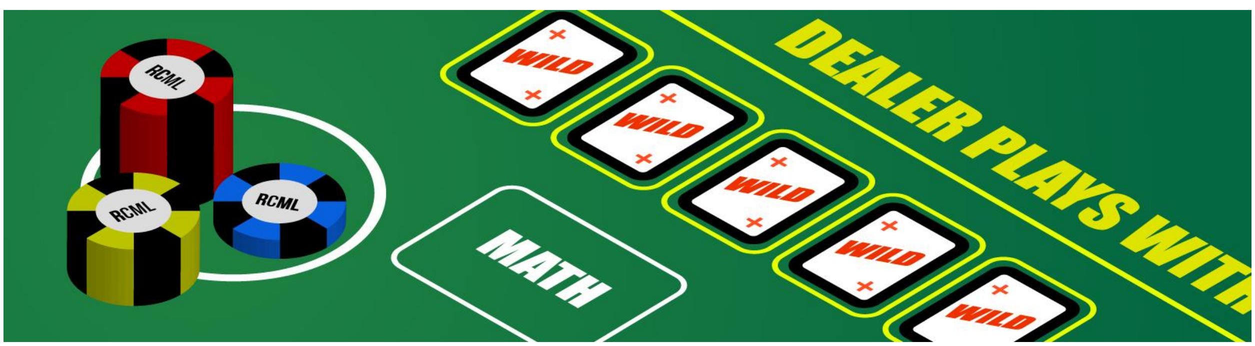
RCML

Proceedings for the 47th Annual Meeting of the Research Council on Mathematics Learning

Increasing the Odds for All Mathematics Learners



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RCML History

The Research Council on Mathematics Learning, formerly The Research Council for Diagnostic and Prescriptive Mathematics, grew from a seed planted at a 1974 national conference held at Kent State University. A need for an informational sharing structure in diagnostic, prescriptive, and remedial mathematics was identified by James W. Heddens. A group of invited professional educators convened to explore, discuss, and exchange ideas especially in regard to pupils having difficulty in learning mathematics. It was noted that there was considerable fragmentation and repetition of effort in research on learning deficiencies at all levels of student mathematical development. The discussions centered on how individuals could pool their talents, resources, and research efforts to help develop a body of knowledge. The intent was for teams of researchers to work together in collaborative research focused on solving student difficulties encountered in learning mathematics.

Specific areas identified were:

1. Synthesize innovative approaches.
2. Create insightful diagnostic instruments.
3. Create diagnostic techniques.
4. Develop new and interesting materials.
5. Examine research reporting strategies.

As a professional organization, the **Research Council on Mathematics Learning (RCML)** may be thought of as a vehicle to be used by its membership to accomplish specific goals. There is opportunity for everyone to actively participate in **RCML**. Indeed, such participation is mandatory if **RCML** is to continue to provide a forum for exploration, examination, and professional growth for mathematics educators at all levels.

The Founding Members of the Council are those individuals that presented papers at one of the first three National Remedial Mathematics Conferences held at Kent State University in 1974, 1975, and 1976.

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DESIGNING FOR A STRUCTURED SMALL GROUP MATHEMATICS LEARNING ENVIRONMENT

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This study of secondary mathematics classrooms illustrates the impact curricular supports can have on students' connections to the math task, peer-to-peer discourse, and social dynamics in small groups. Naturalistic observations of small group work provide illustrative examples of limited group functioning while working on cognitively demanding math tasks and the potential that curricular supports have in strengthening group functioning when added to the same tasks.

Small group learning environments promote opportunities for conceptual learning and powerful mathematical work (e.g., Mercer, 2005). Effective group work depends on tasks that are cognitively challenging for group efforts, discourse that promotes engagement and meaning-making, and peer relational processes that support collaboration (Cohen, 1994). Facilitating small group work is complex for teachers because their presence with any one group is intermittent while multiple groups work simultaneously and independently. The Peers Engaged as Resources for Learning (PEARL) project studies small group learning environments in secondary mathematics classrooms. In the first four phases of PEARL, we worked with teachers and students to identify successes and challenges they encountered during small group instruction. During the fifth phase, we worked with teachers to test embedded curricular supports designed to promote small groups' engagement with the demands of tasks, productive discourse, and positive social dynamics. Teachers first implemented a set of lessons where students worked in small groups without the PEARL curricular supports. The teachers then attended a summer workshop that introduced the supports and features of effective small group work. During the next school year, following refresher sessions for each lesson, they implemented the revised lessons with their students. The study described in this paper focuses on the role the curricular supports played in strengthening small group work in secondary mathematics classrooms.

