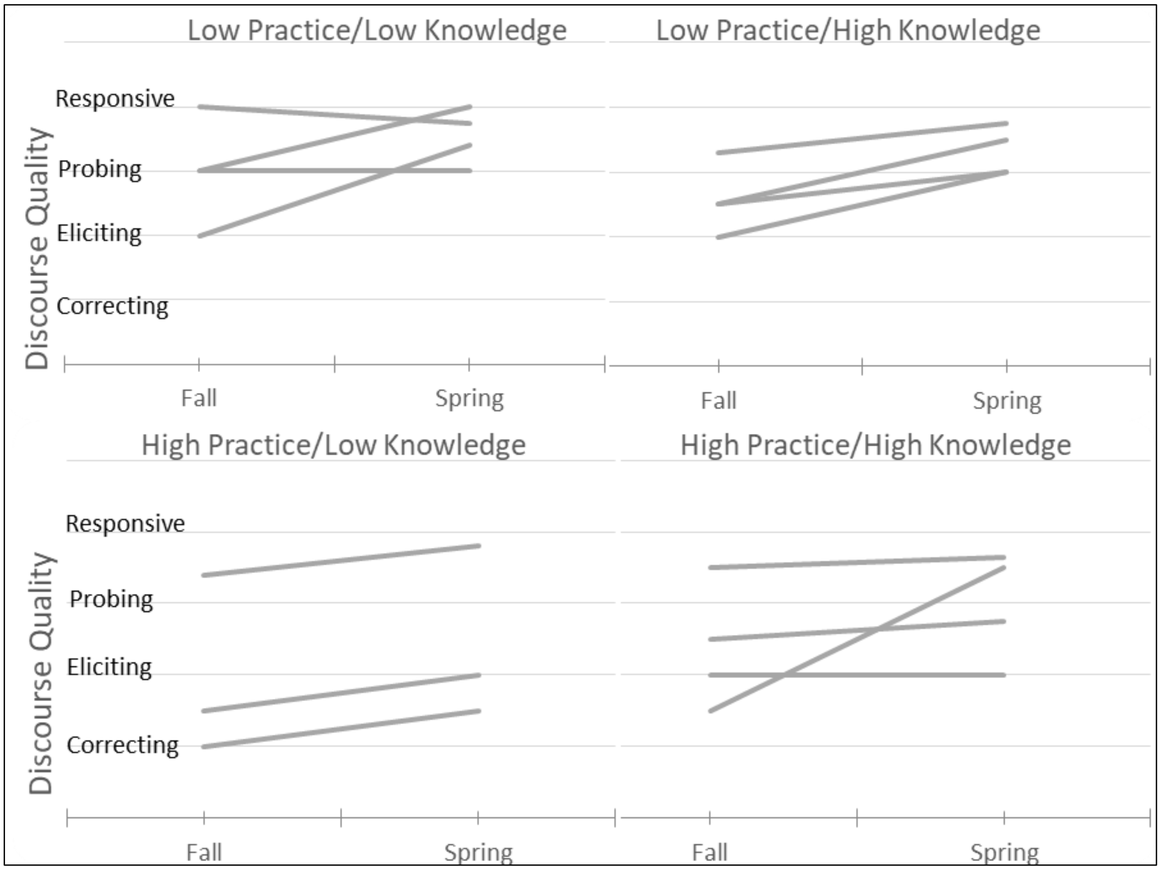

Figure 1. Changes in the overall discourse types from Fall to Spring.

On the other hand, there was improvement in the communication patterns observed in most teachers' classrooms. Only one teacher, whose initial level was lower practice and higher knowledge, had decline in the communication patterns, but remained above the probing discourse. Furthermore, two teachers remained at the same communication-pattern discourse level, but one of them remained at the responsive level. Considering these observations together, this might indicate that adopting strong questioning techniques and fostering richer explanation might be more challenging for teachers than fostering multi-directional communication patterns.

In addition to exploring the change in richness of mathematical discourse, we also attempted to learn if teacher initial knowledge and/or practice level matters for teacher change while participating in the Project AIM PD. Overall, growth in teachers' orchestration of discourse was observed no matter what the teachers' initial knowledge and practice level was; i.e., initial knowledge/practice did not seem to be what defined their growth. A minor exception to that is illustrated in Figure 1; teachers who started with higher practice and lower knowledge, although growing in the observed discourse, did not cross to the next higher level. For example, the two teachers who started at correcting and between correcting and eliciting discourse did not grow beyond eliciting. Similarly, the third teacher in this group started and ended the year between probing and responsive discourse. However, as mentioned previously, the latter started high and stayed at a higher level.

