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Celebrating 50 years of Research on Mathematics Learning

Reflecting on the Past,
Refracting into the Future

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RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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Celebration: Teachers & Partnerships

PROTOCOL: SYSTEMATIC LITERATURE REVIEW OF THE NATURE OF PARTNERSHIP IN MATHEMATICS EDUCATION RESEARCH

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This systematic literature review examines and explores our understandings of the nature of research partnerships in mathematics education. This paper focuses on the protocol and methods of the systematic literature review we are conducting to address the question of what we know, in the field of mathematics education scholarship, about the nature of partnership and partnership processes. To examine this question, we share in this paper research methods for conducting a rigorous systematic literature review, including data collection and screening processes of searching scholarly databases of published work and our data synthesis processes.

Background

In this study, we investigate what we know and then we consider, by contrast, what we as yet do not know, about the nature of partnership processes in mathematics education research. Because processes of partnership--from launching a partnership through to concluding a partnership--are essential to our understandings of mathematics teaching and learning, it is important to our field to reflect on what we know and what we have learned over the decades about how and why effective partnerships come to be.

In mathematics education research, partnerships can take a variety of forms and functions; our field has been informed by, among others, partnerships between inservice teachers and scholars (Raymond & Leinenbach, 2000), by partnerships between preservice teachers and university faculty (Cavanaugh & Garvey, 2012), and by partnerships involving schools, districts, the community, and families (Civil, Bratton, & Quintos, 2005; Sheldon & Epstein, 2005; Wadham, Darragh, & Ell, 2020). As we, in this study, specify a protocol to examine our understandings about the nature of these partnership instantiations, we begin by articulating our notion of partnership.

The essence of partnership, for us, is relationship. Partnerships sometimes, but not always, arise from relationships. Relationships permeate our social existence (Giles, Smythe, & Spence, 2012) and are continually emerging and dissipating. To position our work within this dynamic universe of relationship, we offer our meaning for relationship, for the purpose of this project, as a connection (or connections) between people (interested persons) and a focus of interest (or perhaps, in some cases, a goal). Instances of relationship include, among a range of other instances, the connections students form with academic disciplines like mathematics, as well as interactions arising from attending a particular conference session--session attendees are in relationship with the rest of the session attendees as well as with the presenter(s). Essentially, relationships are formed from patterns in how we channel our energy and attention (or, to some extent, how our energies and attention are channeled); when we channel our energies towards a common interest, and we are aware of this channeling, a relationship emerges between us and also between us and our common interest.

While relationship must be present for the emergence of partnership, not all relationships result in a partnership. Partnership arises from a shared commitment to the existence of a relationship (Casey, 2008; HEA, 2014). Thus, if I were a mathematics student, I would not see myself as in partnership with mathematics because--though I may have a commitment to my relationship with mathematics, I do not have evidence that mathematics shares that commitment. Similarly, I would not typically see myself as in partnership with fellow conference session attendees even though I recognize the relationship we have created by attending the same session--because I would not necessarily have a commitment to sustaining the existence of those relationships. However, if I were to linger for a few minutes following the conclusion of the session and converse with one or some fellow attendees about potential research questions and directions sparked by the session, then the potential for partnership is opened. This potential could be realized if my peers and I continued to turn our energies and attention on a shared focal point; this continuation of contribution of time, energy, and effort is strong (but not the only) evidence for commitment to the continuation of relationship.

Similarly, partnership is also an extension of collaboration—collaboration is the situation of two or more people working on a shared task to accomplish the same goal. Partnership incorporates this situation but also includes an agreement, or commitment, to engage in the collaboration and to work together, often because the partnership is mutually beneficial (Casey, 2008; HEA, 2014). Because of this commitment or agreement to the shared work, the resources that support the work (time, energy, material funds) would typically be the subject of communal decisions rather than the purview of each individual collaborator. Thus, the distinction between partnership and collaboration is empirically meaningful, since the two situations involve different decision-making processes and, likely, different goals. For the purposes of this study, we confine our literature examination to mathematics education research partnership processes. The aim of this study is to better understand what we know in mathematics education about the nature of mathematics education processes; we frame our research question in broad, general terms so as to capture as much insight as we can into partnership processes. The research question that guides this systematic literature review is:

What do we know about conducting research partnerships in mathematics education?

Methods

This protocol describes the research design plan for a systematic review focused on partnership research within mathematics education. As part of a collaborative effort, we will synthesize the research on mathematics partnership to identify the characteristics of these research efforts described in the literature. To answer our research question, we will complete a four-step literature synthesis process outlined in detail below.

Step 1: Literature Search and Retrieval. The systematic review will include primary studies examining and related to partnership research in the mathematics education discipline. The proposed study will include working papers, dissertations, reports, and articles on the subject to be fully comprehensive and useful to other researchers, practitioners, grant funding agencies, and policymakers. Therefore, we will include peer-reviewed articles and gray literature. Regarding the study design type, we will include prior systematic reviews, experimental, quasi-experimental, rigorously designed quantitative or qualitative research.

The target population are members engaged in research partnership within the context of mathematics education, to include but not limited to: researchers, teachers, students, parents, communities as well as other academic and professional entities. Here we operationally define **research partnerships** as empirical investigations that exhibit bidirectional governance and a commitment to the existence of the partnership, multi-directional benefits, and mutual understandings of engagement in the research enterprise. Thus, we will include all participants engaged in partnership as operationally defined.

We will conduct an inclusive and exhaustive search to obtain relevant studies examining and related to mathematics partnership research. We will conduct an electronic search with the help of relevant educational databases. We will use keywords with synonyms for our search. The search databases will include Academic Search Complete, Education Research Complete, Education Resource Information Center (ERIC), and Teacher Reference Center. We will use an extensive set of keywords and Boolean operators to capture all relevant documents. Based on the keyword strategies used in the most recent systematic reviews (Hoekstra et al., 2018), and pilot literature searchers, we developed a list of keywords and synonyms to guide our search. The specific search strings we will apply to search titles and abstracts are as follows: “partnership*” in combination with the terms “Math*” OR “Algebra*” OR “Number concepts” OR “Arithmetic” OR “Computation” OR “Data analysis” OR “Measurement” OR “Geometry”, AND “education*”. Furthermore, we will conduct ancestry searches through the collected studies. Our institutions employ several systematic-review librarians, whom we plan to work with during the search and retrieval processes.

Step 2: Document Screening. Our general inclusion criteria are as follows: (1) study must present at least one stated research question examining an aspect of the research partnership processes, and the study reported on the methodology, analysis, data collection, findings, and interpretations of findings for this research partnership question, (2) study must be published within the last 40 years (since 1982; from 1982 to 2022), (3) study must be empirical in nature within the mathematics or STEM education (pk-20) subject area, and (4) study is available in English. We chose to base the inclusion and exclusion framework on the SPIDER (sample, phenomenon of interest, design, evaluation, research type) formulation rather than the traditional PICO (population, intervention, comparison, outcome) formulation due to the possible body of qualitative involved in mathematics research partnerships. We developed our inclusion and exclusion criteria, based on the framework we present in Table 1. The conceptual dimensions used for coding will reflect key mathematics education research partnerships derived from prior research. These characteristics are presented in the inclusion and exclusion framework and further illustrated in Table 1. These data will be used to guide the initial screening of all abstracts.

Table 1
Common Mathematics Partnership Research Study Characteristics Used to Derive Inclusion and Exclusion Criteria Based on the SPIDER Formulation

Sample (S)	"researcher*" OR "institution*" OR "college" OR "university" OR "teacher*" OR "school" OR "Community" OR "organization" OR "student*" OR "youth" OR "parent" **Inclusion Criteria:** Study must include at least two individuals or entities engaged in collaborative research. **Exclusion Criteria:** Study does not include at least two entities engaged in collaborative research.
Phenomenon of Interest (PI)	"research partnership*" OR "research collaboration" OR "research association" **Inclusion Criteria:** Study must explicitly examine the research partnership. **Exclusion Criteria:** Study does not explicitly examine the research partnership.
Design (D)	"Action" OR "Case Study" OR "Causal Design" OR "Cohort Design" OR "Cross-sectional Design" OR "Descriptive Design" OR "Experimental Design" OR "Exploratory Design" OR "Historical Design" OR "Longitudinal Design" OR "Observational Design" OR "Philosophical Design" OR "Sequential Design" OR "Systematic Review" OR "Meta-analysis" **Inclusion Criteria:** The document is a research study and has an observable research design. **Exclusion Criteria:** The document is not a research study or does not have an observable research design.
Evaluation/Effect Sizes (E)	"Effect sizes" OR "odds ratio" OR "correlation coefficient*" OR "standardized mean" OR "difference" OR "view*" OR "experience*" OR "opinion*" OR "attitude*" OR "percep*" OR "belie*" OR "feel *" OR "know*" OR "understand*" **Inclusion Criteria:** Study reports or has the necessary collected date to generate an effect size or theme. **Exclusion Criteria:** Study does not report or include the necessary collected date to generate an effect size or theme.
Research Type (R)	"Quantitative" OR "Qualitative" OR "Mixed Methods" OR "Research Synthesis" **Inclusion Criteria:** Study reflect a common research paradigm **Exclusion Criteria:** Study does not reflect a common research paradigm

The data management software EPPI-Reviewer 4 will be used to import the citations. The EPPI-4 software uses machine learning to prioritize unscreened articles based on reviewers' previous screening decisions to reduce the time it takes to screen titles and abstracts; specifically, EPPI-4 assesses the frequency of words incorporated into the inclusions versus the exclusions.

Using EPPI-4, two independent reviewers will screen the abstracts and titles. Periodically, reviewers' screening decisions will be checked for consistency, and unscreened references will be refreshed to enable machine learning software to prioritize unscreened items based on relevance. A team consultation will resolve any uncertainty over the inclusion and exclusion of studies.

Step 3: Data Collection. Data management and data extraction will also be performed using EPPI-Reviewer 4 web-based software. Data extracted from included studies will include study aim/objective, study design, data collection method, study participants, data analysis and study outcomes related to the study of research partnerships. Two members of the review team will perform the data extraction process. There will be both quantitative and qualitative findings from the included articles (e.g., effect sizes, themes, subthemes, quotes from participants). A third independent researcher will be consulted in the case of a dispute between two researchers that cannot be resolved through discussion.

Step 4: Data Synthesis. Based on the Joanna Briggs Institute's (JBI) three-step meta-aggregation process, results from selected studies will be synthesized as follows: (1) extraction of findings or categories and quotes and illustrations from primary studies; (2) categorizing findings according to their similarity, and (3) synthesizes of identified categories into a set of synthesized findings (or analytical themes). We will use full sentences to present the categories generated by meta-aggregation to avoid them being less informative or overlapping. Inductive and deductive qualitative analysis techniques will be employed throughout the data synthesis process. In order to generate a synthesized finding that is informative, two independent researchers will conduct all steps independently then cluster their findings together to create synthesized findings.

Discussion

This review protocol outlines our systematic review process for the retrieval and synthesis of research partnership studies within mathematics education. Based on this review protocol, we contend that our approach will result in a series of review manuscripts describing specific aspects of different types of research partnerships. These manuscripts will attempt to address the gap in the literature, with a specific focus and interest in mathematics and STEM education research and practice. As a result of this study, we hope to provide guidance to mathematics and STEM education researchers seeking to engage in the examination of research partnerships.

In conclusion, our protocol as presented provides a methodological design overview for future research. This paper contributes to the methodological refinement of systematic reviews by emphasizing how our empirical approach is logistically feasible, effective, and efficient. These protocols are written to optimize the quality of research; however, they remain elusive in the mathematics education research context. Ultimately, we hope our protocol will contribute to both the teaching and learning of mathematics. It will also contribute to the quality of research design and reporting of the research partnership studies in the mathematics education research literature.

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IMPACTING SCHOLARSHIP: APPROACHES TO GATHER VALIDITY EVIDENCE

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A central aspect of any quantitative assessment is the degree to which the interpretations and uses of test scores are valid. There have been recent calls for synergies between subject matter experts and psychometricians in the assessment development process. A purpose of this manuscript is to provide readers – particularly mathematics educators – with methodological information that may inform their future assessment work.

Tests are broadly used by mathematics educators and researchers for a variety of purposes. For instance, a test may be designed to evaluate the effectiveness of teacher professional development program, or a test may be designed to assess middle-school students' algebraic readiness. Tests are intentionally designed for a purpose that require them to be used in a specific manner and context. Researchers and practitioners alike should consider whether a test appropriately and efficiently measures the construct they are interested in – such consideration is related to validity and validation. Validity is a function of accumulated evidence and is associated with test results and their interpretations, not the test itself (AERA et al., 2014). It is best understood as "the degree to which evidence and theory support interpretations of test scores" for their proposed uses (AERA et al., 2014, p.11). The process of gathering validity evidence is called validation and it involves the process of accumulating relevant evidence to support proposed score interpretations. There has been a recent surge in assessment development and validation within mathematics education contexts; however, it is unclear what approaches are appropriate for gathering validity evidence and how scholars can collaborate on validation scholarship. A purpose of this manuscript is to give readers strategies to collect validity evidence and continue a conversation around pursuing best practices in validation work that follow the *Standards* (AERA et al., 2014).

Related Literature

The *Standards* (AERA et al., 2014) define assessment to be a broad term encompassing "any systematic method of obtaining information, used to draw inferences about characteristics of people, objects, or programs; a systematic process to measure or evaluate the characteristics or performance of individuals, programs, or other entities, for purposes of drawing inferences" (p. 216.) They describe five sources of validity evidence as well as reliability. Those sources include (a) test content, (b) response process, (c) relations to other variables, (d) internal structure, and (e) consequences from testing/bias. Table 1 provides an overview of the five sources. Reliability is described as a measure of consistency in scores (AERA et al., 2014).

Each source of validity evidence supports a particular aspect of the intended interpretation and use of test scores. There are different approaches to gather each source of validity evidence, which can in turn, support a claim (e.g., Respondents communicate responses in ways that are anticipated.) Two examples are provided to operationalize those connections.

Table 1
Description of the five sources of validity evidence (AERA et al., 2014)

Source of Validity Evidence	Brief Description
Test Content	Indicates that test items, or test content, align to the construct a test intends to measure.
Response Processes	Primarily describes the alignment between test takers' performance or behavior and the construct a test intends to measure.
Internal Structure	Indicates the degree to which test items conform to the construct a test intends to measure. Such evidence may be collected through analysis of test dimensionality and item interrelationships.
Relations to Other Variables	Examines the degree to which test scores are, or are not, related to some ancillary variable.

Consequences of Testing	Presents the intended and unintended consequences following the interpretation and use of test scores.

Evidence based on test content might indicate an assessment accurately represents the construct being measured. Evidence based on internal structure might indicate the test items adequately measure a single construct. Unfortunately, “evidence of instrument validity and reliability is woefully lacking” (Ziebarth et al., 2014, p. 115) in the literature. There have been calls for greater discussions of validity in mathematics education scholarship (Carney et al., 2022; Hill & Shih, 2009), yet validation studies and studies describing validity evidence within mathematics education contexts have been rather rare until recently (e.g., Bostic et al., 2021; Kosko, 2019; Krupa et al., 2019; Lavery et al., 2020). Furthermore, validity is often operationalized too narrowly in K-12 education, such as erroneously discussing the validity of a test, and large-scale assessment developers (e.g., state achievement tests and commercially available testing programs) often present validity evidence without discussing how such evidence supports an intended interpretation and use of test scores (Folger et al., 2022). There has been little discussion in current literature of what approaches might be useful for gathering validity evidence and demonstrating support for validity claims. Validation is a study itself such that individuals and teams of scholars can investigate with the aim of supporting hypothesized claims. A claim is a statement supported with validity evidence. For example, conducting a factor analysis on respondents’ test responses is certainly a form of internal structure but without a claim, it can be difficult to discern what the results from the factor analysis communicate to potential users or test administrators.

A recent surge in validity and validation work within mathematics education scholarship suggests a turn towards grounding research in best practices. This manuscript extends this turn further by providing readers with information about viable approaches to gather validity evidence.

It is important that mathematics education scholars are prepared (a) to conduct such work on their own and (b) to raise questions with scholarly partners. Validation includes both quantitative and qualitative data; hence, it is something that scholars with some form of methodological training can accomplish. This methodologically focused manuscript synthesizes research across numerous mathematics education works as well as experts' opinions to frame data collection approaches that may be taken up in a quest to gather and communicate validity for a test or assessment. An aim of this manuscript is to communicate approaches to gather validity evidence with respect to mathematics education research. Thus, we aim to describe evidence types associated with different approaches.