Theoretical Framework

We conceptualize the small group learning environment as comprising three major elements: the mathematics task (Stein, Smith, Henningsen, & Silver, 2000), the discourse related to the mathematics content (Sztajn, Heck, & Malzahn, in press), and the peer social dynamics among the group members (Hamm & Hoffman, 2016). Effective group work depends on tasks that coordinate these elements synergistically (Heck & Hamm, 2016). This study addressed the question: What is the potential of curricular supports to address students' needs to engage productively with the demands of tasks, discourse, and social dynamics in small group work?

We employ the Task Analysis Guide (Stein et al., 2000) to characterize the cognitive demand of a task. The structure and expectations of a written task denotes a set of demands for what students are intended to do. A teacher's implementation may convey demands beyond or instead of those in the written task. In the hands of small groups of students, these expectations may be modified further as the students decide what their work entails.

Tasks that demand conceptual mathematical thinking must be matched by discourse that supports a focus on mathematical ideas and reasoning. The Mathematics Discourse Matrix (Sztajn et al., in press) specifies four key dimensions of mathematics discourse: Explaining, Questioning, Listening, and using Modes of Communication. Variations in teacher and student actions within these four dimensions are tied to four types of discourse—Correcting, Eliciting, Probing, and Responsive—each serving different purposes for instruction and learning.

We view peer social dynamics through the lens of Peer Cultures of Effort and Achievement which reflect the activities, routines, and norms that students develop in interaction with one another, that communicate the acceptability, desirability, and value of effort and achievement (Hamm, Hoffman, & Farmer, 2012). Teachers can influence peer cultures by setting and supporting expectations for social behaviors, scaffolding the kinds of social interactions they would like students to have with classmates (Farmer, Lines, & Hamm, 2011).

Methods

Four lessons were selected from The Math Resource for Instruction for North Carolina Math 1 (2017), created by the state's Department of Public Instruction. The lessons originated from two internationally available sources (illustrativemathematics.org and the Math Assessment Project at map.mathshell.org). To promote student engagement with the demands of the tasks, productive mathematics discourse, and positive social dynamics, we developed and embedded five curricular supports into student materials for the four selected lessons. Figure 1 shows how the supports were embedded into student handouts in a lesson titled Population and Food Supply.

Do this First (A) directs each student's attention to an initial access point, prompting them to begin work toward their individual contribution. Mathematically Meaningfully Roles (B) evenly assign the cognitively demanding aspects of the task and promote active communication.

Discourse Structures (C) are embedded into the lesson to explicitly indicate actions students should use to engage in productive discourse. Helping Prompts (D) are designed as just-in-time supports for students to seek help from and provide help to their peers, often attuned to common misconceptions about the content. A Group Product (E) ensures all students are accountable for individual contributions and promotes collective meaning making that draws them together.