Interestingly, teachers who before the PD began reported lower practice started the school year with generally higher discourse levels as compared to teachers who reported higher practice (see Figure 1). This could be related to different factors; for example, it was possible that teachers who are more aware of the importance of discourse among their students--possibly those who had higher levels of discourse in the Fall observations--were harsher on themselves in the questionnaire responses. It was also possible that involvement in the Summer PD sessions particularly impacted teachers who reported lower frequency of discourse-promoting practice prior to starting the PD.

It was important to note that the observations analyzed in this study are not pre and post implementation of the PD; rather, the Fall observations were conducted relatively early in the implementation of the PD and the Spring observations were conducted towards the end of the PD implementation. Participants had already completed the Summer PD institute when the early (Fall) observations were conducted. Limitations of this study included the relatively small sample size and the source of observation data as written field notes rather than video recordings.

The results of our investigation suggest value in teacher participation in the Project AIM PD for enhancing mathematical discourse in the classroom, regardless of their initial mathematical knowledge level or perception about their practice of orchestrating discourse. This study contributes to PD investigations by enhancing knowledge about teacher change while participating in a research-based PD and by utilizing the Mathematics Discourse Matrix to analyze discourse in mathematics classrooms. Future research can replicate this investigation with a larger sample of teachers and with other mathematics PD programs to learn further if participants' initial knowledge or practice matter for their change while participating in PD.

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THE 8x8 PROJECT: A STUDY OF A PROFESSIONAL DEVELOPMENT PROJECT

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This study reported on the evaluation efforts of a professional development project focused on increasing teacher use of the eight mathematical teaching practices to stimulate student use of the eight standards of mathematical practice. The evaluation sought to answer the questions: How will an inservice professional development workshop series focused on mathematical teaching practices affect (a) teacher views toward mathematics teaching, (b) K-12 teachers' mathematical knowledge for teaching, and (c) teacher support for the standards of mathematical practice with their own student? The study showed significant improvement in teacher views toward teaching mathematics as evidenced by the Draw a Mathematics Teacher test, showed no significant improvement in mathematical knowledge for teaching on the Teacher Knowledge Assessment Survey, and showed significant improvement on the number of standards of mathematical practices being fostered meaningfully in the participants' classrooms when comparisons were made between videos recorded at the beginning and end of the program.

Background

This study investigated the effectiveness of an inservice professional development workshop series focused on mathematical teaching practices to improve teacher pedagogical content knowledge and use of classroom best practices for mathematics teaching. The 8x8 Project studied the mathematical teaching practices outlined in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) to equip teachers with techniques for supporting student use of the standards of mathematical practice outlined in the Common Core State Standards for Mathematics (NGA Center & CCSSO, 2010). The project used a Teacher Needs Based (TNB) model of professional development training researched by Lee (2005). This model responds to the call for teacher professional development "that is responsive to the intrinsic needs of teachers" (Lee, 2005, p. 41) by surveying the perceived needs of teachers throughout the project, facilitating collaborative lesson planning, relying on active learning activities, and incorporating meaningful reflection opportunities through video lesson studies and reflective journaling. The TNB model has proven to change beliefs about the nature of mathematics, increase understanding of the need to change how mathematics is taught, increase the use of best practices for teaching mathematics, and increase understanding of content standards (Lee, 2005).

Throughout the 2016-2017 and 2017-2018 academic years, the project provided 18 full-day workshops for K-12 teachers with support from Mathematics Science Partnership funding. While the workshops often brought all of the participants together to learn strategies, the participants also spent time within their grade-level specific cohorts (K-5, 6-8, 9-12). Each cohort was led by a university mathematics educator and a district master teacher. Each cohort was also supported by a university preservice student assistant. Participants were surveyed throughout the project to determine the order of study for the teaching practices and to give voice to changes needed in the structure of workshops to better meet the needs of the teachers in the group.

The first workshop introduced participants to the eight mathematical teaching practices and the eight standards of mathematical practice. Also, pre-assessment data was collected in this session. The subsequent workshops were structured in sets of two workshop cycles where a mathematical teaching practice was explored, later implemented, and then that implementation was analyzed. Each mini cycle began with a reading assignment from *Principles to Actions* (NCTM, 2014) where the participating teachers explored a new mathematical teaching practice. Then at the next workshop, cohort leaders would facilitate tasks that expanded on the ideas in the reading and present practical classroom strategies for implementing the highlighted practice. Teachers were then asked to implement the tasks and strategies learned during the workshop in their own classrooms. The teachers were also asked to video record at least one class session that showcased their use of the target strategy. These videos were then watched during the next workshop within cohort groups; fellow teachers and cohort leaders gave feedback on the practice implementation seen in the video. The "Teacher and Student Action" charts at the end of each mathematical teaching practice sections of *Principles to Actions* were used as rubrics for implementation and guided video feedback (NCTM, 2014). Teachers were also taught feedback frames such as "I noticed," "I wonder," "Would you consider," and "What you did gave me an idea" to enrich the feedback given and to avoid evaluation of the videos with frames such as "I liked" and "I didn't like."