Methods

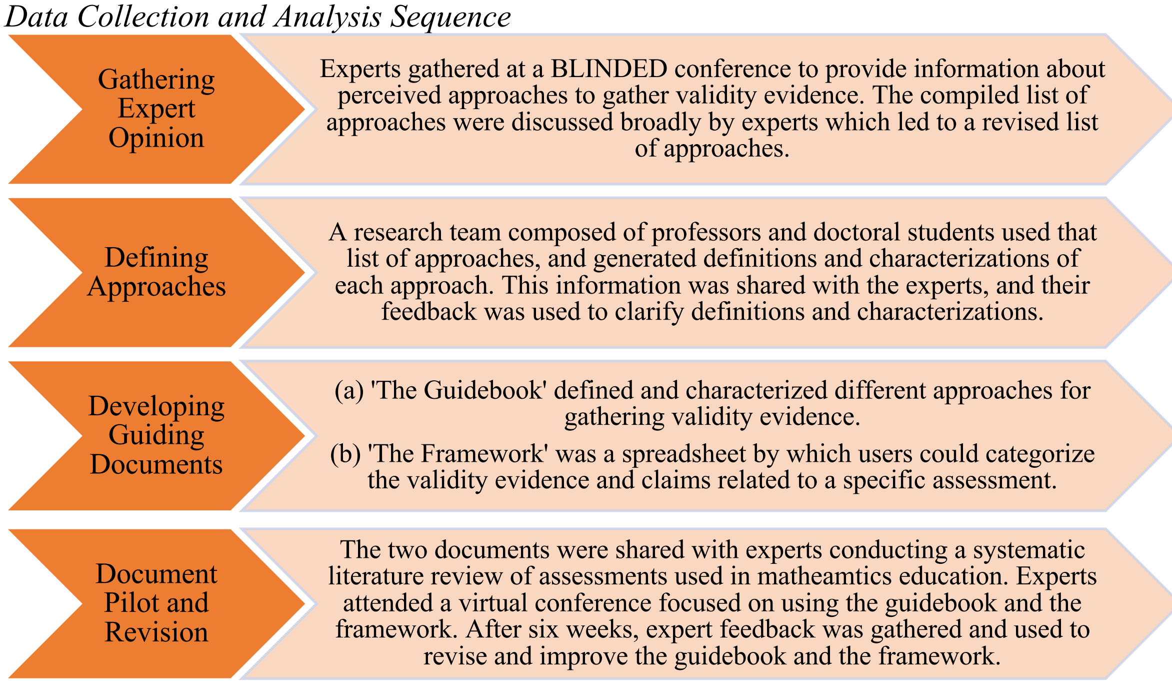
Context & Participants

This study stems from the Validity and Measurement in Mathematics Education (V-M^{2}Ed) project, which aims to describe current validity evidence of mathematics education scholarship between 2000-2020. Participants from V-M^{2}Ed included a diverse group of 35 scholars representing mathematics education, psychometrics, policy, and special education graduate students, academic faculty, and scholars who have published, presented, or worked on projects that related to validity and mathematics assessments. These scholars initially met at the V-M^{2}Ed conference to discuss validity, validation, and mathematics tests and assessments (e.g., observation protocols and surveys). Follow-up meetings occurred online through emails and Zoom meetings. A team of two mathematics education faculty, who organized V-M^{2}Ed, and three graduate students conducted the research in this study.

Data Collection & Analysis

Our team drew upon a design-based research perspective (Middleton et al., 2008) to generate a comprehensive list describing approaches to collect validity evidence related to each source. In addition, a key aim of this study was to clearly characterize numerous approaches for gathering validity evidence. Readers should refer to figure 1 for the sequenced process. This multi-step process provided an opportunity to demonstrate fidelity and validity that our results were drawn appropriately and logically from data sources. A team of five scholars from V-M^{2}Ed qualitatively analyzed data using inductive analysis (Creswell, 2012) and shared findings with the experts who gathered at the V-M^{2}Ed conference.

Figure 1
Data Collection and Analysis Sequence



Results

We selected results from one source of validity evidence, response processes, to highlight the broader findings and demonstrate a connection to the prior literature. Regarding response process validity evidence, the *Standards* state that such evidence characterizes "*the fit between the construct* *and the detailed nature of the performance or response actually engaged in by test takers" (AERA et al., 2014, p. 15).* In other words, evidence based on response processes demonstrates alignment between the construct and the ways in which test takers respond to items or tasks. It is an essential validity source because disconnects between the construct and test takers' behaviors/actual responses may demonstrate a major flaw when interpreting test outcomes. Table 2 defines and characterizes the agreed upon approaches identified by experts participating in the study.

Table 2

Approaches to gathering response process validity evidence

Approach	Definition
Cognitive Interview	Explores a participant's comprehension of an item or task (Leighton, 2017).
Error related to response patters/CTT	Classical Test Theory (CTT) predicts the true score of a test-taker by accounting for and calculating an error score (Novick, 1966). Standard error, as seen in CTT, may highlight ideas about potential noise in the data that could in turn, highlight respondents' performance or engagement.
Eye tracking and/or physiological data	Physiological data involves the empirical observation of variables pertaining to the functioning of systems in the human body. Eye tracking data are collected by measuring the movements of the eye, including fixation and duration. It is inferred that eye movement is a proxy for attention and in turn, evidence of participants' responses. These data provide developers with evidence of comprehension processes that take place while reading (Paulson & Henry, 2002).
fMRI (Functional Magnetic Resonance Image)	Neuroimaging that measures brain activity by detecting changes in blood oxygenation and flow. fMRI data could be used to measure changes in brain activity in response to an assessment item (Byars et al., 2002).
Focus groups	Interview technique of collecting data by orchestrating discourse with a small group of individuals. Participants of the focus group discuss feelings or thoughts regarding various elements of a test item. Item developers can use these data to examine the psychological processes of test-takers (Padilla & Benitez, 2014).
Generalizability-theory related evidence	Generalizability theory provides a conceptual and statistical framework to model multiple sources of error in assessment data. This enables assessment developers to quantify and address inconsistencies in observed scores that could develop over replications of the assessment. G-theory evidence related to response processes includes variance in how test-takers respond to items, and rater-error (Lane, 2019; Brennan, 2001).
Log data	Provides a record of actions taken while working through an item or task (Stadler et al., 2020).
Predicted response patterns/processes based on Learning Trajectories	Learning Trajectories refers to the progression of student thinking during the learning of mathematics. This coincides with progress levels that can be used to make generalizations of student thinking (Confrey et al., 2019).

Rater agreement	The consistency of scores assigned by two or more independent raters of the same performance. Evaluation of rater reliability is needed to promote valid score interpretations and uses. Construct-irrelevant variance may occur as a result of poor rater agreement/reliability. Sample quantitative forms of rater agreement/reliability include using ICC, rwg, Kappa, and percent agreement (Lane, 2019).
Rater training and calibration	Developing rater agreement by establishing consistency among raters. Construct-relevant variance is affected by aspects of rater training such as training materials, training procedures, and rater calibration (Lane, 2019).
Sorting tasks	The process of grouping a set of items into categories based on meaning. Sorting tasks can be used to elicit the knowledge structure of a respondent (Tang and Clariana, 2017).
Response times	Measure of time required to complete a task provide evidence of how respondents engage with the item/set of items in desired or undesired ways. Typically, this is in regards to response times focus on the relationship between response time and cognitive demand of the item (Padilla & Benitez, 2014).
Think alouds	Interview procedure used to measure problem-solving processes by asking the participant to articulate their thoughts in response to an item or task. Think alouds provide item developers the opportunity to examine the cognitive processes of test-takers (Leighton, 2017; Padilla & Benitez, 2014).
Written work	Any written work documented by the test-taker that can be used to identify solution strategies and provide evidence of their thinking. This includes "scratch-work" as well as written justifications (Watson, 1995).

Discussion

An aim of the broader study was to define and characterize approaches to data gathering related within validation scholarship, with an intentional focus on response processes. A goal was to provide researchers with approaches for gathering validity evidence, which are grounded in both expert opinion and prior literature. These findings may also be of interest to the mathematics education community at large. For instance, educators may conduct think-alouds to collect validity evidence for tests used in their classrooms. Validation scholarship may be better influenced and more easily accessed by a mathematics education audience with it located in one manuscript, as done here. One aspect that is clear regarding response process work is that much of the data are collected and analyzed qualitatively. Effective assessment development requires thoughtful teamwork between individuals who can engage in qualitative and quantitative methods. Teams are encouraged to draw upon different knowledge – whether it includes a team of mathematics educators with various training or a team inclusive of mathematics educators, methodologists, and others. This is an opportunity for scholars to work together and draw upon their methodological assets rather than assert one methodological approach as dominant over another. An intended outcome of this work was to increase conversations around validation, which can have broad implications for those quantitative mathematics education research.

Different Sources

Validity has become a developing topic within quantitative research published in mathematics education journals. Limited validity evidence has been published until quite recently; however, this may be remedied with more focused attention and more robust understanding by scholars. Thus, this manuscript has potential to inform researchers about validity and approaches for their research. It is keenly evident from this list for response process validity evidence that these approaches are largely qualitative in nature. Gathering response processes evidence is a practice that can bring together scholars from diverse methodological training and expertise around a shared goal.

Strategies

Throughout V-M^{2}Ed work, as well offshoots from it, there have been multiple researchers who have conducted assessment development (Kosko, 2019), validation work (Krupa et al., 2019; Pellegrino et al., 2016), and assessment reviews using a validation lens (Bostic et al., 2021). One notion that remains is that the results and their interpretations are keenly grounded in the quality and quantity of validity evidence (Carney et al., 2022; Folger et al., 2022). Greater forms of evidence across more validity sources leads to stronger claims that can be made from results and interpretations (AERA et al., 2014). For example, a standardized test that has high-stakes implications should have strong validity evidence across five sources. On the other hand, an in-class mid-semester survey of students' perceptions of a class has low-stakes implications and may only require one form of validity evidence. It is important for users to carefully gauge the implications of their assessment during selection or instrument design. Carney et al. (2022) provided two examples of instrument and use summaries, which help to convey to readers what an instrument does, and the sorts of validity evidence gathered to support its claims. This proceeding may help others reflect on the assessments they use and consider the degree to which their results and interpretations are grounded in claims and validity evidence.

Final Thoughts

This manuscript is intended to be educative; fostering the surge of validation scholarship and addressing Ziebarth et al.'s (2014) gap in validation scholarship within mathematics education contexts. It aims to foster collaboration across a team (e.g., mathematics educators, qualitative researchers, and psychometricians) and promote synergy around a shared vision. Validation work is qualitative and quantitative; it requires deft knowledge of content and analytical approaches. By providing researchers with ideas about how to gather validity evidence, then it is more evident how researchers with different strengths can leverage their strengths. Put simply, validation work is an opportunity for qualitative and quantitative researchers to work together and this can happen with knowing what approaches can be applied.

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EFFECTS OF USING THE CLASS-BASED CRITERIA FOR PROOF

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In this paper we investigated how using the communal proof-writing criteria over a semester-long course can affect their proof perceptions. Through the use of the class-based criteria, the results from this study suggest that the majority of students develop a better understanding of what makes an argument a proof and what the essential elements are needed for a proof. Furthermore, the results from this study suggest that the use of communal rubrics for proof help students better develop their communication skills. Implications for future research and teaching and learning proof by using the class-based crier for proof are also discussed.

Introduction

Increased attention has been given to proof in K-16 mathematics curriculum because it plays vital roles in mathematics, including verification, explanation, communication, discovery, and systemization (e.g., de Villiers, 1990; Knuth, 2002). Additionally, proof serves as a means to deepen learners' understanding of mathematics and enhance learners' skills in communicating mathematical ideas (Stylianides, 2007). At the undergraduate level, in particular, mathematics majors, including pre-service secondary mathematics teachers, are expected to be able to construct, evaluate, and validate proofs in proof-intensive courses (Committee on the Undergraduate Program in Mathematics, 2015). However, research has shown that undergraduate students have considerable difficulties with proof (e.g., Bleiler, Thompson, & Krajčevski, 2014; Ko & Knuth, 2009, 2013; Selden & Selden, 2003). Holding the *authoritarian proof scheme* is one challenge for undergraduate students to recognize what makes an argument a proof (e.g., Harel & Sowder, 1998). Undergraduate students typically observe their proof instructors presenting a completed and polished argument, commonly referred to as the *authoritative* perspective. More recent research promotes a *communal* approach to teaching and learning proofs, where undergraduate and graduate students, including pre-service secondary mathematics teachers, are actively and collaboratively involved in negotiating and making sense of arguments (Bleiler, Ko, Yee, & Boyle, 2015; Boyle, Bleiler, Yee, & Ko, 2015; Ko, Yee, Bleiler-Baxter, & Boyle, 2016; Stylianides & Stylianides, 2009; Yee, Boyle, Ko, & Bleiler-Baxter, 2018). Moreover, a communal method of proof instruction is defined as individuals actively and collaboratively involved in developing class-based criteria for proof through constructing, evaluating, negotiating, and making sense of arguments (Bleiler et al., 2015; Boyle et al., 2015; Ko et al., 2016; Stylianides & Stylianides, 2009; Yee et al., 2018).

While Bleiler et al.'s (2015), Boyle et al.'s (2015), Ko et al.'s (2016), Stylianides and Stylianides' (2009), and Yee et al.'s (2018) studies show a successful implementation of a communal approach to teaching and learning proof, their work focused on pre-service elementary or secondary mathematics teachers' understanding of what counts as a proof and their ability to construct and evaluate arguments. Yet, to date, we do not know how engaging in creating the class-based criteria for proof under a communal teaching approach affects undergraduate students' proof perception over the semester. To address this research gap, this study investigates how undergraduate students perceived creating the class-based criteria has impacted on their perceptions of proof.

Theoretical Perspective

In the mathematics community, mathematicians communicate their thoughts and negotiate their productions before their arguments are counted as proof. Building on communal aspects of proof, Stylianides (2007) defined proof as follows:

Proof is a mathematical argument, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:

1. It uses statements accepted by the classroom community (set of accepted statements) that are true and available without further justification;

2. It employs forms of reasoning (modes of argumentation) that are valid and known to or within the conceptual reach of, the classroom community; and
3. It is communicated with forms of expression (modes of argument representation) that are appropriate and known to, or within the conceptual reach of, the classroom community. (p. 291)

Stylianides' (2007) definition of proof incorporates a focus on the set of statements (i.e., definitions, theorems), the appropriate forms of argumentations (e.g., direct proof, proof by cases), and representations accepted (e.g., pictorial or diagrammatical formats) and understood within a particular mathematical community, which shows the general case will be always true without exception. In addition, proof "involves the subjective negotiation of not only the concepts concerned, but implicitly also of the criteria for an acceptable argument" (de Villiers, p. 22, 1990). Taken together, we build on Stylianides' and de Villiers's (1990) work which demonstrate proof as a product of a social communication to design a proof course using a communal teaching method.

Figure 1

Directions for Undergraduate Students' Second Part of the Final Take-Home Exam

Math 320: Final Take-Home Exam Part 2-Due on Thursday, December 9, at 12:00PM (EST) via Blackboard

Note. This take-home exam is worth 10%.

The instructional approach in our class is most likely different from any mathematics classes you have taken in the past, since you are asked to read textbooks, watch videos, and complete course preparation assignments outside of our classroom. Also, you are actively engaged in creating our class-based criteria for a mathematical proof, as well as doing mathematics and presenting and explaining your solutions rather than being told how to solve problems in class. The purpose of this assignment is for you to reflect on our class-developed criteria for a mathematical proof, as well as your experiences evaluating mathematical arguments produced by yourself and your peers. Please limit your typed reflection paper to 3-5 double-spaced pages, while also making sure to fully answer the questions provided below (to use as a guide while writing your reflection).

- Suppose one of your friends (a non-math major) were to ask you, "What is mathematics?" How would you respond?
- Suppose this friend was really curious, and they wanted to know more about your study of mathematics. They have heard about "proof" in mathematics, so they ask you, "What is a mathematical proof?" How would you respond?
- How has creating our class-based criteria for a mathematical proof impacted your understanding of what counts as a proof?
- How has using our class-based criteria for a good mathematical explanation to evaluate your and your peers' mathematical work increased your writing abilities in mathematics?
- How has using our class-based criteria for a mathematical proof to evaluate your and your peers' mathematical arguments increased your abilities in writing and evaluating mathematical proofs?
- How has evaluating your own constructions and your peers' arguments increased your ability to critique the reasoning of others?

Methods

Twenty undergraduate students who enrolled in a Discrete Mathematics course—a transition-to-proof course—at a Midwest University in the United States participated in this study. A transition-to-proof course in the United States is designed to help undergraduate students progress from non-proof intensive courses, such as Calculus I, II, and III, to proof-focused courses, such as Abstract Algebra and Real Analysis. Regarding the participants' backgrounds, three were middle school mathematics teaching majors, six were mathematics teaching majors, seven were mathematics majors, and the remaining were mathematics minors. The course met two times a week for 75 minutes over the 15-week long semester and covered topics such as propositional and predicate logic, different proof methods (e.g., direct proofs, proof by cases, proof by contrapositive, proof by contradiction, and proof by mathematical induction), sets, relations, functions, and combinatorial methods. In designing the course, the instructor (the first author) drew upon the existing proof studies with a focus on a communal method of proof instruction aimed at providing opportunities for her students to engage with proof as a social, negotiated, and sense-making process (Bleiler et al., 2015; Boyle et al., 2015; Ko et al., 2016; Stylianides & Stylianides, 2009; Yee et al., 2018).

The primary source of data for this paper was the undergraduate students' written responses with a focus on how using the class-based criteria for proof affected their understanding of what counts as proof. This question (Question 3) was included in the second part of the final take-home exam, and Figure 1 shows detailed directions provided to the undergraduate students. Because two undergraduate students (one is a mathematics teaching major, and the other is a mathematics major) chose to take an in-person final exam, both authors individually read 18 undergraduate students' written response to Question 3 first. Then the first author followed the principles and techniques of grounded theory (Glaser & Strauss, 1967; Strauss & Corbin; 1990) to develop coding categories, and the second author sorted undergraduate students' responses individually. In addition, all the participants' names used in this paper are pseudonyms.