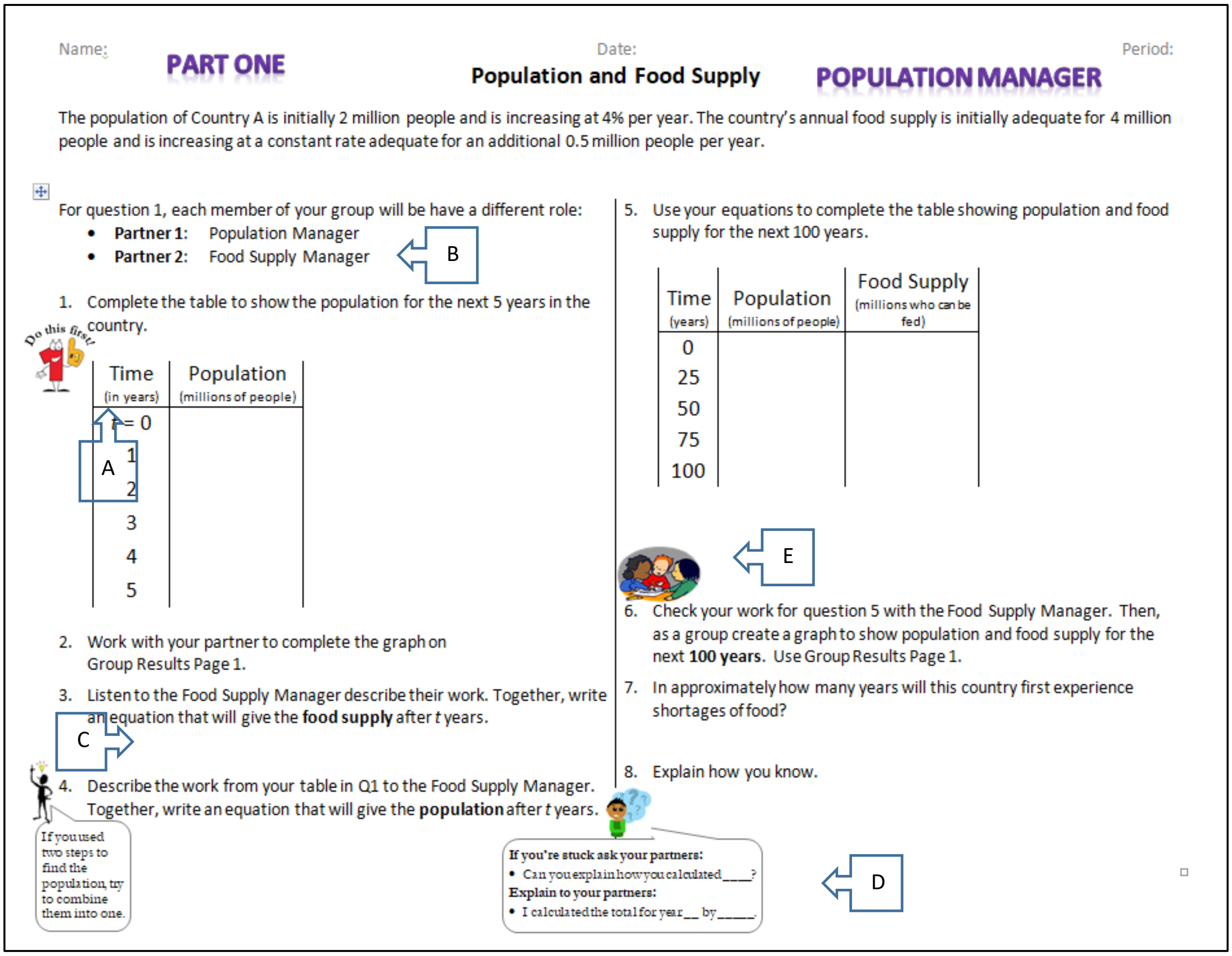

Figure 1. Embedded Curricular Supports for Small Group Work.

One 8th and two 9th grade teachers of High School Math 1 participated as volunteers. Data collection took place in two waves. The teachers first implemented unedited versions of the four lessons, with the only expectation that students work in small groups. Following a summer workshop to develop facility with the curricular supports, the teachers implemented updated versions of the lessons in comparable classrooms during the subsequent year.

We observed 17 lessons in the first wave and 14 in the second wave. The work of all consented small groups of students was audio recorded and observers took field notes, attending to non-audible evidence that would support analysis of audio recordings. Group sizes ranged from pairs to trios, with two to fourteen groups recorded per lesson.

For illustrative purposes, the Population and Food Supply lesson was selected to compare the small group environment in the same teachers' classrooms from the two waves of data collection. Using the three frameworks described above as a basis, one researcher (second author) reviewed the audio recordings from the first wave of data collection noting any evidence of students demonstrating a need for additional support to engage productively. Then, the researcher reviewed audio recordings from the same teacher's class working on the same lesson in the second wave of data collection, noting evidence of students' needs being met via the supports. All identified evidence was confirmed with the second researcher (first author).

Results

Throughout the PEARL study, we have identified challenges students face when working in small groups. They may have a hard time knowing how to get started meaningfully on tasks, how to ask for or provide help when necessary, how to manage multiple responsibilities, and how to have productive mathematical discussions. The examples that follow illustrate the potential of curricular supports to aid students in addressing these challenges.

Do This First

In wave 1, we saw evidence that students had a hard time beginning with the tasks. For example, one group of students indicated that they needed to get started multiple times, but it took them several minutes to move past just reading the question. They spent this time distracted with off-topic discussions until one told the group that they "need to make an equation and need to make another equation and need to do it fast." The students then worked to quickly interpret the context of the problem and write an equation for population increase.