The teaching practices targeted during the first year of the project were "Implement Tasks That Promote Reasoning and Problem Solving," "Support Productive Struggle in Learning Mathematics," and "Build Procedural Fluency from Conceptual Understanding." The practice of "Establish Mathematics Goals to Focus Learning" was investigated during a three-day summer intensive workshop between the two academic years. The teaching practices targeted during the second year of the project were "Use and Connect Mathematical Representations," "Pose Purposeful Questions," "Facilitate Meaningful Mathematical Discourse," and "Elicit Evidence of Student Thinking.

Methodology

Throughout the project, a variety of data was collected to answer the questions: How will an inservice professional development workshop series focused on mathematical teaching practices affect (a) teacher views toward mathematics teaching, (b) K-12 teachers' mathematical knowledge for teaching, and (c) teacher support for the standards of mathematical practice with their own student?

Lee (2005) explained that TBN professional development programs "can be assessed in the areas of teacher knowledge and skills, as well as teaching practice" (p. 43). The Draw a Math Teacher Test (DAMTT) and classroom videos were used to assess changes in teaching beliefs and practice. The Teacher Knowledge Assessment Survey (TKAS) was used to assess changes in teacher knowledge. All of the assessments were given at the beginning and end of the project.

Draw a Math Teacher Test

To understand the teachers' views toward mathematics teaching, the *Draw a Math Teacher Test (DAMTT)* was used (Utley & Showalter, 2007). Both at the start of the first workshop in the series and the close of the last workshop in the series, teachers were given 10 minutes to draw a mathematics teacher. Also, for teachers that joined the project in year two, these teachers were given the prompt at the start of their first and close of their last workshop in the project. The project evaluator and student assistants scored these drawing according to the *Draw a Math Teacher Test Checklist* (DAMTT-C) (Utley, Reeder, Redmond-Sanogo, Showalter, & Adolphson, n.d.). The checklist assesses eight aspects of the drawings, each on a three-point scale representing where each aspect falls on the DAMTT-C Teaching Styles Continuum. The attributes were teacher activity, teacher position, student activity, student position, classroom environment - arrangement, classroom environment – instructional tools, and classroom-environment – interactions. A score of one indicated a characteristic that is teacher focused, a score of three indicated a characteristic that is student focused, and a score of two indicated a characteristic that is transitional between teacher and student focused. Scores on the DAMTT-C can range from 8 to 24. This instrument was chosen for this project because Lee (2005) cited shifts toward a more student focused, learner centered classroom in a TBN professional development model, and because this instrument measures six attributes on a scale from teacher focus to student focus.

Classroom Videos

Classroom videos of “typical” lessons were collected four times throughout this project at the beginning and end of each school year over the course of the project. The beginning of the year videos was recorded within the first three weeks of the school year and the end of the year videos are recorded within the last three weeks of school. The teachers were asked to record an entire mathematics class session that typified their mathematics instruction.

The project evaluator and student assistants analyzed the videos with the use of the *Revised SMPs Look-for Protocol* that specifically look for student use of the eight standards of mathematical practice and teacher support of those practices (Bostic, Matney, & Sondergeld, 2017). Prior to analyzing the videos, the project evaluator held a meeting to train the student assistants on how to use the protocol to look for evidence of the standards of mathematical practice. The evaluator also reviewed a subset of the videos to check for rating consistency and adherence to the protocol.

In each video, evidence of student use of and teacher support for each of the standards of mathematical practices was rated as none/minimal, developing, or proficient. This protocol was chosen because it specifically measured student use of the standards of mathematical practice and therefore allowed the project evaluator to assess the degree to which teachers were empowered throughout the project to support the use of these practices by their students.