Results and Discussion

Table 1 displays the frequency of which undergraduate students' responses were placed in terms of perceptions of proof. As seen in the table, the three most referenced coding categories are Understand what is makes an argument to be counted as proof, Discuss the essential elements needed for an argument to be counted as proof, and Develop communication skills. From creating class-based criteria, all undergraduates in the course noted that they understood what makes an argument a proof (18 cases, 100% of students). In responding to how creating the communal criteria enhanced her understanding of proof, Rachel stated, "I believe this has impacted my understanding of what counts as a proof by setting the standards for our proofs." In a similar vein, Allan explained, "I understand that there is a difference in what truly counts as a proof." Creating the class-based criteria showed Caroline what she needed to do for her arguments to count as proof, because "[she] knew what was expected of [her] at the beginning of the semester (before even becoming too familiar with proofs themselves). [Also], it was easy to apply [the] qualities to the proofs." Another student, Ava, responded that the class-based criteria helped he understand the requirements for an argument to be qualified as a proof. Whether the participants thought of the community-agreed rubrics in regard to understanding the essential elements or what was expected, the criteria seemed to help them generate a better understanding of proof overall. These results are consistent with Boyle et al.'s (2015) findings that developing the communal criteria lead students to develop a better understanding of what makes an argument count as proof.

Table 1

Coding Categories for Impact on Perceptions of Proof Identified by Undergraduate Students

Coding Category	Number of Students
1. Understand what makes an argument to be counted as proof	18 (100.0%)
2. Develop communication skills	9 (50.0%)
3. Make a proof understandable by people in our mathematics classroom community	8 (44%)
4. Understand a proof that provides certainty for all cases	1 (6%)
5. Discuss the essential elements needed for an argument to be counted as proof	15 (83%)
6. Expand thinking beyond a specific format that a proof needs to follow	6 (33%)
7. Take ownership for learning	1 (6%)

To create the class-based criteria, there was extensive discussion on the essential elements needed for an argument to count as a proof. According to Table 1, Discussing the essential elements for an argument to be counted as a proof was the next highly identified category by the students with 15 cases (83% of students). Throughout discussing and negotiating what makes an argument a proof, the agreed-upon criteria shared a common understanding of the important characteristics for a proof with a classroom community as opposed to merely satisfying an instructor's opinions. For example, Jenny explained,

> Within this class setting came other ideas, like developing our own class criteria for writing a mathematical proof, I had never done something like that in a class, normally the teacher is responsible for that. I was completely taken back that we as a class were developing this rather than it being given to us.

Another student, David, stated that "[their] groups had some matching core ideas, but also had a few others of their own that weren't the same." The smaller group discussions generated a more diverse class discussion, which showed various perceptions on what an argument needs to be counted as proof. The full class discussion brought the main ideas to light, leading to the initial class-based criteria.

The discussion of criteria was not only at the beginning of the semester, however. "Through a short process of a class discussion, we changed [our] class-developed criteria," Shella recalled. The evaluation of the students' and their classmates' work prompted the updated criteria. One student, Isabella, stated, "The class-based criteria combined with the evaluation of peer and personal work placed a much heavier emphasis on the ability of mathematical work to be communicated." It is important to note that Isabella references how evaluating proofs using the class-based criteria improved her communication skills. As seen previously, half of the 18 students identified that creating the agreed-upon rubric improved their mathematical communication skills. For instance, Abigail "gained insight on how to adequately represent my mathematical data when generating a proof" through the creation of class-based criteria. According to Rob, "Being a part of deciding what would qualify a proof as a good proof helped me understand why it is important to be clear and concise and make a proof understandable for everyone." He further pointed out that creating the criteria made him recognize the importance of communicating his work to more than his classmates. In fact, students in this class were not given any explicit instruction on de Villiers' (1990) five roles of proof in mathematics. Therefore, it is encouraging to note that the creation of class-based criteria improved students' communication skills through constructing mathematical proofs.

As seen previously in Table 1, the least two frequently coded categories were Taking ownership for learning and Understanding a proof provides certainty for all cases. For instance, Jenny noted that creating the communal rubrics for proof helped students "take ownership for their learning and define their own standards." In similar fashion, Shella was the only student who indicated creating the class-based criteria made her understand a proof provides certainty for all cases. Some of the students referenced the categories they came up with as a class, but Shella went into further detail stating you must "show that the proof will work no matter what the case is." Although having a correct conclusion was one of the class-based criteria, Shella was the only student who indicated that creating the criteria helped her understand this rubric. Perhaps most of the students were confident in the verification role of proof from previous courses, as it is a typical role that plays in the teaching and learning of proof (de Villiers, 1990; Knuth, 2002). It could also be possible that the participants thought the class-based criteria was more impactful in other roles of proof, such as explanation, communication, discovery, and systemization (Ko, Johnson, & Rose, Under Review).

Conclusion

In summary, this study shows that the communal teaching method for proof in mathematics has an impact on students' learning and understanding, which is consistent with Bleiler et al.'s (2015), Boyle et al.'s (2015), and Ko et al.'s (2016) findings. Moreover, creating the class-based criteria greatly improved students' understanding of what makes an argument count as proof. This is an important result as available research has demonstrated that undergraduate students have considerable difficulty determining what constitutes proof (e.g., Bleiler et al., 2014; Ko & Knuth, 2013; Selden & Selden, 2003). Continued explorations creating the class-based criteria proof with respect to teaching and learning proof over a semester-long or a year-long course will help us better understand how to support undergraduate students in comprehending, constructing, evaluating, and validating proofs.

With emphasis on creating and using the class-based criteria, the undergraduate students in this study also noticed development in their communication skills. Further research needs to investigate how the use of communal rubrics for proof has impact on students' understanding of other roles of proof suggested by de Villiers (1990), including verification, explanation, discovery, and systemization. Finally, one participant, Jenny, noticed that the creation of class-based rubrics made students "engage in higher order thinking such as critical thinking, analysis, and evaluation." This is one of the main reasons for using a communal method of proof instruction aimed at supporting students to learn proof in a meaningful way.

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Celebration: Teachers & Pedagogy

EXPLORING TEACHER KNOWLEDGE AND NOTICING WITH EYE TRACKING AND 360 VIDEO

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Professional noticing involves attending to and interpreting children's mathematical reasoning. In a similar manner, pedagogical content knowledge (PCK) is defined as the knowledge teachers need to interpret and respond to children's reasoning. The present study reports on an initial exploration of the relationship between these two constructs using eye-tracking technology and the PCK-Fractions measure. Results suggest a relationship between where teachers spend time focusing and their PCK scores.

Introduction

Professional noticing is an important skillset that involves attending to students' mathematical actions, interpreting their reasoning from those actions, and deciding how to respond next (Jacobs et al., 2010). Many examine a myriad of factors affecting how teachers attend to and interpret students' reasoning through teachers' professional beliefs, knowledge, and experiences (Jong et al., 2021; Yang et al., 2021b). Scheiner (2021) argued that beyond being influenced by specifically cognitive factors (such as teacher knowledge), noticing is "embodied, cultural and positional in important ways" (p. 90). Rather, it is the interplay between cognitive, cultural, and embodied experience (i.e., experiences mediated by physiological input) that may best explain the complexities inherent in teachers' professional noticing (Jong et al., 2021; Kosko et al., 2022).

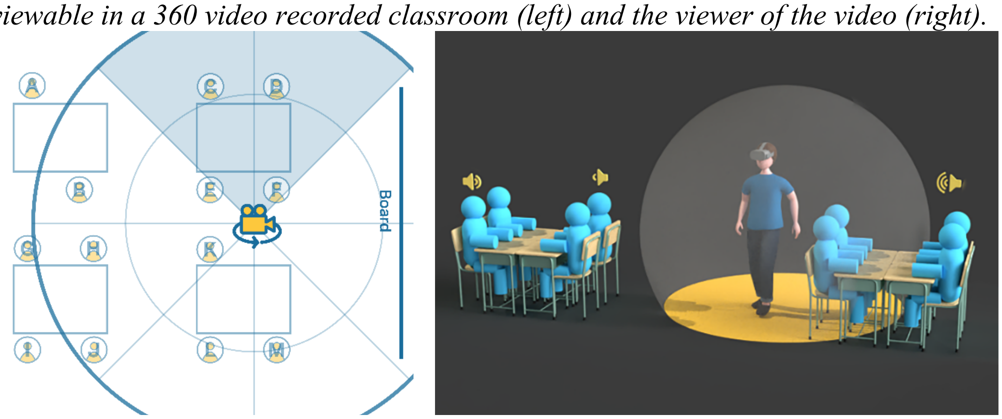
The present study seeks to bridge the gap between embodied and cognitive domains to better understand the nature of teachers' attending to and interpreting of children's mathematics. Specifically, we follow scholarship suggesting that teachers' change in eye-gaze (Huang et al., 2021) and field of view (Kosko et al., 2022) are associated with how teachers interpret students' mathematics. Such scholarship suggests *that* noticing is embodied, and there is mounting evidence for this connection. Yet, there is less description of *why* or *how* such noticing is embodied. Thus, the present study reports on analysis of preservice teachers' (PSTs) pedagogical content knowledge for fractions and how PSTs' embodied actions when viewing a 360 video are associated with such professional knowledge.

Theoretical Framework

Teacher Noticing

Professional noticing involves attending to key pedagogical events, interpreting these events, and deciding how to act based on such interpretations (Jacobs et al., 2010; Santagata et al., 2021). PSTs may initially attend to only generic aspects of a classroom recording such as the teacher's classroom management or students' behavior and engagement (Jacobs et al., 2010). Over time, they can transition to focusing on students' answers and then their procedures. Eventually, a teacher may improve the quality of their noticing such that it focuses on students conceptual reasoning. Quite often, such variations in quality of noticing are associated with the years of experience a teacher has (Yang et al., 2021). However, other factors can affect teachers' noticing. Although there are a multitude of factors that may explain a teachers' noticing (Scheiner, 2021), the present study focuses on two such factors for sake of simplicity: teachers' professional knowledge and physical (embodied) actions.

Figure 1
What is viewable in a 360 video recorded classroom (left) and the viewer of the video (right).



Evidence of the embodied nature of noticing stems from two areas of scholarship: eye-tracking research and use of virtual reality (VR) including 360 video. Scholars using eye-tracking technology tend to indicate that experienced teachers look at more students and have a more even gaze distribution (Huang et al., 2021; van den Bogert et al., 2014). Rather, experienced teachers "exhibit fewer task-irrelevant fixations with shorter durations, they also direct more fixations to student" (Huang et al., 2021a, p. 11) and across more students. Research utilizing 360 video has found corollary results. Specifically, 360 video records omnidirectionally and allows the viewer to choose where they may look from a fixed point in the classroom (see Figure 1). Studying PSTs' noticing with 360 video, Kosko et al. (2021) recorded their viewing sessions and compared their written interpretations of students' mathematics with which tables (i.e., groups of students) PSTs focused in their field of view (FOV). Findings indicated a relationship between how much time PSTs spent focusing on particular tables and the sophistication of their noticing. Later, Kosko et al. (2022) observed that PSTs who positioned students more centrally in their FOV were more likely to describe their conceptual reasoning than PSTs who placed the classroom teacher more centrally in their FOV.

There is significant scholarship seeking to examine the relationship between professional knowledge, such as mathematical knowledge for teaching (MKT), and noticing. Unfortunately, scholarship relating MKT and noticing often produces mixed results indicating MKT either has a negative, a positive, or no relationship with quality of professional noticing (Jong et al., 2021; Yang et al., 2021). One possibility for such mixed results is that other factors may mediate how teachers operationalize their professional knowledge in the act of noticing. Jong et al. (2021) suggest that focusing on subdomains of MKT to examine professional noticing may illuminate otherwise "hidden" (p. 162) relationships. Likewise, Scheiner (2021) suggests a similar approach. Given the mixed findings regarding the relationship between MKT and professional noticing, the present study follows such recommendations by focusing on PCK, but also by examining the role of PCK in noticing via embodied activity of PSTs when viewing 360 video.

Pedagogical Content Knowledge

PCK involves teachers' knowledge of classroom instruction and students' reasoning. As part of the MKT framework, Hill et al. (2008) specified different subdomains for PCK. Most relevant to the current paper is *knowledge of content and student* (KCS), which focuses on knowledge of students' mathematical thinking and their errors. Although many scholars examining MKT focus on both CK and PCK, there is often specific attention on KCS when studying PCK particularly (Tröbst et al.,2018; Zolfaghari et al., 2021). In constructing their initial construct map for PCK for fractions, Zolfaghari et al. (2021, in review) defined four hierarchical levels for teachers' understanding of students' fractions reasoning. At Level 1, teachers are able to assess whether children can partition a whole into a given part, but are not able to assess children's part-whole reasoning until Level 2. At Level 3, teachers can assess children's relational thinking of how to coordinate multiple fractions occurs at this level (e.g., $\frac{2}{3}+\frac{5}{8}$). Lastly, at Level 4, teachers can assess children's knowledge of fractions of fractions (i.e., what is $\frac{1}{3}$ of $\frac{2}{5}$). Attending to children's strategies and noticing their misconceptions is considered part of the professional noticing construct (Jacobs et al., 2010). There is emerging evidence that teachers at lower levels of PCK-Fractions attend less to students' actions when they work with such fractions (Kosko, 2022). The present study seeks to explore this relationship in more detail using the PCK-Fractions measure (Zolfaghari et al., 2021).

Summary and Context of Current Study

The relationship between professional noticing and PCK is both intuitively logical and advocated by the field. However, efforts to establish empirical evidence for this relationship have produced mixed results with some scholars finding positive associations, negative associations, or no observable relationship (Jong et al., 2021; Yang et al., 2021). By contrast, there is growing evidence for the role of embodied activity in professional noticing. More sophisticated noticing is associated with more focused attention (longer sustained durations) on students (Huang et al., 2021; Kosko et al., 2021). This study is exploratory and seeks to examine whether there is any relationship between PSTs' embodied actions, as measured by eye-tracking activity in viewing a 360 video, and their PCK for fractions. Thus, we sought to answer the research question: *Is there a relationship between PSTs' assessed PCK and their duration of eye-gaze behavior?*

Methods

Sample & Procedure

Participants included a convenience sample of 33 PSTs. Participants predominately identified as White and female (76.5%), with other participants including one Black female, five White males, one Hispanic female, and one Hispanic male. Participants were evenly divided between those preparing to teach upper elementary mathematics (51.5%) and those majoring in other educational disciplines (48.5%). Following recruitment, participants engaged in a 45-minute session where they first completed a demographic survey and the PCK-Fractions assessment. Validity evidence for PCK-Fractions has been collected across multiple studies (i.e., Zolfaghari et al., 2021; in review). The measure includes 17 multiple-choice items that assesses a teachers PCK for teaching and learning fractions. Dichotomously scored items are logistically converted to a continuous variables using Rasch modeling (M = 0.17, SD = 0.84).

Next, participating PSTs were provided an overview description of the 360 video they were about to watch using an eye-tracking enabled VR headset (Pico Neo 3 Eye). They were told that they would watch the video to assess children's mathematical thinking:

In this episode from Ms. M's fourth-grade class, several weeks have passed since students first learned about equivalent fractions. In this clip, Ms. M reviews equivalent fractions with students by having them use fraction strips to find an equivalent fraction to 5/6 and then 3/8.

PSTs were provided with a set of fraction strips to explore the topic. Next, PSTs put on the VR headset, calibrated the eye-tracking sensors to their eyes, and watched the 360 video of Ms. M's class. Afterwards, participants wrote the key concept they believed students were learning about, and then to "describe 2-3 moments that showed a child's thinking about the key idea." Eye-tracking data from viewing sessions were collected and analyzed with a machine learning algorithm developed to identify pupil fixations on specific individuals within the 360 video.

Analysis & Results

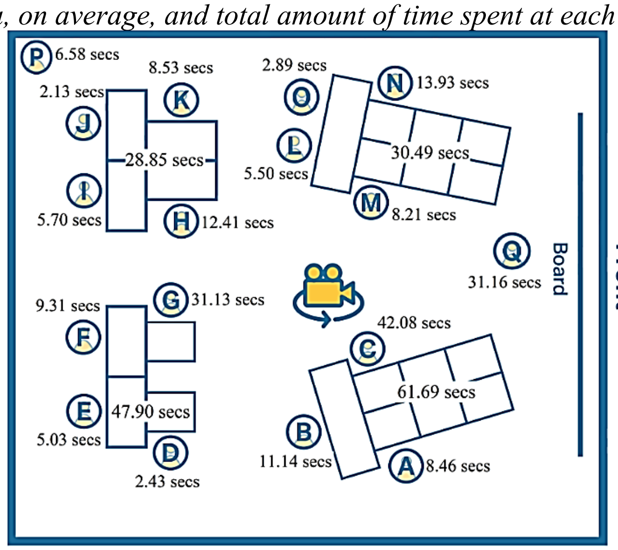
Using raw eye-tracking gaze data, summary statistics were computed by summing the number of seconds each PST looked at a specific student or teacher. For example, if the gaze data reported that a PST looked at student A twelve times, the sum of those 12 occurrences was taken. The sums per student were then taken collectively to report the number of seconds spent looking at a specific table (e.g., back left table) and the total amount of time looking at the two teachers in the room. Figure 2 displays the average amount of time looking at each student, teacher, and the sum of the seconds at each table. From the classroom map (Figure 2) PSTs, on average, spent more time looking at the right front and right back tables and students G, C, and teacher Q. Thus, for each participant there were five reported variables (back-left table, front-left table, back-right table, front-left table, teachers).