The Do This First student support was designed to enable students' access to the task so each will immediately engage with the mathematics. As shown in Figure 1 (A), the Do This First support instructed one student to complete a table to show the population increase over 5 years. When teachers implemented the revised lesson, specific students were able to engage with the task right away by completing this table of values. The following example is illustrative:

S1: Population of Country A is initially 2 million people and they increase at 4% per year.

S2: Initially. Per year. So, zero is going to be 2 million because you start off with it.

S1: Okay.

S2: Just put 2 mil. And then for 1, so you increase by 0.4

S1: It went up by 0.4

S2: So 2 million – two zero zero zero zero zero zero times 0.4 …

Students complete the table for the population after t years, eventually correcting 0.4 to 0.04.

Mathematically Meaningful Roles

Students often struggle to coordinate mathematical demands of tasks and social demands of group work, and productivity suffers (Barron, 2003). Without guidance, many students resort to unproductive strategies, such as a divide-and-conquer approach without coordination or allowing one group member to do all of the work. For example, when students worked on the Population and Food Supply lesson in wave 1, we often heard students say things like "I can't do any of this, I'm just going to ask to go to the bathroom" and "Just tell me what to write for this one."

Strategically assigned roles can support individual engagement and collaboration. We provided two roles for pairs of students, one for population data (The Population Manager) and another for food supply data (The Food Supply Manager). In order to complete a graph and determine when there will be shortages of food, meaningful participation from both was required. In the example below, individual responsibilities ensured that both students had a role in completing the graph and prevented one student from taking over the other's responsibilities.

The Population Manager has graphed data and the Food Supply Manager is adding to it.

Food Supply Manager: We got to do them on the same graph don't we?

Population Manager: Yeah.

Food Supply: So numbers of years, 1.

Population: It's going to be a straight line, right?

Food Supply: Yeah.

Population: So 1 will be at 4 million.

Food Supply: No, you have to put it at 4.5.

Population: No no no you start –

Food Supply: Yes, 1 will be at 4.5.

Population: Hold on, wait wait wait.

Food Supply: Yes, because look, 1 is right there, 4.5 million.

Population: It's going to be in the half, about right here.

Both students continue to work together to make sure the data points are graphed precisely.

Discourse Structures

Teachers expect students to discuss their mathematical thinking with their groups. However, students often struggle to have discussions that go farther than just telling their partner an answer or recounting a procedure. For example, when teachers implemented the Population and Food Supply lesson that did not include any discourse structures, students were generally observed either not talking to one another at all, or only talking when they had a final answer to share.

The discourse structures embedded into the lesson gave students explicit instructions to promote discussions, both explaining their own work and listening to their peers' explanations. In this example, the Population Manager and the Food Supply Manager are instructed to describe how they developed an equation to represent their situation. These explicit instructions offered the students an opportunity to formalize and share their thinking in a socially productive form.

Food Supply Manager: Alright, so now I have to describe my work from my table and we have to come up with an equation. So I saw that the initial value was 4 million people and it's increasing 0.5 million per year. So I did a slope equation: f(x) = 0.5... for the 0.5 million people per year, and then my initial value was 4 million, so that's the y-intercept.

Population Manager: Yeah, I get that.

Food Supply: Alright, so put that in for number 3.

Population: Just write that?

Food Supply: Yeah, just write the equation that I wrote. And now you describe your work from your table and we'll determine what the equation should be.

Population: Okay, well, we start at 2 million and there's a 4% growth each year and this is a whole number of 1 because, you know, you have to keep the 2 million.

Food Supply: Right.

Population: And that's the 4% growth and this is the number of years.

Helping Prompts

We have observed that it is challenging for them to ask for help or provide help when misunderstandings surface. For instance, when students indicated that they were stuck on part of the Population and Food Supply lesson in wave 1, their group members might either not respond or respond by only telling them an answer, leaving misunderstandings unaddressed. As an example, in one group of three, two students were having trouble graphing and sought help, "I'm kind of lost. [Name], are you going to help us out?" The third student attempted to help, but was unable to do so effectively, resulting in the comment, “Here, write it out for me because you just confused me,” as she ended up taking their paper and graphing the data for her peers.