Teacher Knowledge Assessment Survey

To measure potential changes in mathematics content knowledge for teaching, participants took the Teacher Knowledge Assessment Survey (TKAS) (LMT, 2011). The TKAS not only measures grade level content knowledge but assesses pedagogical content knowledge needed to help students arrive at mastered understanding of the grade level standards (LMT, 2011). There are several different TKAS assessments geared toward different grade levels and content areas. Since the participants of the 8x8 Project span grades K-12, different cohorts responded to different assessments. Teachers in the K-5 cohort took the assessment for Numbers, Concepts and Operations for elementary school teachers (EL_NCOP). Teachers in the 6-8 cohort took the assessment for Numbers, Concepts and Operations for middle school teachers (MS_NCOP). Both the 6-8 and 9-12 cohorts took the assessment for Patterns, Functions and Algebra for middle school teachers (MS_PFA). The TKAS was administered online in August 2016, Summer 2017, and June 2018. The intent is for all participants to complete a pre and post assessment as well as gain mid-project information for those participants who were in the program for both years.

Results

Results from the three instruments used in this project help answer the question of how the teaching practices and content knowledge of teachers in the 8x8 Project were affected by the project emphasis on the eight mathematical teaching practices.

Draw a Math Teacher Test

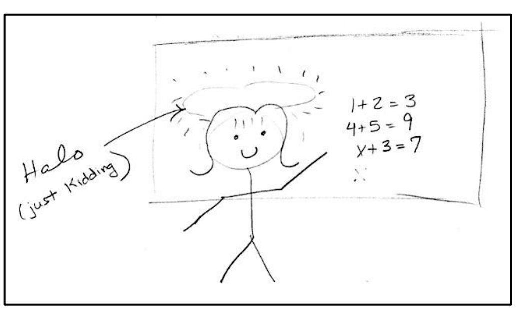
A paired-samples *t*-test was conducted to compare pre and posttest scores on each of the eight aspects of the DAMTT-C and the overall drawing score (see Table 1). There was significant improvement in six of the eight subscales and in the overall total. Of the 29 teachers who completed both pre and posttest drawings, 15 of them completed one year of the program and 14 of them completed two years. While those who completed two years saw more improvement, with an average increase of 2.86 points, compared to those who completed one year, with an average increase of 2.27 points, that difference was not statistically significant ($t = 0.27$, $df = 27$, $p = 0.39$).

Table 1

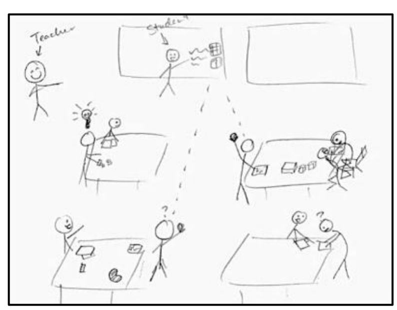
Pre and Postscores for the Drawing Prompt

$n = 29$	Pre	Post	Mean Change	t	*Sig. (1-tail)*
Teacher Activity	1.86	2.45	0.59	3.28	0.001
Teacher Position	1.93	2.38	0.45	2.32	0.014
Student Activity	1.76	2.21	0.45	2.25	0.016
Student Position	1.76	2.07	0.31	1.49	0.074
Classroom Arrangement	2.07	2.21	0.14	0.65	0.261
Instructional Tools	1.48	1.59	0.10	0.58	0.282
Interactions	1.66	2.17	0.52	3.14	0.002
Overall	**12.52**	**15.07**	**2.55**	**2.42**	**0.011**

Figure 1 shows an example of a typical drawing produced at the beginning of the project compared to a typical drawing produced at the end of the project.



Pre



Post

Figure 1. Comparison of typical drawings at the beginning and end of the project

Interestingly, the 40 participants who completed a pretest drawing but not a posttest drawing had an average pretest drawing score average of 9.33, which is significantly lower than the pretest drawing score average of those who completed at least one year of the program (t = 2.49, df = 67, p = 0.008). This may indicate that the teachers most in need of this professional development project were more likely to drop the program.

Classroom Videos

During year one of the project, 25 participants submitted both beginning and end of the year videos. These 25 teachers showed significant improvement on the number of standards of mathematical practices being fostered meaningfully in their classrooms ($\chi^2 = 92.09$, $df = 2$, $p < 0.0001$). In the pretest videos, the participants averaged 5 standards with no/minimal use, 2.6 standards of the practices at a developing level, and 0.4 of the practices with proficient use. In contrast, in the posttest videos for year one, overall, participants demonstrated no/minimal use of 1.9 of the practices, developing use for 2.8 of the practices, and proficient use of 3.3 of the practices, on average. The change in proficiency was highest for the K-5 cohort teachers.