When assessing the relationship between a PSTs PCK-Fractions score and where they attended, results suggest correlations between a PSTs PCK and the back right table ($r(32) = 0.17$, $p = 0.349$) and the front left table ($r(34) = 0.16$, $p = 0.376$) were not statistically meaningful.

However, analysis revealed a statistically significant positive correlation between PSTs' PCK score and the amount of time looking at the back left table ($r(32) = 0.38$, $p = 0.029$). In comparison, there is a statistically significant negative correlation at $\alpha = 0.1$ level between a PST's PCK score and the front right table ($r(32) = -0.32$, $p = 0.074$). This insinuates that a PST with a low PCK score is more likely to look at the table by the camera while a PST with a higher PCK score is likely to turn around and look behind them.

Figure 2

Breakdown of gaze data, on average, and total amount of time spent at each table.



Note. P and Q represent the two teachers in the room.

Discussion

Results are preliminary but suggest a correlation between PSTs' PCK and the amount of time they spent focusing on different groups of students in the classroom. PSTs with higher PCK scores tended to spend more time focusing on the back-left table in the classroom. Important in interpreting this result is that halfway through the video, the teacher has a brief class discussion regarding equivalent fractions to $\frac{5}{6}$. At one point (2:26 to 2:47), students at the back-left table comment that they initially thought to simplify the fraction by dividing but didn't because "5 is prime." PSTs with PCK scores above 1.00 spent an average of 6.29 s in the 21 s interval gazing at this table, whereas PSTs with scores below 0.00 spent an average of 1.69 s. Notably, prior research with eye-tracking data suggests that more experienced teachers spend more time looking at students further away while novices focus only on those proximally close to them (Huang et al., 2021). Another factor is that higher PCK for fractions is associated with teachers' ability to assess children's arithmetic actions on fractions (Kosko, 2022). Given that students at the back-left table were describing their use of multiplication to find equivalent fractions, less time looking at such students may associate with a lack of assessing these students' mathematics.

Results here suggest that teachers' PCK may play a role not only in noticing students proximally further away, but also which such students are attended and when. In this particular case, PSTs with higher PCK scores focused on students describing their use of multiplication to find equivalent fractions, whereas students with lower PCK scores spent significantly less time doing so. This paper presents preliminary results suggesting a relationship between higher PCK-Fractions scores and more time attending to students who described their strategies for finding an equivalent fraction. Future study is needed to extend and elaborate on these findings. However, results suggest an important interaction between professional knowledge and noticing.

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ASSESSING FUTURE TEACHERS' MATHEMATICAL WRITING: AN ASSESSMENT TOOL AND INSIGHTS

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Mathematical writing (MW) plays an important part in students' mathematical learning and is one way students are expected to communicate their thinking to others. However, for teachers to effectively support students in this work, they must have an understanding of and be able to generate high quality MW responses themselves. Yet, little is known about future teachers' (FT) MW competencies and ways to assess FTs' MW. In this study, we used a K12 MW assessment tool in undergraduate elementary mathematics methods courses to understand the utility of the tool and FTs' MW competencies.

Mathematical writing (MW) plays a critical role in mathematical learning because it promotes reflection and clarification of ideas via explanations, descriptions, definitions, and critiques and can promote students' mathematical identities (Boaler, 2002; Freeman et al., 2016; Ivanič, 1998; Marks & Mousley, 1990; National Council of Teachers of Mathematics, 2000). Yet, writing is often overlooked in mathematics classrooms and teacher preparation programs (Powell et al., 2021). If teachers are to effectively foster and scaffold students' MW, teachers must have an understanding of and be able to generate high quality MW responses themselves. However, little is known about future teachers' (FTs') MW competencies and ways to assess FTs' MW.

Mathematical Writing

MW is "a writing activity in which students write about mathematics concepts or procedures" (Powell et al., 2021, p. 418) that can vary based on purpose, formality, audience, structure, and required language (Chval et al., 2021). Within MW, multiple genres exist (e.g., explanatory, argumentative; Casa et al., 2016), however, explanatory writing is the primary focus of teachers, curriculum, and state assessments (Casa et al., 2019; Gillespie et al., 2014). Explanatory writing occurs when writers “reason about concepts” (Casa et al., 2016, p. 9) as they explain (e.g., strategies, connections) or describe (e.g., observations, representations). Given the prominence of explanatory writing in teachers’ work with students, it is important to understand the extent of FTs’ explanatory writing competencies in an effort to better prepare FTs to support students’ explanatory MW.

Future Teachers’ Mathematical Writing

Scholars (e.g., Magiera & Zambak, 2020) have called for more research focused on FTs’ MW that goes beyond proof-related work. In the small body of work that focuses on FTs’ MW outside of proof, much of this examines how FTs’ reflective writing impacts metacognition, beliefs about mathematics, and pedagogical content knowledge (Kilic & Dogan, 2020; Kuzle, 2013; Namakshi et al., 2022). Findings from the few studies that have examined FTs’ explanatory MW indicate FTs’ conceptual understanding is strongly related to their own MW and their ability to critique students’ MW (Magiera & Zambak, 2020). The majority of research that examined FTs’ MW in general has focused upon FTs’ mathematical content knowledge with little examination of their MW mechanics or organization.

Assessing Future Teachers’ Mathematical Writing

We are aware of no broadly available assessment tools designed to evaluate FTs’ MW competencies. As a result, scholars have developed their own scoring frameworks for their respective studies (e.g., Magiera & Zambak, 2020). Although these assessment tools have served their purposes in the context of a specific study, they have not had their technical adequacy (e.g., reliability and validity) examined nor been applied beyond the context of a specific study. Furthermore, these assessment tools often omit writing and writing mechanics assessment (e.g., organization, grammar), which can impact the perceived quality of mathematical understanding and inform future instruction.

Within the context of K12 mathematics classrooms, one group of scholars (i.e., Namkung et al., 2020) have sought to identify MW assessment tools that evaluate both mathematics and writing mechanics as well as demonstrate technical adequacy. Namkung et al. examined four potential assessment tools (i.e., holistic, analytic rubric, elements scoring, and MW sequences) for upper-elementary students. The authors determined that all four scoring methods were moderately correlated with criterion measures of mathematics and writing while confirmatory factor analyses indicated satisfactory construct validity. Furthermore, all tools demonstrated adequate reliability with the exception of increased variability for the grammar dimension of the analytic strategy. Although Namkung et al. recommended use of a holistic scoring approach, holistic scoring has minimal instructional utility as it does not indicate specific areas of need, has little use as formative assessment to measure growth across time, and is primarily useful only for summative evaluations. For formative assessment purposes and instructional utility, analytic scoring strategies may be the most beneficial. The analytic rubric employed by Namkung et al. demonstrated significant correlations with criterion measures of math fluency ($rs = .33$), word problem solving ($rs = .66$), and essay composition ($rs = .36$). The rubric was also scored efficiently with acceptable inter-rater reliability (IRR) and provided targeted information that could identify student strengths/needs in either mathematics content, writing, or both.

This study fills the aforementioned research gaps by (1) providing an assessment tool for MW and (2) shares insights into FTs' MW competencies.

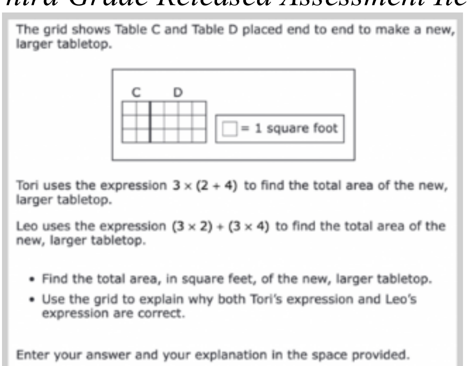
Methods

This study is part of a larger project examining the impact of a MW module on FTs' MW and their ability to assess elementary students' MW. Data was collected from 119 FTs at three U.S. universities. All FTs in this study were junior or senior undergraduates enrolled in a mathematics methods course that was designed to prepare elementary FTs seeking initial licensure in elementary education, special education, or elementary and special education.

Mathematical Writing Task

Data for this study was composed of FTs' responses to an elementary grade level task completed prior to engaging with an online module on MW. The task was a third-grade released assessment item from New Jersey (see Figure 1). The audience for these responses were the instructors of the FTs' methods course.

Figure 1
Third Grade Released Assessment Item Given to Future Teachers and Scoring Rubric


The grid shows Table C and Table D placed end to end to make a new, larger tabletop.

Tori uses the expression $3 \times (2 + 4)$ to find the total area of the new, larger tabletop.

Leo uses the expression $(3 \times 2) + (3 \times 4)$ to find the total area of the new, larger tabletop.

- Find the total area, in square feet, of the new, larger tabletop.
- Use the grid to explain why both Tori's expression and Leo's expression are correct.

Enter your answer and your explanation in the space provided.

Math Content	5: Complete Understanding of Math Content and Procedures	4: Adequate Understanding of Math Content and Procedures	3: Partial Understanding of Math Content and Procedures	2: Limited Understanding of Math Content and Procedures	1: No Understanding of Math Content and Procedures
Math Vocabulary	5: Accurate use of formal math vocabulary	4: Adequate use of formal math vocabulary	3: Use of formal math vocabulary	2: use of informal math vocabulary only	1: No use of (or incorrect use of) math vocabulary
Writing Organization	5: Clear progression of ideas, effective transitions, and fluent sentences	4: Adequate progression of ideas, effective transitions, and fluent sentences	3: Weak progression of ideas, ineffective transitions and connectors	2: Limited progression of ideas, limited use of transitions and connectors	1: No progression of ideas, limited use of transitions and connectors
Writing Grammar	5: Free of grammatical errors	4: 1 or 2 grammatical errors	3: Several errors, but they do not interfere with comprehension	2: Many errors that interfere with comprehension	1: Grammatical errors make comprehension impossible
Clarity and Precision	5: Conveys math concepts and procedures clearly and precisely	4: Conveys math concepts and procedures clearly but not precisely	3: Only some concepts and procedures are conveyed clearly and precisely	2: Most concepts and procedures conveyed are unclear and imprecise	1: Completely lacks clarity and precision

Analysis

FTs' MW was scored using an analytic rubric adapted from Namkung et al. (2020) that assesses MW across five dimensions (math content, math vocabulary, writing organization, writing grammar, clarity and precision) with a rating from 5 (ceiling) to 1 (floor) (see Figure 1). Clarity and precision were added to Namkung et al.'s rubric based on expert feedback. Two researchers with over 20 hours experience with the MW rubric independently scored all FT responses. Consensus

IRR and correlations between rubric dimensions are reported. FT mean, standard deviation, and frequency of rubric performance within each dimension was analyzed for patterns of strengths and weaknesses.

Findings

MW Assessment Properties

We report the consensus approach (percent of exact agreement) and consensus within 1-point (indicates scorers assessed in the same range) for IRR, where 70% or better is considered acceptable (Namkung et al., 2020). IRR for math content was .59 (.97 within 1-point), math vocabulary was .47 (.83 within 1-point), writing organization was .50 (.93 within 1-point), writing grammar was .54 (.93 within 1-point), and writing clarity and precision was .50 (.89 within 1-point). The exact agreement IRR was consistently lower than Namkung et al. (2020) and well below 70%, but IRR within 1-point approached 90%. Although IRR rates here are not suitable for high-stakes decision-making, the within 1-point IRR rates were similar to those found with K12 students and allow for broad interpretations for formative assessment purposes. The analytic rubric was not time intensive and could be used in a typical classroom.

Correlations between the rubric dimensions and text segments (words, values, and expressions) are shown in Table 1. In general, correlations between dimensions were moderately strong to strong and suggest the rubric assesses highly related but distinct dimensions of MW, which could be useful for differentiating instruction. Mathematics vocabulary was correlated more strongly with writing clarity and precision (.71) than mathematics content (.59), while mathematics content correlated the strongest with writing clarity and precision (.85). The low correlations for writing grammar indicate this dimension needs revision. Although grammar is important, and perhaps even more so for younger students, it may be best assessed as an "acceptable/not-acceptable" checkbox on the rubric rather than a scaled score. One potential revision is to replace the writing grammar dimension with text segments.

Table 1.
Correlation Table of Mathematics Writing Assessment Dimensions

Dimension	1	2	3	4	5	6	7
1 Mathematics content	*	.59	.80	.11	.85	.89	.68
2 Mathematics vocabulary	.59	*	.61	.24	.71	.83	.43
3 Writing organization	.80	.61	*	.16	.82	.88	.73
4 Grammar	.11	.24	.16	*	.17	.35	.06
5 Clarity and precision	.85	.71	.82	.17	*	.92	.65

FTs' Performance

Generally speaking, FTs scored similarly across dimensions. Mean scores (see Table 2) across all dimensions indicate FTs' have a partial understanding of the mathematics content embedded within the task (i.e., operations, number) and are not yet able to construct a written response that demonstrates mastery in mathematics *and* writing. Consequently, there was not a ceiling effect on the FTs' MW scores even though the task was at a third-grade level.

Table 2
Mean, Standard Deviation, and Frequency of Assessment Scores for Future Teachers' Mathematics Writing

Dimension	Mean (SD)	Frequency				
		1	2	3	4	5
Mathematics content	3.03 (1.09)	4	37	27	31	20
Mathematics vocabulary	2.66 (1.00)	13	29	44	28	5
Writing organization	2.92 (1.13)	14	19	40	30	16
Writing grammar	3.73 (0.83)	3	3	27	53	33
Clarity and precision	2.54 (1.03)	15	38	35	25	6

Note: To calculate frequency we rounded the average score to the nearest whole number.

When looking across the students who scored a 5 on content, their responses followed a typical structure of first identifying the total square feet of the table top (first prompt) followed by the FT's reasoning of why Tori's and Leo's expressions matched the situation. These responses often included an explicit reference to the two tables represented in the grid and discussed the equivalence of the two expressions, similar to the following

> The total area of the tabletops is 18 sq. ft. Both students have written a correct expression for the problem. Tori's expression focuses on the tabletops as a whole. She said the width (3) times the length (2+4). This expression equals 18 sq. ft. Leo's expression focuses on the tabletops as individuals. He has expressions for the area of each tabletop, and then he adds those expressions together. The answer is 18 sq. ft. If a person counts the squares on the inside of the tabletops, they would find the answer 18 sq. ft.

In contrast, FTs who scored a 1 or 1.5 on mathematics content typically did not answer both prompts embedded in the question and their response lacked specificity (e.g., "Both of their answers are correct because ultimately they would both end up with 6x3 which is 18"). This appears to be somewhat related to scores received on mathematics vocabulary, since responses that received a 1 on mathematics vocabulary were brief and included little, if any, mathematics vocabulary (e.g., square feet, length, width). Differently, responses that were scored a 5 were lengthier and included frequent use of mathematics vocabulary and symbolic notation, such as the example above. Consequently, it seems that FTs who are able to produce more text generally performed better than peers who wrote less.

There was an average of 46 text segments and a range of 1 to 288. Of all respondents, only five FTs provided a text segment of 3 or less (e.g., "18"). When FT's overall score was rounded to the nearest whole number, FTs' who scored a 4 or more wrote an average of 83 text segments whereas those who scored a 2 or below averaged 18 text segments.

FTs' performed lowest on the clarity and precision dimension, with the majority of FTs earning between a 2 or 3. Responses in this range were typically of average length, complete, and used mathematics vocabulary. However, what was different about these prompts compared to those receiving higher scores was a lack of precision, like

> The answer is 18. Tori used the height of the grid to come up with 3, and she added the length of table C and table D together because they make up one side of the bigger tabletop. The width (3) is then multiplied by the total of table C and D. Leo uses a similar strategy, but he multiplies table C and D by the width separately.

In this example, the FT lacked precision and clarity in their explanation by leaving off units (e.g., square feet) and did not connect their explanation with the expressions.

Discussion & Conclusion

In this study, we used a MW assessment tool previously identified for use in K12 school settings in the context of undergraduate elementary mathematics methods courses at three U.S. institutions. The analytic rubric used was efficient and demonstrated acceptable within 1 point IRR for formative assessment purposes, but the rubric needs revision for further research or high-stakes decision making. Yet, use of a common rubric in MW, or a close facsimile, can promote generalization of findings across research studies so that the field may develop a better understanding of what FTs and K12 students need to be successful with MW. Future studies may consider examining whether or not the rubric is responsive to student learning (i.e., sensitive to growth), what kinds of instructional decisions teachers make based upon student performance on the rubric, and if the rubric can be readily used by students to self-evaluate and revise their MW.

Findings from this study indicate that FTs' need additional instruction in their teacher preparation program on mathematical concepts and vocabulary—some of which they had already received in their teacher preparation program (e.g., operations and number)—along with clarity and precision in writing. Given prior research, we hypothesized FTs would likely perform better on the writing dimensions (at or near ceiling) than the mathematics dimensions and were somewhat surprised when this did not occur. Such findings highlight the need to simultaneously bolster FTs' writing competencies alongside mathematics. To support FTs in crafting MW responses, instruction should incorporate writing strategies, such as mnemonic strategies (e.g., PSOLVE, Hauk & Isom, 2009) and graphic organizers, alongside mathematics while leveraging FTs' literacy practices. Furthermore, the planning and organization components inherent in effective writing instruction (e.g., PSOLVE, Hauk & Isom, 2016) could also support students' problem-solving skills. Furthermore, instruction focused around MW should include explicit attention to identifying what academic language is embedded in task statements and why it is important to use academic language to clearly, coherently, and precisely communicate mathematical thinking to others.

Since MW is a form of mathematical discourse and represents one way students communicate their mathematical understandings, it is critical FTs can generate high-quality MW responses if they are to support students in this work. Moreover, given the prominence of MW on standardized high-stakes assessments, ensuring FTs are sufficiently prepared to support students is a matter of educational equity.

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THE DEVELOPMENT OF SECONDARY MATHEMATICS TPACK DURING THE FIRST YEAR OF COVID TEACHING

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At the onset of the COVID-19 pandemic, there was much speculation that the abrupt shift to online and hybrid learning modalities would serve as a catalyst for teaching and learning with technology. Now, evidence suggests that learning decreased during the COVID pandemic. This qualitative study uses the TPACK framework to investigate the development of technological knowledge of secondary mathematics teachers during the first year academic year after the onset of COVID-19. While teachers gained significant technological knowledge, little knowledge of technological content or technological pedagogical content knowledge was developed.

Introduction

Technology integration into secondary mathematics classrooms lags behind expectations (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010; Ertmer & Ottenbreit-Leftwich, 2010). Students report that they mainly use technology to write assignments, conduct traditional research online, or check grades, while teachers report using technology to complete administrative tasks or communicate with colleagues and parents. As online and hybrid forms of learning spread during the COVID-19 pandemic, some speculated these paradigms of learning would serve as the catalyst needed for more meaningful technological integration; yet, current studies have documented learning loss resulting from the pandemic. While much of the field is focused on accelerating mathematics learning post-pandemic, it is important to examine what *did* happen during the pandemic and why. To that end, this study investigated the extent to which mathematics teachers in one state developed new or deeper knowledge of technology integration.

TPACK Model

The effectiveness of technology integration ultimately depends upon the choice of the classroom teacher. Teachers make decisions about what technologies to use, who is able or required to use them, and how they are used (Drijvers, 2015). As such, teachers' attitudes, beliefs, and knowledge about technology, teaching, and learning influence the ways in which students use technology in their classrooms (Drijvers, 2015; Stoilescu, 2015). Thus, it is critical to investigate whether teachers of mathematics developed new or deeper understandings of how to integrate technology to support high-quality learning during the COVID-19 pandemic.

One framework for understanding the different types of knowledge teachers must develop to effectively integrate technology is the technological pedagogical content knowledge (TPACK) framework (Mishra & Koehler, 2006). The TPACK framework builds on the concept of pedagogical content knowledge introduced by Shulman (1986, 1987). TPACK (originally TPCK) stands for technological, pedagogical, and content knowledge (Getenet, 2017; Mishra & Koehler, 2006) and introduces a third domain, technology, to the knowledge basis needed for effective teaching. It therefore elevates technological knowledge as equal in importance to content and pedagogical knowledge, thereby acknowledging that knowledge of technology is necessary for effective teaching (Niess et al., 2009).

Teachers must not only develop knowledge of content, pedagogy, and technology but be able to "explore the interactions among these components" (Stoilescu, 2015, p. 515). That is, in addition to developing technological knowledge, teachers must also develop technological content knowledge, technological pedagogical knowledge, and technological pedagogical content knowledge, or TPACK (Koehler & Mishra, 2009; Mishra & Koehler, 2006). Technological knowledge consists of knowing how to operate hardware, software, and applications. Technological content knowledge, on the other hand, includes knowledge of the constraints and affordances offered by technology as it relates to particular concepts while technological pedagogical knowledge is knowledge about how the tools are related to teaching and learning tasks and what tool might be most suitable for a particular educational purpose (Koehler & Mishra, 2009; Mishra & Koehler, 2006; Niess et al., 2009). Finally, technological pedagogical content knowledge is the application of technological pedagogical knowledge to specific concepts and topics within a content area. Qualitative studies using TPACK may serve to illuminate the challenges and opportunities of teachers as they develop technological, content, and pedagogical knowledge (Patahuddin, Lowrie, & Delgarno, 2016).

Methodology

After a full academic year of COVID-impacted teaching, we conducted a case study of secondary mathematics teachers in one southern plains state to investigate the technological learning and experiences of teachers. The study was bound both geographically and in time; because teachers' belief structures are subject to change, we aimed to capture teacher perceptions a year into COVID. Data were collected through Zoom interviews conducted in the summer of 2021 after the first full year of teaching during COVID. Each participant selected for interviews had experienced fully virtual, in-person, and hybrid teaching and learning models.

Data Collection

Data collection began with an online survey that was sent to all teachers identified as grade 6 through 12 teachers of mathematics on a publicly available state database of teacher information; 2213 invitations to participate were sent in April of 2020; eighty-two teachers completed the survey. As part of the survey, participating teachers were asked if they would be willing to participate in a Zoom interview to further investigate their use of various technologies during the COVID-19 pandemic. Eleven participants agreed to engage in interviews. The participants represented a wide range of experiences, from a single year to twenty-two years of experience, with an average of just under ten years of teaching experience. Five participants worked in suburban schools, three worked in rural schools, and three worked in urban schools, thus the participants are representative of a wide variety of schools. Additionally, according to state records, approximately 20.3% of teachers in the state hold alternative certifications; in this study, three of the eleven participants held alternate certifications (27%).

Using semi-structured protocols (Seidman, 2005), we investigated participants' perceptions of both how they used various technologies throughout the first year of teaching during COVID and how they learned, grew, and adapted the use of technology throughout the year. Participants first identified technologies that they began using or changed their use of in response to COVID. For each identified technology, participants were asked how they used the technology and why, as well as what they had learned about the technology. Each interview was audio-recorded and transcribed verbatim. Transcripts were then subjected to two rounds of coding. In the first round, the first and second authors coded the interviews using the TPACK framework to investigate teachers' knowledge of the technologies. In the second round of coding, the first two researchers independently examined the statements coded under each of the four TPACK categories above. Each previously coded statement was assigned one or more codes that described the purpose or motivation described by the participant when developing the knowledge described in the statement. Thus, if a participant described multiple goals or motivations, then the statement was assigned multiple codes. All coded statements were independently subjected to this second round of coding. After independently coding, the three researchers met to discuss and define the emergent codes. The third author, who was a participant researcher, provided a member check, which allows for participants to provide feedback and refine the coding and findings of the study (Merriam & Tisdell, 2016). Full agreement about emerging codes was reached.

Participants' Development of TPACK during the First Year of COVID Teaching

Across all eleven interviews, 174 statements demonstrating the development of participants' technological, technological pedagogical, technological content, and technological pedagogical content knowledge were coded. Table 2 shows over 90% of statements were coded technological or technological pedagogical knowledge. Three themes emerged from statements coded developing technological knowledge: teacher communication, user-friendliness, and classroom management. Three themes emerged from technological pedagogical statements: access and workload, elicit student thinking, and assessment and feedback. Seven statements were coded technological content knowledge; seven were coded technological pedagogical content knowledge; thus, while the researchers coded these statements, no themes emerged.

Table 3
Codes & Themes

TPACK code	Frequency	Emergent Theme	Frequency
Technological	69	Teacher Communication	21
		User-Friendliness	18
		Classroom Management	17
Technological Pedagogical	91	Access & Workload	27
		Elicit Student Thinking	19
		Assessment & Feedback	13
Technological Content	7	None	
Technological Pedagogical Content	7	None	

Almost a quarter of the statements coded for technological knowledge referred to using technology to improve an aspect of classroom management, be it organizing assignments and materials, assuring engagement, or documenting student data. While many of these statements acknowledge the need for new classroom management techniques as participants navigated virtual and hybrid instructional spaces, most referred to the need to learn how to use learning management and presentation systems adopted by their schools and/or districts at or soon after the onset of COVID. Participants described school or district expectations for the use of such tools. For example, participant 1 said, “Everything had to be turned in on Canvas, all of our grading had to be done on Canvas, and then synced over to Infinite Campus.”

In addition to classroom management, participants focused on developing means of communication between themselves and students who engaged in class virtually. For example, participant 4 discussed using an iPad with Notability as a digital whiteboard and then being able to export PDF of notes so that “I could then post the filled versions online and I would also use that to create instructional videos using screen recording through the iPad to go through notes and have instructional videos for every lesson that we did.”

Like all the statements coded as improving communication, these statements are teacher-centered: the focus is clearly communicating the teacher’s thoughts and work to the student with no reference to student communication either to the teacher or to other students. These statements did not focus on what students would do with the information nor did they focus on communicating mathematical knowledge. Thus, these statements represent the development of technological knowledge.

Finally, user-friendliness emerged as a third theme in participant responses related to technological knowledge. Unlike the first two themes, these statements refer to participants’ motivations for choosing a particular technology to learn rather than the teachers’ purpose for using the technology. These statements referenced the perceived ease with which the participant or their students could engage with the technology. For example, when discussing the use of Desmos and Geogebra, participant 10 said the following:

> The one thing that I don't like about [Geogebra] is that it is- it's slower. It’s slower for Chromebooks. It's not- it does have, like, a classroom feature too. That's relatively new- where

you can kind of have a class code and you have all students working on the same thing and you can see student work.

More statements were coded as technological pedagogical statements than technological statements. Approximately 30% of the statements coded as technological pedagogical knowledge focused on student access and workload in online and hybrid environments. These statements focused on the appropriate quantity of experiences to provide for students to best learn, and thus moved beyond the technical knowledge of how students could engage with the tool to considering how best to use the tool for learning. Many participants, like participant 1, explained how they came to recognize that expectations set at the beginning of the year were not in students' best interests:

> we wanted something for students to do every single day whenever we were virtual and what that turned into was, every teacher gave an assignment every single day. So, if you think about it, a student has seven classes so now we're at seven, so thirty-five different assignments a week and we were all trying to figure out, well how long would this take? How is this the best thing for us to do?

The statements coded as access and workload demonstrate the participants' recognition of the paradigm shift students were experiencing along with their teachers. As such, the predominance of this code may be a result of the specific circumstances of COVID; a more gradual turn to online and virtual learning spaces may not have required focus on issues of access and workload.

Assessment and feedback emerged as another theme in technological pedagogical knowledge. These statements were concerned with how to ensure that students did not inappropriately use technology on assessments while others were concerned with how to appropriately capture student thinking while assessing with technology. For example, participant 10 was cautious about the quality of feedback provided by a technological tool:

> Feedback is good; really good whenever it's interpretive. Whenever it’s not, it’s still pretty good, because obviously sometimes in practice you want to know if you're right or wrong. But there's other times where I don't want them to know. I want them to think about it.

Finally, participating teachers focused on ways they could use technology to engage students in communicating their thinking. For example, participant 9 explained that they chose to use Pear Deck because “I can project it and show their answers without anyone knowing it's their answers and we can go through and have discussions.” Like this example, quotes coded as communicating thinking specifically focused on mathematical thinking; these tools might have been used in the same way by teachers in other content areas. Statements focused specifically on mathematical learning were rare. With few statements focused on technological content knowledge or technological pedagogical content knowledge, we cannot say that a theme emerged from these data. However, the paucity of such statements is notable.

Discussion and Implications

This study asked whether teachers of mathematics in the state developed new or deeper understandings of how to integrate technology to support high-quality teaching and learning. Results indicate that participants’ development of TPACK was largely limited to technological and technological pedagogical knowledge. Participants rarely demonstrated development of either technological content or technological pedagogical content knowledge. Thus, while participants developed understandings of how to integrate new technologies, these understandings were often limited in scope and simply allowed participants to replicate current pedagogies, as described by previous studies (Handal, Campbell, Cavanagh, Petocz, & Kelly, 2013; Loong, 2014; Niess et al., 2009). Indeed, many of the statements made by teachers revealed that they framed their work with technology by considering how the technology might be used to replicate traditional in-person experiences. With this goal in mind, it is not surprising that mathematical pedagogies were not transformed during this experience.

This study was limited to the teachers of a specific state within a U.S. context. More studies are needed to determine what influences teachers' development of TPACK both within and beyond the COVID pandemic. Additionally, study is needed to determine the most effective methods for facilitating the development of TPACK knowledge. What is clear from this study is that additional research on the processes of developing TPACK is necessary.

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DEVELOPING MATHEMATICS TEACHERS' PEDAGOGICAL DESIGN CAPACITY

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In the current education climate, well-prepared teachers are more important than ever. Given that much of the country is faced with teacher shortages that force them to hire underprepared teachers, we conducted a PD designed to help develop teachers' pedagogical design capacity. The goal was to meet teachers where they were and help them identify, modify, and implement mathematics tasks that have a high-cognitive demand. Based on results, teachers indicated an increase in confidence levels in finding a high-level mathematics task, adapting a lower-level mathematics task so that it is high, and implementing a high-level task in their classroom.
Keywords: pedagogical design capacity, mathematics, teachers

Research has shown that teaching is the most important school-based factor that impacts student success (Center for Education Policy Research, n.d.). We argue, and research corroborates, that effective instruction and high expectations are, in part, dependent upon the curriculum being used but are more importantly dependent on implementing curriculum with fidelity and with learner-centered practices (Stephan, 2020). Unfortunately, despite using rigorous mathematics curricula, many teachers feel their curriculum is not enough as many students are still underperforming despite being pulled for intervention. As such, these teachers are often utilizing the internet as a source for supplemental resources to help meet the diverse needs of their students (Hunter & Hall, 2018). Research has shown that many teachers do not know how to determine whether these resources are appropriate regarding content and research-based expectations for instruction (Peterson et al., 2019). This is not surprising as many teachers are not skilled in designing curriculum. That is, teachers are often lacking the pedagogical design capacity (PDC) (Brown & Edelson, 2003) needed to know how best to implement and refine these additional resources in a way that supports the learning goals for their classroom.

PDC is not necessarily a component built in teachers' educational training or in professional development (PD). However, given teachers' breadth and wealth of knowledge, we believe that developing teachers' PDC through carefully designed PD opportunities will be a beneficial endeavor. This opportunity can leverage teachers' current understandings and knowledge and develop their abilities as it relates to PDC for mathematics teaching. Given the pervasiveness of free curricula resources combined with teachers underdeveloped PDC, there is an immediate and critical problem that needs to be addressed. This study is a pilot study for a larger study aimed to address this critical need. For the present study, we conducted a week-long professional development (PD) and subsequent follow-up PD days, details below. Our research question is *How does this PD develop teachers' pedagogical design capacity (PDC) for mathematics teaching? How does this PD affect sustained use of skills and knowledge learned in this PD?*

Brief Literature

Brown and Edelson (2003) stated that teachers are inherently designers. "Teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs and devise strategies – all in the pursuit of their instructional goals" (p. 1). This is not surprising as many teachers often take a curriculum and make adaptations to it as they see fit for their particular students, instructional goals, and school contexts. The ways in which teachers offload, adapt, and improvise their curriculum and resources is what Brown (2009) called pedagogical design capacity (PDC). More specifically, he stated that PDC is "teachers' capacity to perceive and mobilize existing resources in order to craft instructional contexts" (p. 5). How teachers use the curriculum "is likely to be influenced not just by the nature of the curricular artifact, but also by their understanding of the relevant subject matter, their familiarity with the recommended instructional strategies, their knowledge of student understanding and their beliefs about teaching and learning" (Brown & Edelson, 2003, pp. 1-2).

Amador (2016) examined how mathematics teachers' PDC changed from lesson planning to lesson implementation. Amador was able to classify teachers' PDC and make comparisons to how it was implemented in the classroom. Interestingly, Amador found that when "teachers' capacities from lesson-to-lesson implementation" changed, it was "always in the direction of less curricular reliance and increased self-created lesson components" (p. 78). In all cases, the teachers' capacities changed because of "what they perceived to be important for students to know for assessments" (p. 83) based on how students were responding to the lesson.

This literature review informs our research in three important ways. First, it demonstrates that developing PDC is important. Remillard (2018) found teachers' PDC is often lacking in certain areas yet robust in other areas. As a result, teachers' implementations of curricula vary and may not always vary in a way that is productive for students' mathematical learning. As such, this work has the potential to positively impact the teachers involved by developing their PDC through utilizing their current robust capacities and informs the larger study. The outcome of this project has the potential of presenting the research community with a better understanding of what robust capacities teachers have, what capacities still need to be developed, and how best to do so when some teachers either no longer have a set curriculum or supplement their adopted curriculum with other resources found either online or from other physical sources. Second, the research identifies components of PDC that one should focus on when developing this capacity such as developing teachers' mathematical content knowledge as well as their pedagogical content knowledge for teaching mathematics. Another is considering ways in which teachers offload, adapt, and improvise resources.