Table 2

Pre and Posttest Average Number of SMPs at Each Level for Year One

	K-5 (n=11)		6-8 (n=6)		HS (n=8)		**Overall (n=25)**	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
None/minimal	4.8	0.2	4.2	2.5	5.8	3.8	**5.0**	**1.9**
Developing	3.2	3.4	2.2	1.7	2.3	2.9	**2.6**	**2.8**
Proficient	0	4.5	1.7	3.8	0	1.4	**0.4**	**3.3**

Similarly, for year two, the 32 participants submitted pre and posttest videos and showed significant improvement on the number of standards of mathematical practice fostered in the classroom (χ^2=47.82, $df = 2$, $p < 0.0001$). On average, in the pretest videos, the 32 participants demonstrated no/minimal use of 3.1 of the standards of practices, developing use for 3.0 of the practices, and proficient use of 1.9 of the practices. In contrast, in the posttest videos for year two, overall participants demonstrated no/minimal use of 1.4 of the practices, developing use for 2.5 of the practices, and proficient use of 4.1 of the practices, on average.

Table 3

Pre and Posttest Average Number of SMPs at Each Level for Year Two

	K-5 (n=10)		6-8 (n=11)		HS (n=11)		**Overall (n=32)**	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
None/minimal	1.7	0.5	3.7	1.3	3.9	2.5	**3.1**	**1.4**
Developing	2.5	1.4	3.3	3.5	3.2	2.5	**3.0**	**2.5**
Proficient	3.8	6.1	1.0	3.2	0.9	3.0	**1.9**	**4.1**

Twelve of the participants (three from the K-5 cohort, five from 6-8 cohort, and four from HS cohort) participated in both years of the program. Their pretest video scores from year one were compared to the posttest video scores from year two (see Table 4). These two-year participants also showed significant improvement ($\chi^2 = 70.86$, $df = 2$, $p < 0.0001$), with their pretest videos year one demonstrating no/minimal use of 5 of the standards of practices, developing use for 2.5 of the practices, and proficient use of 0.5 of the practices, on average. In contrast, in the posttest videos for year two, overall, participants demonstrated no/minimal use of 1.1 of the practices, developing use for 2.2 of the practices, and proficient use of 4.7 of the practices, on average. It should be noted that there was no significant difference in the pretest video scores of the participants who finished the project and those that withdrew from the project.

Table 4

Comparison of year-one prevideo and year-two postvideo scores for two-year participants

n = 12	Pre	Post
None/minimal	5.0	1.1
Developing	2.5	2.2
Proficient	0.5	4.7

Teacher Knowledge Assessment Survey

Scores for the TKAS were reported on a standardized scale, with a range of approximately -3 to +3, with a score of 0 representing the average score. Paired-samples *t*-tests were conducted to assess participant improvement in mathematical knowledge for teaching between pre and posttest scores (see Table 5). Changes in the EL_NCOP, MS_PFA, and MS_NCOP were not significant. The table provides more detail.

Table 5

Post – Pre Score Changes on Teacher Knowledge Assessment Survey

	One-year change				Two-year change			
	Mean	*t*	*df*	Sig. (1-tail)	Mean	*t*	*df*	Sig. (1-tail)
EL_NCOP	-0.02	-0.07	11	0.53	0.05	0.21	7	0.42
MS_PFA	0.01	0.02	19	0.49	0.24	0.97	6	0.18
MS_NCOP	0.33	0.86	4	0.21	0.76	2.33	2	0.07

Further Study and Implication

The data collected throughout this project provided evidence called for by the TBN model for professional development of the teacher knowledge, skills, and practices developed within project participants. Drawing prompt data gathered throughout this project shows a significant shift from teacher-centric classrooms to student-centric classrooms among the project participants. Video data showed a significant increase in teacher support of the eight standards of mathematical practices in their classrooms. Furthermore, the data suggested a possible (though not statistically significant) increase in mathematical knowledge for teaching especially for teachers that participated in the project for two years. Additionally, these improvements seem to be most pronounced for the K-5 teachers and/or teachers who spent two years in the project. All of this data supported the claim that teachers participating in this TBN professional development project focused on increasing use of the mathematical teaching practices increased teacher support for student use of the standards of mathematical practice and a shift toward more learner focused classrooms.

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DESIGNING FOR ORGANIZATIONAL SENSEMAKING OF MATHEMATICS STANDARDS AT SCALE

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This mixed methods study explores the interactions of individual users as they engaged with an online professional development module about mathematical tasks. Using a theoretical lens of organizational sensemaking in conjunction with instructional vision, the findings indicate that instructional vision may mediate the ways in which high school mathematics teachers and district leaders engage in intersubjective meaning making.