Methods

Professional Development and Context

The overarching goal for this project is to support the development of teachers' PDC for teaching mathematics with an innovative PD and identify in what ways our PD is effective. The motivation for this project reflects the reality that most teachers modify their curriculum to meet the needs of their students, but "insufficient attention has been paid to how to delineate [PDC] and help teachers develop [it]" (Remillard, 2018, p. 73-74). The project supported K-4 teachers in modifying and enhancing their mathematics curriculum combined with implementing it with learner-centered practices so that teachers ultimately provide rigorous, high-quality mathematics instruction engaging students in doing mathematics. The objectives for engaging teachers with this PD are to develop their PDC for teaching mathematics and determine how these opportunities affect sustained use of skills and knowledge learned in this PD.

To design and facilitate effective mathematics instruction, teachers need to possess the pedagogical content knowledge that is needed to choose appropriate tasks that have high cognitive demand, to facilitate these tasks so that the level of domain remains high, and to analyze student thinking during and after the task. The goal of our PD was to aid in participants' ability to identify strengths and weaknesses in teachers' current teaching practice and curriculum, collaborate with other teachers to create revised lesson plans that focus on school-specific curriculum, and learn to implement lessons in ways that align with learner-centered teaching practices (Stephan, 2020). Additionally, the PD worked to create an environment where time was devoted to working directly with teachers design and implementation of their own curriculum and answer questions. We conducted a summer four-day PD with follow-up days in two subsequent semesters. Each day of the summer portion focused on a unique area of teaching and pedagogical design capacity:

Identifying tasks as high and low level and analyzing them; analysis of localized resources (curriculum used within the school) to identify strengths and weaknesses of lessons within grade-band and improve each lesson collaboratively among teachers; norm setting in the classroom in conjunction with identification of student reasoning levels and building on student misconceptions in the classroom; and creating a modified lesson plan with material discussed in previous PD days. Throughout the school year, teachers implemented and videotaped their modified lesson plan to improve and reflect on their teaching. At PDs throughout the year, teachers reflected on peers' teaching and task implementation. We worked with 18 K-4 teachers from a local school district (5 different schools; 4 Kindergarten, 2 1st, 4 2nd, 3 3rd, 1 4th grade teachers, 3 interventionists, and 1 district curriculum specialist). Teaching experience ranged from 1 year to approximately 20 years, and there were approximately 350 students in all these teachers' classes.

Data Collection and Analysis

To answer the research questions, both qualitative and quantitative data were collected. The purpose of collecting both types of data was "to see if the two types of data show similar results but from different perspectives" (Creswell & Plano Clark, 2011, p. 6). Thus, the approach taken was a fixed mixed-methods approach, particularly a convergent parallel design so both qualitative (e.g., open-ended survey item questions, lesson revision artifacts, video observations) and quantitative (e.g., survey items) data will support the research to bring positive change to the mathematics teacher community (Schensul & LeCompte, 2016). In addition, this approach provides triangulation, completeness, credibility, and illustration. As we collected and analyzed the quantitative and qualitative data separately but combined results during interpretation, we utilized a convergent parallel design (Creswell & Plano Clark, 2011).

Qualitative data collected from teachers included discussions of teachers videotaped modified lessons, observations of PD days, and open-ended survey items on teachers' development of PDC. We collected survey data from teachers using validated instruments, as well as some additional survey items. Specifically, we collected data using the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). MTEBI is a validated instrument and measures teacher's beliefs regarding mathematical instruction, students' ability to learn mathematics, and teacher's sense of efficacy in teaching mathematics (Enochs et al., 2000). As it is difficult to obtain individual student data from districts, and correlate that with teachers participating in the PD, we are not able to determine if this PD has improved student performance by looking at test scores. Thus, we will ask the teachers about their perceptions regarding their students' performance because it may be an important factor that determines how much of the skills and knowledge that they transfer from the PD into their normal teaching practice.

The research team analyzed qualitative data by reading through and organizing open-ended survey items and transcripts of video observations in a meaningful way. During this open-coding process, we purposefully selected a sample from each type of data collected and coded these samples. We then meet to discuss codes and look for reliability among the data and how it was being coded. Once this baseline was established, the rest of the samples were coded. Any disagreement on codes between researchers were discussed and themes created. These themes were then detailed and discussed to illuminate any connections or relationships among themes. Quantitative data summarized in terms of frequencies of responses to Likert scale questions.

Results

Here, we present results from various surveys and discussions of videotaped modified lessons. Future analyses will involve interviews. We gave a survey at the beginning of the summer PD, the end of each day of the summer PD, and overall end of the summer PD, and the end of the follow-up PD day in the semester following the summer PD. Each survey contained both Likert-scale and open-ended items. Our sample size is small to conduct meaningful quantitative analyses thus we report overall frequencies here.

For the majority of the MTEBI items, participants either strongly agreed or agreed if the statement was positive about their teaching and strongly disagreed or disagreed if the statement was negative about their teaching. An item that had more distributed responses was “The low mathematics achievement of some students cannot generally be blamed on their teachers” with 0 strongly disagreeing, 4 disagreeing, 5 neutral, 8 agreeing, and 1 strongly agreeing. Related items “If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching” and “When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher” had comparable responses.

Before the summer PD, participants were asked: Rate your confidence level on the following statements (with 10 being the most confident): Item 1) I feel prepared to find a high-level mathematics task OR adapt a lower-level mathematics so that it is high-level. Item 2) I feel prepared to implement a high-level mathematics task in my classroom. They were asked the same question again after the summer PD except it began with “Following the summer PD…” Table 1 shows the frequencies (levels 1-3 grouped as there were 0 instances of each).

Table 1. Frequencies of confidence levels.

Confidence Level	**Item 1**		**Item 2**	
	Before	**After**	**Before**	**After**
1-3	0	0	0	0
4	1	0	1	0
5	1	0	2	0
6	5	0	3	0
7	4	0	4	1
8	6	4	7	4
9	0	10	0	9
10	0	4	0	4

As seen in Table 1, there was a shift in confidence following the summer professional development. Teachers noted that they feel more prepared to find a high-level mathematics task, adapt a lower-level one, and implement a high-level task in their classroom.

Teachers reported gaining knowledge regarding "MULTIPLE *(sic)* ways to implement discussions, strategies, questioning techniques, and wrong answers that produce higher level thinking in students," as well as valuing "being able to work with people from my school, being able to work with others who teach my grade, being in a 'safe environment' so that I could share ideas." At the beginning and end of the PD, we asked participants to define good mathematics pedagogy. After the PD experience, teachers defined good mathematics pedagogy as "engaging, student voiced, safe, and higher level in student thinking", "includes the process - right and wrong - of computation" and "being flexible and able to meet student needs by differentiation".

In the follow-up PD day, we watched clips from several teachers but focused on two larger clips from two teachers (one Kindergarten and one 1st grade). These teachers' clips were chosen because they explicitly acknowledged what about the lesson from their curriculum, they modified in the lesson itself or in the implementation of it. The Kindergarten teacher said,

> I believe I gave higher-level tasks to the students than what the lesson called for. For example, the lesson said "Get out 5 counters, now get out 3 more. How many is that?" I asked the students to get out 8 counters and then show different ways (hidden partners) to make 8. Another instance is when the lesson said, "Does your friend's group of 8 look like yours?" I added "How is it the same? How is it different?".

After observing this clip, teachers discussed how this modification to the lesson afforded students opportunities to show multiple representations of their thinking. They further discussed how the activity also engaged students in each other's mathematical thinking.

The 1st grade teacher is in her second year of teaching. She said the following,

> I took two videos one of my small group and one of my whole. My small group went very well with the ways they were able to explain their thinking and show their thinking. My whole group was mainly engaged with being able to "showdown" their work.

This teacher has a joined classroom (51 students doing mathematics at the same time, with the other teacher helping and they switch for different content). Thus, she was focused on her implementation and making sure that even in that context students were explaining their thinking. This is overall summed up by a quote from the 2nd grade teachers about modifying lessons: "Our focus has switched to being intentional with creating higher order thinking questions and implementing lessons with more manipulatives."

Conclusion and Future Research

To design and facilitate effective mathematics instruction, teachers need to choose appropriate tasks that have high cognitive demand, facilitate these tasks so that the level of domain remains high, and analyze student thinking during and after the task. The goal of our professional development is to aid in participants' ability to identify strengths and weaknesses in their current teaching practice and curriculum, collaborate with other teachers to create revised lesson plans that focus on their curriculum, and learn to implement lessons with learner-centered teaching practices (Stephan, 2020). The work presented here is from the pilot study.

In these results we focused on teachers shift in confidence, their shift in what good mathematics pedagogy is, and highlights of what they learned. We have one additional PD day with the teachers (at the time of this writing) and following that we will be able to do larger analyses of teachers development of their mathematics pedagogical design capacity. We worked with 18 teachers (both classroom and intervention teachers and the district curriculum specialist). In our future work, pending funding, we plan to expand our work with this district and work with 70 K-8 mathematics teachers about their pedagogical design capacity. We are encouraged by the teachers' commitment to their students having a deep conceptual understanding of mathematics. By focusing on how teachers can take their curriculum and make it more high-level (cognitive demand and implementation), we are able to support their growth as a teacher.

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Celebration:
Diversity in Teaching

EATING BY NUMBERS: PROBLEMATIZING NUMERICAL NEGOTIATIONS BETWEEN FOOD, BODY, AND MOVEMENT

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We unpack how some school mathematical word problems can perpetuate and (re)construct anti-fatness in the classroom. We explored NCTM practitioner publications which incorporated activities encouraging students to mediate their relationship to food and bodies using school mathematics and found anti-fatness was present as a subtext in many of these publications. We endeavor to understand how to identify anti-fatness in mathematics activities.

The political power of mathematics has been used throughout modern history for incredible things, but it has also been used to perpetuate and justify harm (Zuberi, 2001). We cannot assume numbers are neutral objects. The ways we conceptualize and share numbers are informed by social factors (Craig & Guzmán, 2018). When we give our students mathematics exercises incorporating real-world contexts, an opportunity emerges for students and educators to reflect on how their identities inform the conclusions they seek with numbers. We can encourage reflection, and in so doing remind students numbers can sometimes be used to say harmful things. The mathematics classroom's emphasis on quantification makes it a powerful site to trouble this harm. In the case of mathematics and nutrition, we must reckon with how the dominant culture numerically frames health, and therefore, reckon with anti-fatness. We endeavor to engage in conversation on how mathematical activities can strengthen existing social norms of sometimes harmful, problematic, and even violent notions of health and anti-fat bias.

What is Anti-Fatness, and How Do Teachers Fit In?

Anti-fatness (Gordon, 2021), also called fatphobia, refers to "the hatred and fear of fatness. It is a historically-rooted, racism-driven notion that fat bodies are physiologically and morally deficient" (Mercedes, 2021). Anti-fat bias impacts access to adequate or compassionate medical care, equitable education, employment opportunities, and so much more (Wann, 2009). Anti-fatness works at a systemic level, shifting institutions to put fat[1] people at a disadvantage as well as amplifying other inequitable systems (e.g., racism, ableism, sexism; Prohaska & Gailey, 2019). Generally acceptable public rhetoric places the blame on the individual, a practice commonly referred to as fat shaming, and is mentally and physically harmful to fat folks (Chrisler & Barney, 2017). Anti-fatness cannot be solved through individual fat people losing weight, as fat individuals are not responsible for systemic bias. The broader social acceptance of fat-shaming as an appropriate health measure has dangerous consequences for children, but teachers can intervene on fat children's behalf. Size acceptance as a medical and social practice has been shown to produce far more favorable mental and physical health outcomes (Bombak, 2014). Teachers can position fat children as worthy of care and respect as they are. Fat children and their bodies are not cases to be fixed.

Anti-fatness is particularly harmful to fat Students of Color. Anti-fatness is by construction inseparable from anti-Blackness (Harrison, 2021). Modern notions of health within diet culture are by construction inseparable from whiteness and white supremacy (Harrison, 2021). The foods, bodies, and community practices of racially marginalized people, in particular Black, Latinė, and Indigenous people, have historically been excluded by dominant notions of health (Moore & Swisher, 2015). We will continue to explore how and where whiteness shows up in classrooms, and what teachers can do to support fat students.

The mathematics classroom often positions health as binary (bad or good; thin or fat). When foods and bodies are seen as opportunities to practice numerical precision, little room is made for ambiguity or other mitigating factors. Dieting toward intentional weight loss is proven to be ineffective in the long term for the vast majority (Mann et al., 2007). Even if weight loss were the goal for a child (and it need not be), evidence suggests it is not a reasonable long-term goal to set through measured and calculated food choices. Suggesting children should use restrictive food practices to alter the shape of their already rapidly changing bodies is not realistic.

Positionality Statement

Teachers can have a role in supporting fat students, however careful reflection on internalized anti-fatness is necessary for that support, particularly for thin and/or white instructors. Thin bodies derive their political power from the marginalization of fat bodies, and thin folks, in particular thin white women, have historically overtaken the narrative in 'body-positive' spaces (Gordon, 2021). Most practicing teachers in the United States are white women (Leonard & Boas, 2021), and the authors of this piece are both thin, white-presenting individuals. We know it is not our place as thin scholars to become complacent in Fat spaces. Without vigilance and care coming from knowledge of the voices and writing of fat folks (e.g. Marquisele Mercedes, Imani Barbarin, Da'Shaun Harrison, Caleb Luna, Shilo George, Jordan Underwood), we can always default to centering thin experiences.

Our training is in mathematics education research. We do not have the lived experiences of fat people, and it is not our place to introduce new theoretical ideas through Fat studies in mathematics education. Our conversations regarding the disruption of anti-fatness in mathematics spaces are compilations of the voices and writings of fat scholars and activists. We are passing on their words that have not yet made it to mathematics education spaces but need to be here. Anti-fatness permeates all spaces, and mathematics education has great potential to disrupt anti-fat constructions. So much of the power of anti-fatness comes from the political power of mathematics (Mudry, 2009). Our field has a responsibility to reckon with how we have empowered anti-fatness and examine how we can begin to disempower it. Mathematics education needs to hear and read what fat liberationists have already done, and it needs to create actionable work based on that knowledge.

Methodology

We began by conducting a search through NCTM practitioner journals using key terms (calories, nutrition, food, fat, health, healthy, diet, 'overweight', 'obese'), to find papers and tasks asking students to do mathematics regarding food, bodies, and health. Our initial search found 349 articles. As our first round of coding is being conducted, three behaviors have emerged in many of these papers and tasks encouraging students to engage with: shame regarding food choices, 'exercising away' certain foods, and worrying food behaviors. A throughline across the emergent themes is an implicit message of anti-fatness. In this section, we will explore several activities within these themes.

Emergent Themes

Shame Regarding Food Choices

The first set of activities promoted the students to feel shame regarding their food choices. Activities in this theme encouraged students to doubt the validity of their food choices by presenting specific numerical health guidelines from a governing body and asking children how their real or hypothetical food choices compare to what is being established as correct and authoritative. Students use mathematical ideas (e.g., proportions) as a way to make comparisons of their own habits and food consumption to guidelines deemed universal. Those who do not fit or follow the authority are positioned as having unhealthy habits or making inappropriate food choices. There is no discussion in the activities of the critiques of these authorities.

For example, Bush et al. (2012) describes how an entire middle school analyzed their eating habits and compared them to the USDA (United States Department of Agriculture) *My Plate* program as well as to the eating habits of their peers. Students recorded their food intake for one week in a food diary (below we'll discuss how this strategy can encourage dangerous behaviors; Custers, 2015) and tallied their consumption of vegetables, fruits, grains, proteins, and dairy. Class discussions were held to determine how to categorize some foods. In one example given, Pop-Tarts® were decided not to be categorized as grains, but as junk foods. Because the USDA does not mention junk food, the students "realized that the likely suggested, but unstated, proportion of that food group would be zero" (Bush et al., p. 102). The mathematics classroom confirmed foods like Pop-Tarts® should not be consumed at all. Within the confines of the USDA recommendations, some foods that nourish and bring students joy are not enough, and students will have to reckon with this every time they eat something and have no place for it in their journal. Students receive the message that some of their choices quite literally have no place on their plate, and thus may not belong there in the first place.

Once each of the students gathered the data, they developed a metric to convert their tallied consumption of food groups into a measure on a length of cashier's tape. "While constructing the tape representations, students instantly began comparing their proportions of food consumption in particular groups with those of their peers, noting those who ate more, less, or about the same amounts and in which categories" (Bush et al., p. 103). When discussions regarding who ate the most based on tape length came up, the authors did note how much eaten does not correlate with higher number of calories. "We guided the conversation so that students concluded that the number of food servings consumed in a week does not necessarily correspond with caloric intake.…a healthy eater might have a longer tape but actually consume fewer calories" (Bush et al., p. 103). Here we see an attempt to accommodate students and discourage bullying, however, messages perpetuating anti-fatness are still present. A defense of more tape as not indicating more consumption implies more consumption is inherently in need of defending. Caloric intake is still framed as undesirable, resting on commonly perceived links between high caloric intake and fatness (Mercedes, 2021), with fatness as a trait to be avoided.

In an excerpt of student work, the student remarks, “We also conclude that I eat healthy while comparing to the nation’s food plate” (Bush et al., p. 105). The activity hints at a binary resolution. Students whose ratios do not match the USDA have to decide for themselves whether it is okay to not be perfect by the standards and authority of the USDA. School and government authorities employing one measure of good eating may pressure such students into feeling shameful, to doubt their intuition on eating.

Exercising Foods Away

When food is positioned as something to be erased through exercise following consumption, student relationship to food is constructed then as transactional. Food is then earned through movement, and movement is done for the sake of erasing food. We see this play out in Yanik and Memis’ (2016), *What is Your Body Mass Index?* The lesson itself begins by listing the “health risks” (p. 442) of being fat, followed by an introduction to the BMI (Body Mass Index). The authors define the BMI as a number calculated and shared to make patients aware of their possible health risks, however many scholars have recommended the BMI be removed as a measure of overall health (Tomiyama et al., 2016).

After students calculate BMIs given heights and weights of sample children, the activity pivots to how these sample children can achieve ‘normal’ BMIs (those not categorized as over- or under-weight) through lifestyle changes. The students enter the sample child’s information into a mathematical model. “After the information was entered, the simulation calculated the minimum number of calories required to maintain one’s basic life functions. To lose or gain some weight, one needs to burn, or consume, more calories than this number" (Yanik & Memis, 2016, p. 445). Caloric needs in this model maintain "basic life functions" using a metric/authority unnamed in this article. Student need and hunger cues are not mentioned in this simulation, despite food needs varying depending on a multitude of dynamic factors. Cultural, racial, and familial aspects of food are also not included. Eating, consequently, is flattened to a mathematical activity.

Within this model, weight loss is achieved by exercising away food through a preset list of prescribed movements, not accounting the movements students enact already or the movements physically disabled students may be unable to reproduce. Any excess consumed within this simulation must be accounted for with a movement of equal value within the singularly caloric metric. The student understands eating food to be a zero-sum game within a calories-in, calories-out model, as maintenance of a "normal weight" (p. 444) is the overall goal. The students are changing to conform to a world in which size diversity does not exist.

Worrying Behaviors

Our final example activity focuses on the dangerously precise behaviors encouraged by instructors for a class activity. In the activity explored below, students are asked to keep track of precise calorie and nutrient counts of foods in a whole-school project. Other activities we saw commonly had students or characters in word problems measuring foods down to tenths of an ounce. In this section, we try to remove the activity from the context of school mathematical precision to highlight how worrying these behaviors would be in other contexts.

Selmar et al. (2011) focus their work on the USDA *My Pyramid* nutrition metric. In one highlighted activity, students are asked to record the foods they eat over the course of several days using a template. The information needed for each food is found using the USDA *What's in the Foods You Eat* search tool. In the year of writing this (2022) this search tool offers information on a variety of foods, however, it is limited to those known by the USDA and the ratio of ingredients assumed by the organization. This puts to advantage foods commonly eaten in America, with those not known by the USDA having to be broken down by ingredient. For example, hilachas, a common dish in Guatemala, was not found using the search tool.

Once found on this search tool, the quantity of food eaten must be specified based on area, volume, or weight depending on the food. This implies the student has measured their food prior to consumption. All data collected is combined to create pie charts indicating the ratio of calories from protein, carbohydrates, and fat across several days. The data is framed as neutral, something to be used to gain statistical knowledge and, "...a better understanding of healthy eating" (Selmar et al., p. 276). In this case, detailed tracking of food by volume and nutrient content is a part of the lesson on what healthy eating means.

Within the context of school mathematics and the mathematical practices of the Common Core, numerical precision and data collection and comparison are encouraged. It is normal in the mathematics classroom to calculate things like area and volume, to tally totals, to track progression and to compare and contrast to other models. Outside of this context (e.g., homes, parks, playgrounds, kitchen) this behavior of tallying such precise totals might be cause for concern. Many students likely do not ask their mothers how much oil was added to their hilachas, or for the weight of the tomatillos. Sustained behavior like this would warrant concern for disordered eating ("Warning signs", 2021). The prioritizing of quantification in the context of health over student joy and cultural practices erases the humanity in student well-being.

Discussion & Conclusion: Moving Forward

While most mathematics classroom activities do not center food or movement, we hope to turn a more critical eye toward those highlighting these topics. When eating and movement are introduced to the mathematics classroom, they are introduced in a structural context of anti-fatness. If the classroom does not actively reject anti-fatness, anti-fat ideology will remain a subtext of the activities. To move forward we must unpack how and for whom we are defining health as mathematics educators and mathematics education researchers.

In her talk at the 2022 Association for Size Diversity and Health, Barbarin (2022) said "I don't rely on non-disabled people to create my joy because they don't know what it would look like for me." She chooses what joy looks like for her as a fat, Black woman with Cerebral Palsy, and her choice should not be questioned by anyone not living in her body. We can relate her naming of the ableism embedded in the demand she pursues dominant notions of health to the ableism, racism, and anti-fatness embedded in asking students to pursue health through mathematical activity that erases their agency surrounding food and body. We would like to instead pivot to centering food and health sovereignty in future work with students. Brady et al. (2021) situates these ideas of food and health sovereignty as follows:

> …[A]n inherent right to food that must be affirmed and realized with no strings attached. That is, the right to food, and the right to self-determination over the means of producing and accessing it, is intermingled with the right to health, and more specifically the right to self-determination over what that means and looks like in all respects, including what, when, how, why, and with whom one eats. (p. 5)

Children in school are in a process of learning what health means and who decides how they conceptualize it. Students, with particular attention paid to fat students and Students of Color, deserve to be a part of defining health for themselves in flexible and dynamic ways. An emphasis on quantification as a preferred means of defining health can overshadow the flexibility and dynamism existing in more qualitative definitions of health and normalize the consistent pursuit of health as a moral obligation. If the mathematics classroom is to incorporate health and body sovereignty, it too must be flexible and dynamic. There is value and power in the lesson that numerical precision does not belong in every aspect of life.

Notes

We use the term 'fat' in solidarity with those in fat liberation spaces. We elect not to use offensive terms such as 'overweight' (which implies fatness as an incorrect deviation from a thin norm) or 'obese' (which pathologizes fatness; Wann, 2009).

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DECONSTRUCTING BLACK PRESERVICE ELEMENTARY TEACHERS ENTERING IDENTITIES & VISIONS

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In this paper, we present findings of the entering mathematics identities and visions of high-quality mathematics teaching of Black freshman undergraduate preservice elementary teachers. Evidence suggests that race and racialized experiences impacted both their MIs and their vision for mathematics teaching, highlighting the need to attend to these constructs as we support the preparation of Black teachers who can flourish in their humanity and brilliance.

Introduction

Research has progressed to more clearly understand teachers' mathematics identities (MI) and visions of mathematics teaching (VMT) (e.g. Boaler & Greeno, 2000; Munter, 2017); however, this work often does not take into account the ways race or racialized experiences play a role in preservice teachers' MI and VMT. Others have described the ways mathematics learning experiences can be forms of racialized trauma (e.g. Matthews, 2018), and how mathematics education might be conceptualized to rehumanize Black students (e.g. Joseph et al., 2019). Separately, these studies have built an important knowledge base and generate additional empirical questions. In this paper, we share findings from a qualitative analysis of interviews with Black undergraduate first-year students who have declared elementary education as their major to answer the research question: *How and in what ways have K-12 mathematics learning experiences influenced the MIs and VMT of preservice Black elementary teachers?*

Background

We have a rich body of evidence of instructional characteristics, pedagogies, and practices that can support mathematics learners. These practices include relating mathematics to local contexts (e.g. Ladson-Billings, 1995), developing sociomathematical norms and authority (e.g. Boaler &

Staples, 2008), and positioning students as agentic and competent (e.g. Battey, 2013). Other scholars have explored ways to reconceptualize and humanize Black students' experiences with mathematics (e.g. Joseph et al., 2019) and develop clearer conceptions of Black pedagogical excellence (Acosta et al., 2018). While this evidence shows actionable progress, there still exists a stark lack of high-quality mathematics learning opportunities for Black students; with one study of 191 high school classrooms finding that only 13% of Black students engage with mathematics in classrooms that can be characterized as "high quality" (Wilson et al, 2019).

Prior to Brown vs. Board of Education, roughly 35-50% of teachers in America were Black. Currently, only about 7% of teachers are Black (Lee, 2019) – a sobering statistic for Black preservice teachers as they prepare to enter the teaching workforce. Many programs are engaged in work to change these statistics to reclaim Black teachers' rightful roles as aspirational figures and advocates for Black learners, and Minority Serving Institutions play a key role in this work - disproportionately preparing teachers of color (Gasman, et al., 2017). We extend these efforts by exploring how we might create a mathematics education program that acknowledges, honors, and builds from these histories to support the preparation of Black teachers who flourish.

Mathematics Identity

In our work, we draw upon Martin's (2009) definition of MI as the "dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematics contexts and to use mathematics to change the conditions of their lives" (p. 136). This way of describing identity brings into focus different aspects of mathematics teaching and learning, and together can serve as an analytical tool to better understand the ways mathematics learning experiences shape the entering and evolving MIs of Black preservice teachers. Math education researchers have explored K-16 teachers' and students' MIs in many ways. For example, Gresalfi &

Cobb (2011) explored teachers' mathematics teaching identities as they participated in professional development focused on high-quality teaching, while Black and colleagues (2010) explored the relationship between undergraduate students' MIs and their professional career goals, concluding that further research should explore how the power of mathematics can support students in managing their evolving MIs.

Vision of Mathematics Teaching

A persons' VMT is not simply a compilation of static beliefs or teaching philosophy. Rather, it is a set of concrete notions about what mathematics teaching should be that shifts with past and present experiences, knowledge, and context and becomes more layered and sophisticated over time. Seeking to explore shifts in teacher's VMT, Munter (2014) tracked their articulations of their VMT and their alignment toward research-based descriptions of high-quality mathematics teaching. Munter characterized teacher's VMT across three dimensions: the role of the teacher, the role of classroom discourse, and the role of mathematics tasks in supporting student learning. Researchers have used these dimensions in studies focused on prospective teachers (Walkowiak, Lee, & Whitehead, 2015), practicing teachers (e.g. Wilhelm, 2014); and mathematics leaders (Jackson et al., 2015) and have found that teachers with a VMT more closely aligned with high-quality teaching tend to enact instruction more closely aligned their VMT, select more rigorous mathematics tasks, and have greater content knowledge for teaching mathematics.

These studies provide robust evidence relating teachers' visions and enactments of mathematics teaching, however, Munter (2014) acknowledged that they lacked attention to ensuring mathematics learning spaces are spaces where students feel they belong and where their race, gender, or other identities are affirmed. He called for additional research exploring the relationship between his conceptualization of vision for mathematics teaching and one that also attends to "culturally ambitious practices aimed at critical consciousness" (p. 619).

Methods

This study is a part of a multiyear study investigating ways to construct a Black liberatory mathematics education that cultivates the flourishing, humanity, and brilliance of Black preservice teachers (Martin, 2018). Focused on undergraduate elementary education student's mathematics experiences, it explores their entering and evolving MIs and VMT as they progress through their teacher preparation program and redesign efforts to support their learning.

Phase one of the study, which we report here, explores the entering MIs and VMT of students attending a large Historically Black College and University (HBCU) in the southeastern United States. Students were recruited using a convenience sample from a course for entering freshmen who declared elementary education as their major. Of the 21 students who met the criteria, 15 chose to participate in a one-hour semi-structured interview during the Fall 2020 semester, receiving $20 for their participation. 14 of the 15 participants identified as female and one identified as male.

Questions related to *MI* focused on: (1) K-12 mathematics concepts participants found helpful, useful, or difficult; (2) K-12 factors that may have contributed to the ways they think or feel about, relate to, or engage with mathematics; (3) their problem-solving strategies; (4) and whether they felt their race or racialized experiences played a role in their MI. Questions related to *VMT* focused on: what (1) teachers and (2) students should be doing and (3) what kinds of tasks students engaged with in high-quality mathematics classrooms; (4) how their vision has been shaped by prior learning experiences; and (4) whether race plays a role in their VMT.

After all interviews were recorded and transcribed, we used qualitative analysis software and open coding (Corbin & Strauss, 1990) to capture common themes across participants interviews related to mathematics, identity, and vision. Next, we developed a codebook using these themes and coded each interview, reaching 89% - 100% agreement across coders at the individual code level (Table 1). Finally, because we were interested in using these findings for the design of future experiences for students, we looked for patterns related to students' MI and VMT that aligned with and diverged from practices of high-quality mathematics teaching and learning.

Findings

Shown below in Table 1 are the number of instances and percentage of total instances coded across all 15 interviews, listed in descending order. As context, across participants, interviews had an average of 35 codes and a median of 32 codes per participant.

Table 1
Number of Codes and Percentage of Total Codes

Mathematics			Identity			Vision		
Math Content	59	11%	Feelings	48	9%	Teachers	45	9%
Relevance	38	7%	Teachers	44	8%	Students	32	6%
Math Tasks	17	3%	Understanding	44	8%	Race	14	3%
Math Tools	14	3%	Performance	24	5%			
			Problem Solving	22	4%			
			Race	21	4%			

We organize our summary of these findings into two categories: mathematics identity and vision of mathematics teaching. For each category, we first share noticeable patterns that emerged from the data related to traditional ways of describing them and then highlight findings that particularly make salient racialized moments that impacted participants' MI and VMT.

Mathematics Identity: *Teachers, Tasks, and Relevance*

Participants noted many instructional practices that their K-12 teachers used to support their learning, aligned with high-quality instruction, and were positive contributions to their MI. These practices included the use of relevant tasks, providing clear explanations, sharing multiple approaches, asking questions that explored their thinking, and providing individual support. However, two quotes from participants summarize well the challenges Black students often face in mathematics learning spaces. First, in describing their experience with mathematics teachers, one participant stated, "*I want to learn by someone who actually wants to teach me*", which aligned with others who had mathematics teachers who were "*strict*", "*unforgiving*", "*mean*", "*yelling*", and said things such as, "*oh well if you don't get it [math]*". Second, in describing the relevance of the mathematics they experienced, one participant stated, "*all of it is necessary, but then again none of it is necessary*." This statement aligned with others who shared their experiences engaging with mathematics tasks that were "*long drawn-out problems*" or "*textbook teaching*" that was "*not useful beyond basic math*" or "*only useful to know more math*".

Mathematics Identity: *Problem Solving, Understanding, and Feelings*

From the perspective of alignment with the knowledge, skills, and dispositions of high-quality mathematics teaching, participants overwhelmingly indicated a deep desire to learn, understand, and use mathematics – both as a learner and teacher. They stated that solving mathematics problems helped them develop a "*connection to math*" that "*made them feel good and smart*", and that they desired to see and explore visual representations and different approaches to mathematics. They also stated that they wanted to understand mathematics to "*help others*" and to "*know where my [future math] students will go.*" Common themes that negatively contributed to their MI were classrooms that focused on performance and perfectionism - leading to feelings such as, *"everyone else knows it"* and the stress of the "*additive*" nature and speed of learning concepts. These feelings led students to modify their problem-solving strategies to focus on *"following steps,"* using formulas, or "*reversing through a problem.*" Lastly, there was a common theme of needing to "*prove people wrong*" when engaging in mathematics with others.

Mathematics Identity: *Racialized Experiences*

Two themes emerged related to the ways race played a role in their MI. For some, they vaguely indicated that race did not play a role or that they were unaware of it, with statements such as, *"I don't really think race played a part in it. Math is math, like no matter what race you are, either you get it, or you don't.*" For others, they stated that race did not play a role in their MI because, "*I learned in a diverse school*" and, "*maybe it might have had an impact, but I probably just didn't notice.*" Conversely, others noted the power of Black representation with statements such as, "*I've only had one Black math teacher…with her I just felt comfortable*." While others had explicit descriptions of the ways that race impacted their MI. Related to teachers' beliefs and practices, participants made statements such as: "*we are at a disadvantage on how well our teachers think we can do…maybe if I was in a different person's shoes then maybe the teacher would try to build on what skills I have more*" and "*I did not feel comfortable asking questions and it was hard to find teachers who did not just talk down to us.*" Finally, related to teachers, we share the following quotes to demonstrate the impact some White teachers had on Black preservice teachers' MI.

> *Honestly, I think she was racist. We were the only two Black kids in the class and she was a White lady. I feel like she wasn't as supportive because we were Black. She wouldn't help us, would just give us the answer instead of teaching us. When we did ask questions, she would have an attitude. It's like why ask the question. I'm aggravated because I'm a student trying to learn and get into college. I need a good grade.*
>
> *The black kids in regular math classes and the white kids in the honors. It was like they just knew how to do it because they actually taught and helped them. Teachers just gave us stuff saying, "here's stuff, do it, learn it, if you don't oh well." I felt like my teachers didn't care enough to show us how to do it. Why should I care enough to like learn it.*

Vision of Mathematics Teaching: The Role of the Teacher, Tasks, and Students

When articulating their VMT, participants provided vague descriptions of what teachers should be doing, with statements focused on teachers being "*supportive so that students aren't confused,*" ensuring that students "*feel comfortable*" or safe to ask questions, and a general focus on student compliance over mathematical understanding when students were engaging with mathematics. When describing their vision for what students should be doing, they noted that they should be collaborating, but again focused on compliance or following the teachers' expectations, with comments focused on ensuring that students "*do their work.*" However, there were three instances stating that students should be engaged in "*exploration*" and "*discovery.*"

Vision of Mathematics Teaching: Racialized Visions

Two themes emerged related to the ways participants articulated whether race played a role in their VMT. First, like MI, some participants shared what might be described as a colorblind or equality approach to their vision with statements such as: *"I would teach all the kids the same"* and *"I feel like when we… put race into a classroom, it hurts the child more than it hurts the teacher...just straight down the middle."* Second, and most prevalent were quotes from participants relating their VMT to representation and past mathematics learning experiences.