Introduction and Background

Large scale educational reform has traditionally been unsuccessful and has only led to modest change in classrooms. One challenge associated with reform at scale is that many innovations, such as high school mathematics standards, trickle through levels of the educational system which allow for multiple interpretations by users that create inconsistencies with the innovation designers' intentions across classrooms, districts, and throughout an entire state (Spillane, Reiser, & Reimer, 2002). One promising solution to limit multiple interpretations of mathematics standards was to promote systemic coherence. The National Research Council (2012) described coherence as creating a shared understanding of the goals which undergird the standards being implemented. One approach to building systemic coherence around mathematics standards was to promote a shared vision of high quality mathematics instruction (VHQMI) as such a vision will necessarily mediate the ways in which standards are interpreted and acted upon. In an effort to limit multiple interpretations of newly revised state-wide high school mathematics standards, our research team designed supports that embody VHQMI to promote organizational sensemaking. One such support was a set of online professional development (PD) modules made available to every high school mathematics teacher in the state, which provide the context for this study.

Context

The online PD modules consisted of content and discussion board pages that were designed to promote VHQMI. Each module consisted of approximately 15 pages and was designed to take users about one to two hours to complete. Upon completion, users were given a completion certificate for which they could submit to their district for continuing education credit. The content pages provided information about research-based instructional strategies through the use of text, images, and videos. The discussion board pages provided opportunities for teachers to respond to open-ended discussion prompts, read others' posts, and respond (see Figure 1 for examples). In the hopes of removing potential barriers to participation, the research team made responses to discussion prompts optional.

Two Discussion Board Prompts from the Mathematics Tasks Module

example 1: Analyzing Math Tasks Prompt

Consider the tasks being used in the content modules this month. Post your answer to these questions and share your thoughts in response to some of your colleagues. *In what ways are these low floor - high ceiling tasks? Why or why not?*

example 2: Adapting Math Tasks Prompt

Pick one of the tasks provided [*one task is provided to the right*] and share in your post ways you would adapt it to increase its cognitive demand. Read and respond to others' responses to see how your colleagues have adapted the tasks. Use the like feature and comment feature to engage with others about adaptation of the tasks.

Find the slope using two points and write an equation.

x	y
0	5
1	9
2	13
3	17
4	21

Figure 1. Two Discussion Board Prompts

Theoretical Perspectives

Two theoretical constructs informed the design of the online modules. First, we conceptualized the process of implementing new mathematics standards using organizational sensemaking (Weick, 1995). Sensemaking was a process which begins when individuals encounter a cue, an event or piece of information, that is novel, ambiguous, or differs from their expectations in such a way that problematizes their current understanding. Sensemaking was subjective, meaning that it only occurs when a cue prompts an individual to actively seek out clarification. Within organizational sensemaking this is referred to as, *constructing intersubjective meaning with actual, implied, or imagined others* (Maitlis & Christianson, 2014, p.67). When sensemaking unfolds among multiple individuals within an organization, such as the set of mathematics teachers in a state, the process shifts from the individual to the collection of individuals as the organization tries to make collective sense of the cue. This perspective has been used by other researchers to investigate teacher learning during the implementation of science standards (Allen & Penuel, 2015).

Second, we used research around instructional vision to articulate learning goals for the modules. Hammerness (2001) defined one's instructional vision broadly as, "teachers' images of ideal classroom practice" (p.143). To date, researchers have investigated the ways instructional vision relates to changes in teacher practice and explored how a shared VHQMI promotes coherence in large scale improvement efforts (Munter & Correnti, 2017). Munter (2014) describes three dimensions of high quality mathematics instruction, one of which is the role of mathematical tasks.

Drawing on organizational sensemaking processes and instructional vision, the online PD modules were purposefully designed to promote VHQMI. One key feature of the modules was the ability to share aspects of one's vision through discussion boards. Given that the presentation of information alone was inadequate to support teachers in the development of new forms of practice (Cobb & Smith, 2008; Coburn & Russell, 2008), we conjectured that the discussion board prompts and responses would act as cues to trigger uncertainty and provide opportunities for intersubjective meaning making that would lead to a more sophisticated and shared VHQMI. This study investigated this initial design conjecture with two specific research questions: *(1) To what extent did users engage with features of the module that were designed to promote VHQMI?* and *(2) In what ways does instructional vision mediate engagement in intersubjective meaning making?*

Methods

This mixed methods study utilized a sequential explanatory design (Creswell, Plano Clark, Gutmann, & Hanson, 2003) to conduct a two phase analysis of quantitative (Phase 1) and qualitative data (Phase 2) focused on one of the online modules, the *Mathematical Tasks* module, and its use over a 15-month period. Recognizing the importance of mathematical tasks for student learning, our research team developed the *Mathematics Tasks* module by drawing heavily on Smith and Stein's (1998) cognitive demand framework as a way to provide opportunities for mathematics teachers across the state to learn and create a shared vision of high cognitive demand mathematical tasks. The module consisted of content and discussion board pages that were designed to introduce ambiguity related to mathematical tasks with a low-floor (accessible to all students) and high-ceiling (potential to lead to further discovery) for the purpose of engaging users in intersubjective meaning making.