> *I want students to see "that it's possible that they have a Black, educated teacher that understands what they're talking about. [I] should've seen more people that look like me doing things like this."*
>
> *"Just because you had a bad experience in high school with your math teachers because of the way you were Black … doesn't mean you gotta let somebody else go through that same situation."*

"I've seen teachers put Black students on one side and Whites on another... I was in it. The teacher was like "any questions." I've left not knowing before and get home still confused. My goal is to never let a child leave confused. I don't care how long it takes."

"I feel like I'm going to have to work a little harder than everybody else. It's my duty and my responsibility to reach out to young black students ... because the world is going to knock them down and make them feel like their bad students or whatever else they have to say about us. But I feel like you know it's really going to be my job to build them back up and know ensure them that they are just as good as the next person."

Discussion and Conclusion

The purpose of this study was to inform our efforts to design learning opportunities for Black preservice elementary teachers that might positively impact their MI and VMT so that they can both experience and enact a "Black liberatory mathematics education … that allows Black learners to flourish in their humanity and brilliance" (Martin, 2018). Results from our analysis suggest that Black preservice elementary teachers possess a strong desire to learn, understand, and use mathematics and desire mathematics experiences that center visualization and multiple approaches, consistent and authentic support, and a safe space for exploration. To support their development of a more robust MI and VMT, we aim to leverage these desires in our future design to perturb their focus on compliance over student- and discourse-focused exploratory mathematics teaching. While disheartened to be reminded of the myriad of ways Black learners are often negatively positioned and treated in mathematics learning spaces, we are taking these findings forward in our design to explore ways we might use the constructs of flourishing, humanity, and brilliance alongside justice-forward mathematics learning experiences that center the power of representation of Black liberatory mathematics and mathematicians.

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MATHEMATICS & IDENTITY: LIVED EXPERIENCES OF PROSPECTIVE TEACHERS OF COLOR

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We explore the lived experiences and identities of prospective Teachers of Color and how they influence perceptions of themselves as doers and teachers of mathematics. Using Sfard and Prusak's (2005) conceptualization of identity as a narrative, our goal to amplify the voices of our participants is supported by sharing visuals and quotations directly from data sources. Preliminary findings suggest opportunities for reflection and discussions about lived experiences and identity support prospective teachers in thinking about how they can make mathematics a humanizing and meaningful experience for their future students.

Introduction and Literature Review

Prospective teachers (PTs) have unique identities, lived experiences, and beliefs about mathematics, influencing their decisions to become educators. Not only do these experiences impact future career choices, but they also play a role in how elementary school teachers perceive themselves as teachers of mathematics. Gee (2000) defined *identity* as "the 'kind of person' one is recognized as 'being,' at a given time and place, it can change from moment to moment in the interaction, can change from context to context" (p. 99). In considering elementary school contexts, we must acknowledge that teachers and students come to school with lived experiences and identities too often ignored or made invisible, especially in mathematics classrooms.

Berry (2021) illuminated teachers' limited understanding of the identities and lived experiences of historically excluded learners and reminded us that "teachers must know and understand learners' identities, experiences, and cultural contexts and consider using them to connect students meaningfully with mathematics" (p. 163). For students to develop positive and meaningful relationships with mathematics at a foundational point in their educational trajectory, teachers must find ways to bring their lived experiences and identities into everyday mathematics learning experiences. This work supports the call to make space for PTs to engage in critical conversations and activities supporting teaching mathematics with a sociopolitical lens (Gutiérrez, 2013). We argue that for teachers to know their students as doers of mathematics and make school mathematics a more humanizing experience, PTs must know themselves and reflect on the experiences that have shaped their mathematics identities.

Identity in educational research is conceptualized and studied in various ways (Aguirre et al., 2013; Gee, 2000; Lutovak & Kaasila, 2013; Martin, 2009; Sfard & Prusak, 2005). In defining mathematics identity, Martin (2009) shares how it refers to "the dispositions and deeply held beliefs that individuals develop about their ability to participate and perform effectively in mathematical contexts and to use mathematics to change the conditions of their lives" (p. 326). Previous experiences with mathematics influence one's relationship with the subject. It is essential to consider multiple facets of students' identities, such as race, language, and gender, when considering ways to support students as doers of mathematics in school contexts.

The design of this course and study created space for four participants, all of whom identify as Women of Color, to share their identities in ways they have not previously experienced in a mathematics class. The following research questions guide this work: (1) How do lived experiences of Women of Color influence how they view themselves as prospective elementary mathematics teachers? (2) How have prospective teachers' mathematics identities and perceptions of themselves as doers and teachers of mathematics evolved?

Theoretical Framework

Multiple facets of identity, including race, family, culture, gender, sexuality, and language, influence mathematics identity development. Engaging in critical reflection through storytelling about lived experiences can support PTs in connecting multiple identities (Aguirre et al., 2013), mathematics identity, and professional identity. Sfard and Prusak (2005) equate identity with narrative, meaning one's identity is shared through the act of storytelling. We are using the notion of *identity as a narrative* (Sfard & Prusak, 2005) as a theoretical framework for this study, aiming to center the voices of our participants through the stories they share.

In thinking about the decisions PTs make throughout their lifetime, we agree with Sfard and Prusak's (2005) conceptualization that "identity talk *makes us able to cope with new situations in terms of our past experiences and gives as tools to plan for the future*" (p. 16, emphasis in original). Sharing stories about their past experiences and future plans in multiple settings provided space for PTs to talk about their identities, lived experiences, and beliefs about mathematics and to consider how they work together to influence their decision to become teachers. Through these conversations and assignments, PTs also had opportunities to consider the mathematics teacher they hope to be for their future students and how their prior experiences and identities have impacted their perceptions of themselves as teachers.

Methods

Context and Data Collection

In the Spring 2022 semester, 16 students were enrolled in an Elementary Mathematics Methods course at a large public university in central Texas. This portion of the study explores PT's understanding of how they can use their lived experiences and identities to challenge traditional ways of teaching and learning mathematics. Out of the 16 students enrolled in the course, seven volunteered to participate in the study. This work focuses on four of the seven participants, Andrea, Maisy, Mia, and Simone (all pseudonyms), who are navigating a teacher education program. We selected Andrea as the representative case because of the unique and multimodal ways she presented her story through class assignments. Additionally, her story conveyed similar themes to the stories Maisy, Mia, and Simone shared.

Primary data came from one semi-structured interview during the Spring 2022 semester and multiple class assignments acting as artifacts. These assignments were flexible, and students were encouraged to create something meaningful for their teaching journey. For example, at the beginning of the semester (January 2022), students wrote a Mathematics Experiences Reflection (MER), through which they shared their experiences as mathematics learners and reflected on how these experiences shaped their attitudes towards mathematics. A second assignment, the Mathematics Autobiography, was completed at the end of the semester (May 2022) and asked students to share a video-recorded reflection. Responses from the MER supported us in answering the first research question, while responses from the autobiography video helped answer the second research question. Andrea chose creative ways of responding to both class assignments, including her use of poetry and illustrations.

Preliminary Findings

Following Sfard and Prusak's (2005) articulation of identity as a narrative, we share Andrea's multiple identities and lived experiences through storytelling as the representative case for this paper. We include excerpts from her interview, MER, and autobiography; and illustrations directly from her autobiography video to center Andrea's voice, lived experiences, and identity (see Figures 1-6). Challenging the notion of telling Andrea's story from our perspective as researchers, we aim to amplify her voice and her journey to becoming an elementary school teacher.

Meet Andrea

Andrea is a college student who grew up in southern Texas with her mom, dad, and two younger sisters. She identifies as Hispanic and refers to her primary language practices as

"Spanglish." In her transition from her hometown to college, she shared her excitement about being surrounded by new cultures, languages, and experiences, pointing out that she went from being considered part of "the majority" to now one of "the minority." When asked to tell us about herself and her identity, she spoke of her hometown, family structure, and love of trying new foods. Being the oldest of three siblings played a significant role in her finding her way toward becoming an elementary school teacher, primarily because both of her parents worked full-time, and she acted as a caregiver for her sisters. She spoke briefly about her parents' careers and how they influenced her enjoyment of mathematics while admitting they were not home often to see the mathematics she was learning at school.

Figures 1 and 2: Excerpts from Andrea's interview and drawings from Andrea's autobiography video

Decision to Become a Teacher

In interviewing Andrea, we learned she did not fully admit to herself or her family and friends that she wanted to pursue teaching as a future career until she was already in college. The COVID-19 pandemic was a significant turning point in her decision to switch to the teaching program. Andrea's parents pressured her to attend college and choose a major that would lead to a financially stable career, such as a doctor or lawyer. Although she began her college journey as a biology major, she subconsciously knew she was meant to be a teacher and slowly progressed through various majors in her college journey.

She spoke about how her parents felt when she shared that she had switched her major to Early Childhood Education. Because of her family's experiences of struggling financially throughout her childhood, money was a leading reason why her parents expected her to choose a major that would lead to a STEM career. She went on to share that although they were upset, they were not completely surprised by her decision. Andrea's family conceptualized this opportunity to "break out" of her hometown and attend college to support herself financially and worried she was wasting her opportunity to be an elementary school teacher. Their changing realization came from Andrea's parents seeing how her demeanor improved and how happy she was on this new pathway.

"I was a bio major. I work hard. I'm smart. I can do several things. My dad says, you know, you can do whatever you want. They always wanted me to be something that would make a lot of money financially because we always struggled. So that was a doctor, lawyer, you know, the big things that pay a lot of money. So that's why I came in bio and then very quickly I was like, mmmm. I'm doing okay in the classes, but I just wasn't in it." *(Andrea, Interview 1, April 2022)*

"I've always wanted to be a teacher. I just never said it full chest. Cause you know, financials, the politics of it all, like it's not something you go into lightly and especially if you want a family down the line. If that's something you wanna do. Do you wanna travel? It's not something that you're like, oh yeah, a teacher can financially support me. So that's something that had always weighed on me." *(Andrea, Interview 1, April 2022)*

"So very small amounts of us, like breakout… Very few of us break out of that. And then to just become a teacher was like, why aren't you doing more? Why come out all the way over here just to become a teacher? But you get a whole other life over here, the perspectives of everyone and you see so many different types of cultures and everything. Now they're into it. They support me. They know I'm happy." *(Andrea, Interview 1, April 2022)*

Figures 3 and 4: Excerpts from Andrea's interview and drawings from Andrea's autobiography video

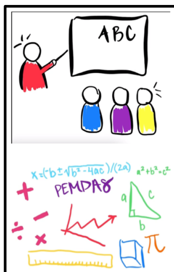
Mathematics Experiences and Perception of Herself as a Mathematics Teacher

In talking specifically about her childhood mathematics experiences, Andrea explained how she has always loved mathematics, highlighting some of her teachers as role models for her enjoyment and success with the subject. Throughout this mathematics methods course, she understood that not all students love the subject, and many will come into her class feeling dread and unease. She acknowledged that her role as a mathematics teacher is to help students see mathematical connections outside the classroom. When asked why math is her favorite subject, Andrea responded,

> There's no one answer. You can answer it a bunch of different ways. And I think, as a teacher, I know that. And so, I would go about teaching it in several ways. Growing up, they always taught one way, and I struggled…it was always like, we're gonna grade you off of this. You need to do it our way. And so, I had to come up with a different way that worked for me in order for me to understand it, and then I could do it their way. (Interview, April 2022)

Andrea discovered what worked for her as a doer of mathematics but acknowledges that many students may not have this realization on their own. As a result, she wants to be a teacher who makes mathematics meaningful for her students.

Andrea also spoke of how various facets of her students' identities, such as race and language, will significantly influence how they engage with and participate in math. Through these conversations, she shared the importance of making her classroom a space where students want to participate and share their voices. However, Andrea admitted that this had not been at the forefront of her mind until taking this course.



"I actually really like math! I've had some amazing teachers in my past that really cared more about you as the student than the material. They made an environment that put me first and made me want to learn...I realize my experience with math has made me biased. I understand that others may have a harder time with it and won't like it. Because of this, I plan on having math throughout my class and try to make it feel less like math if that makes sense. I want them to grow into it to be able to see different approaches to see the way numbers can help us when they actually are used and how they can hold a lot of information. I want them to see the connection to the real world." *(Andrea, Math Experiences Reflection, January 2022)*

We are two girls that look very different
I am Hispanic and she is white
Our math experiences do not intersect
But try to understand each other, we might

Carpet time and exploring
Something I had never seen
Infinite freedom and self-expression
Was only just a dream

One language only heard
One type of person present
Not knowing and seeing others
That is not pleasant

(Andrea, Oral History Interview Reflection, April 2022)

"As I go through this, as I become a teacher, I want no student to be quiet in my class, you know? Of course I was always gonna create a safe environment. I was always gonna let my students be who they are. But it's never been the forefront of my mind, like wow. The Students of Color that I have, I need to make an effort to make sure that they are heard every day in my space. Like, you know, I've never, it was never at the forefront of my mind before." *(Andrea, Interview 1, April 2022)*

Figures 5 and 6: Excerpts from Andrea's interview, autobiography video, and a class assignment

Discussion and Conclusion

From our data, a narrative is portrayed of Andrea's journey toward becoming a PT and practicing teacher. Andrea was raised in an environment of similarly identifying people where she considered herself the "majority." During this time, her college and career goals were oriented toward community and familial expectations. She felt she needed to pursue a career that is well known to be prestigious and lucrative (i.e., physician, lawyer). Once Andrea began attending her university, she started to identify as the "minority" as she surrounded herself with people from varying cultures and backgrounds. Andrea shares that, due to the COVID-19 pandemic, she changed her college major four times (see Figure 3) and ultimately decided to study Early Childhood Education. The pandemic became a turning point that allowed Andrea to put herself first and follow her dream of becoming an elementary school teacher. She has even considered what else she can pursue once she graduates that will allow her to support systemic changes, such as policy work or administration.

One's college major and career decision can be intimate and personal. When considering her future, Andrea went through some trials that challenged the ideas she thought she had about her future and ultimately led her to find her true career passion. Her lived experiences and multiple identities impacted the journey she took in deciding to become an elementary teacher. Andrea leans on her previous experiences when speaking about the teacher she hopes to become. Throughout the semester, she expanded her ideas of how to support students, especially those with marginalized identities, in the mathematics classroom. In her interview and autobiography, Andrea shared that this course has helped her establish new goals of amplifying the voices of Students of Color and ensuring they are taking up space in the mathematics classroom.

Each participant shared about their individualized journeys in the stories they told. Although there were fundamental differences, some significant themes connected the participants. All four PTs also shared various turning points or epiphanies (Denzin, 1989) that impacted their decision to pursue a career in elementary education. For example, Andrea spoke about how the COVID-19 pandemic pushed her toward finding a major that brought her happiness rather than pursuing the career path expected by her parents. However, family pressure played a prominent role in original decisions regarding career choice for all four participants. Some participants, such as Andrea, felt unsupported in deciding to become a teacher, while others had much more positive reactions from their family members. Either way, family members, friends, and communities impacted each of their journeys to becoming a teacher.

Additionally, all four participants articulated what they would like to bring from this course to their future mathematics classrooms. Many of them admitted they were surprised the mathematics methods course did not feel like a "normal math class," and they received many ideas and strategies about ways to connect with their students. Gutiérrez (2013) reminds us of the importance of adopting a sociopolitical lens to rethink "terms such as 'mathematics' [and] 'who is good at mathematics'" (p. 51). In leading a mathematics methods course with a sociopolitical lens, PTs were able to make connections to how they will challenge traditional ways of teaching, learning, and participating in mathematics, which, in turn, influenced how their mathematics identities evolved throughout the course. The autobiography video assignment allowed students to explain how their mathematics identities evolved.

This research aims to think more deeply about how teacher educators can design mathematics methods courses that provide opportunities for PTs to connect their and their students' lived experiences and identities to mathematics. "When we acknowledge a sociopolitical perspective on mathematics education, it raises questions about whether PTs are receiving the kinds of knowledge and skills they need" (Gutiérrez, 2017, p. 19). For teachers to connect meaningfully with their students, they must acknowledge and work toward learning about and genuinely understanding mathematics' sociopolitical nature. This study allowed us to learn about the varying lived experiences and the multiple identities of four Women of Color and how they plan to lean on and learn from their experiences to support all their students, no matter how they identify. "When individuals are seen as enacting their identities and actively negotiating schooling, we are able to view the mathematics classroom as more than a site for enculturation or social reproduction" (Gutiérrez, 2013, p. 51). Opportunities for reflection through storytelling (Sfard & Prusak, 2015) provided space for PTs to consider their own lived experiences and mathematics identities and how they intend to create a mathematics classroom that humanizes mathematics and makes their students' full and multiple identities visible.

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