For Phase 1, we collected and analyzed quantitative data using the online platform analytics to determine to what extent users engaged with features of the module. The *Mathematics Tasks* module was made available to all high school mathematics teachers in the state, approximately 4500 teachers. A total of 231 people accessed the module and 85 people completed the entire module. Data included rates of completion for the *Mathematics Tasks* module, the number of people who engaged with the module, the number of page views for each page within the module, and specific to the discussion board pages, the number of likes, posts, and responses. Phase 1 analysis aimed to identify patterns of joint engagement among users to determine if users engaged with features of the module designed for intersubjective meaning making.

For Phase 2, we narrowed the focus to discussion board posts that elicited responses from one or more users as responses provided tangible evidence of intersubjective meaning making with actual others, rather than implied or imagined others (Maitlis & Christianson, 2014). We referred to these discussion threads (one post and one response) as episodes. Across the entire *Mathematics Tasks* module, 20 users engaged in 13 episodes, sometimes as the individual making the initial post and sometimes as the responder. Note that some users engaged in multiple episodes. All users were given pseudonyms.

At the beginning of this module, users were asked to narrate their vision of mathematics tasks in a statement by responding to the prompt, “When you hear the term, “math task” what do you think of?” Of the 20 users, 18 provided a vision statement for mathematical tasks which further restricted the sample from 13 episodes to 10. Each of the 18 users’ vision statements were scored using Munter’s (2014) rubric for vision of mathematical tasks, on a hierarchical scale of 0 (lowest) to 4 (highest).

Table 1

Munter’s (2014) Rubric for Vision of Mathematical Tasks

Vision Score	Description
0	The user does not view tasks as varying in quality
1	The user sees tasks as a set of procedures that the teacher needs to share with students
2	The user focuses on “real-world” contexts
3	The user attends to the need for tasks to have multiple solutions
4	The user looks to generalize the mathematics beyond the general context

To establish internal reliability, the vision of mathematical tasks statements were scored separately by two researchers. Reliability was reached with initial agreement of 94% (Miles & Huberman, 1994), any disagreements were resolved through discussion. Using the vision scores, the 10 episodes were split into two groups in relation to the score of the initiator and the respondent (see Table 2). We then looked for patterns across these episodes to look for instances when a user, either the initiator or respondent, was exposed to aspects of instructional vision for mathematical tasks that were outside of their current understanding. Instances of this interaction were coded as productive because new ideas were surfaced. When there was a lack of new ideas surfaced, an unproductive code was given.

Table 2

Episodes Coded as Different of Same

Grouping	Description	Number of Episodes
Different	The initiator and respondent had different vision scores.	6
Same	The initiator and respondent had the same vision score.	4

Findings

Table 3 shows the type and quantity of engagement across the four discussion boards. The data suggest that discussion boards, as a design feature, prompted organizational sensemaking. Making an initial post is optional, yet over 50% of users posted to each of the discussion boards, thus giving evidence that the discussion boards served as cues and users potentially constructed intersubjective meaning with implied others. There are a small number of episodes, but there was a large percentage of repeated visits (number of views) which provides evidence of triggering sensemaking through cues and a few instances of observed intersubjective meaning making with actual others (the number of episodes). These findings imply that discussion boards, as a feature of the module, were effective at triggering sensemaking through the use of cues. There is less direct evidence of intersubjective meaning making, though the high number of repeat visits suggests visitors are using others' posts to make new meanings of tasks.

Table 3

Online Platform Analytics

Discussion Prompt	No. of People	No. of Views	Max No. of Views for Single User	No. of Likes	No. of Initial Discussion Posts	No. of Episodes
Analyzing Math Tasks	120	358	11	7	65	3
Adapting Math Tasks	94	246	9	16	63	8
Sharing Resources	86	215	11	5	62	1
Implementation Challenges	85	204	7	17	68	1

Phase 2 findings from the qualitative analysis suggest that there are two patterns of interaction occurring across episodes. First, productive interactions only occur in episodes where users had different levels of vision. Secondly, different levels of vision did not guarantee productive interactions (see Table 1 for distinctions in vision level). Two episodes, one productive and one unproductive, are provided to illustrate an existence proof for potential opportunities for users to construct intersubjective meaning. Both of these episodes are responses to the discussion board prompts in Figure 1.

A Productive Episode - Different Vision Scores Initial Post - Charles (Vision Level 4) It seems like the learning goal is for students to be able to write a linear equation given a table or set of ordered pairs. Maybe you could give the students a table but instead of labeling the columns x & y, you could label it hours and cost (or units from a different context). And instead of giving so many points where the x increases by 1 every time, give them a couple points, like 1 hr costs $9, and 4 hours costs $21. Then ask them how much it would cost for 15 hours, 40 hours. Then ask if they could create a rule (equation) that could be used to find the cost for any n hours. Response - Jack (Vision Level 1) I like this approach, Charles. When money is involved it seems to be more engaging. Real world problems will get them to initiate the problem solving on their own.
Note. This episode was in response to the "Adapting Math Tasks" discussion board

Figure 2. A Productive Episode - Different Vision Scores

The exchange shown in Figure 2 is an example of a productive interaction in which Charles and Jack had different vision scores. In his post, Charles suggests that the given task be modified to increase its cognitive demand by removing some of the scaffolds, setting the problem in a real-world context, and allowing for students to find the cost at 15, 40, and *n* hours without privileging a specific strategy. These suggestions are consistent with Charles's vision score of Level 4. In Jack's response, he attends specifically to the real-world context of Charles's post, but ignores Charles's suggestions that attended to the mathematics of the problem (e.g. the cost at 15, 40, and *n* hours). Jack's vision score is a Level 1, yet his response to Charles's post is consistent with a vision score of Level 2 because he attended to the real-world aspects of Charles's suggestion, indicating that Jack appropriated new sophistication into his vision.

An Unproductive Episode - Different Vision Scores Initial Post - Sue (Vision Level 1) The[y] each have some guiding questions to help get kids started. Response - Lisa (Vision Level 2) my thoughts exactly!
Note. This episode was in response to the "Analyzing Math Tasks" discussion board

Figure 3. An Unproductive Episode - Different Vision Scores

The exchange shown in Figure 3 is an example of an unproductive episode in which Sue and Lisa had different vision scores. In Sue's post, she describes the tasks as being low-floor, high-ceiling because they have guiding questions. This is consistent with Sue's Level 1 vision that focuses on procedures that need to be applied as opposed to a Level 2 vision that privileges real-world problem solving. Lisa's vision score is Level 2, yet in her response to Sue, she did not offer any new information and only agreed with Sue's post. This episode provides evidence that having users with different vision scores interact does not guarantee productive interaction.

Episodes in which users had the same vision score played out similarly to this example unproductive episode. No new information surfaced, as a result all four episodes that were grouped as *Same* were coded as unproductive. Of the six episodes grouped as *Different*, there were three instances of productive interactions (e.g. Productive Episode) and three instances of unproductive interactions (e.g. Unproductive Episode).

Discussion and Implications

Based on the findings from Phase 1 and 2, there are two important design implications of this study. First, findings from Phase 1 suggest that the current design of the discussion boards, guided by the organizational sensemaking framework (Maitlis & Christianson, 2014), is effective at triggering uncertainty and allowed for users to potentially engage in intersubjective meaning making with imagined or implied others. However, the low number of episodes suggest that the design of the discussion boards may need to be more explicit in prompting users to respond to one another, as this provided opportunity for productive intersubjective meaning making with actual others, as evidenced by Phase 2.

Second, when trying to promote intersubjective meaning making between individuals, it is important that individuals with different levels of vision have opportunities to jointly engage and build understanding. Like-visioned individuals are unlikely to have vastly different resources and ideas with which to build new meanings. However, having individuals with differing vision interact potentially allows for new information to surface, thus allowing the opportunity for individuals to build new meanings together. Our findings suggest that individuals with less sophisticated vision benefit from interacting with others who have more sophisticated ones when remediating their understanding. This seems to satisfy the criteria put forth by Cobb and Smith (2008) who mention the need for interactions of depth with more accomplished others in order to support teachers' development of new practices. This implies that finding the perfect balance of teacher interaction is important because we know from other research that without this interaction it is unlikely change will occur (Coburn & Russell, 2008). Further research is needed to understand this phenomenon in more detail. Specifically, we are left wondering why having different instructional visions only provided opportunities for productivity instead of guaranteeing it? Moreover, did engagement in productive episodes allow for individuals to appropriate more sophisticated instructional visions?

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