Helping Prompts were designed to guide seeking and providing help. They offer just-in-time support for students to use if they are stuck. In the Population and Food Supply lesson in wave 2, students were more likely to ask their partners specific questions to help resolve their misunderstandings and partners were more likely to provide assistance that helped them learn rather than prioritizing final answers without explanation. For example, students asked questions like “How do you know?” and “Linear means that it’s going to be a straight line, right?” and students provided explanations like “The food will run out between years 75 and 80 because I know that when the lines cross there will be more people than food.”

Group Product

When working in small groups on a cognitively demanding task, students often struggle to bring all of their work together to make sense of it. For example, in the Population and Food Supply lesson, students observed in the first wave of data collection might type two equations into a calculator, find the intersection point, and finish the task without sharing their ideas or making sense of their work in a more meaningful way.

Including a group product in a lesson promotes shared effort and meaning making that draws on individual contributions. Each student is responsible for answering their own questions, but they also need to collaborate with their partners to create a group product and make sense of the work that was done individually. In the example below, two students were examining population and food data graphed together. They concluded it would be many years before a food shortage.

Food Supply Manager: I mean, you see yours, they’re like 2.08 or …

Population Manager: They don’t even go above 2.5!

Food Supply: Oh gosh, yeah. They’re all going to be very close.

Population: It’s just going to have to look the way it is.

Food Supply: It’s fine. Yeah, I mean, yours are all very close and mine are all spread out.

Population: Thankfully, it doesn’t look like we’re going to run out of food any time soon.

Conclusion

Comparisons between the two small group work environments illustrate the potential of the curricular supports to address specific needs and strengthen group functioning. The purposively selected case analysis illustrated students’ need for and use of supports when they encountered challenges related to the three major elements of the small group learning environment: mathematics task demands, productive discourse, and positive peer social dynamics. The student curricular supports that were embedded into the lesson mutually reinforced the coordination of the different elements of small group work and show potential for helping students improve their group functioning. In our ongoing work, a complete and systematic analysis of the small group environments in both waves of the study, using time sample coding and sentiment analysis related to the three frameworks, will further establish how and how well the curricular supports and other factors affect the functioning of the small group learning environment.

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GIRLS' CONSTRUCTION OF MATHEMATICAL DISCOURSE IN SINGLE-SEX CLASSROOM ENVRIONMENTS

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This study seeks to examine and illuminate the ways in which girls in middle grades single-sex mathematics classrooms construct mathematical Discourses. We implement a holistic case study design by providing an in-depth description of how girls engage in constructing mathematical Discourses both with other students and their teacher.

Background

Single-sex education offerings increased since the change in regulations to Title IX in the United States (Federal Register, 2006), and single-sex public education options recently emerged (Carter, Kombe, & Che, 2014). Because of this recency, research on single-sex public classrooms in the U.S. is limited (Bracey, 2006). Scholars have demonstrated the lack of evidence to support single-sex education (Mael, Smith, Alonso, Rogers, & Gibson, 2004; Smyth, 2010); specifically in mathematics education (Carter et al., 2014; Che, Wiegert, & Threlkeld, 2012). There are gaps in our understanding of the affordances and drawbacks of these classroom spaces. It is imperative we continue to research how these spaces impact and affect student learning because of our limited understanding in how students experience and learn in single-sex mathematics classrooms.

Following the influential work of Fennema (1974), it is imperative we facilitate success for all students in mathematics. All students have the ability to be successful in mathematics and it is our job to facilitate mathematics learning that allows students to be successful, whatever that success means for each student. Our study seeks to contribute to our understandings of the affordances and drawbacks of single-sex classrooms, by examining the processes in which girls construct mathematical Discourses (Moschkovich, 2003) in single-sex mathematics spaces. To explore and further understand these classroom spaces, the following questions guide this study:

1. In what ways do girls in middle grades mathematics classes engage in constructing mathematical Discourses in single-sex environments?
2. What forms of positioning do girls utilize as they construct mathematical Discourses with other students in their classroom?

3. What forms of positioning do girls utilize as they construct mathematical Discourses with their teacher in their classroom?

Theoretical Framework

We focus on the construction of mathematical Discourses (Moschkovich, 2003) for girls in single-sex environments. We follow Gee's (1999) definition of Discourse because language alone will not be sufficient for understanding how mathematical Discourses are constructed. Moschkovich (2003) points to Gee's definition of Discourse (Gee, 1999) and builds on the definition to define mathematical Discourse:

> Mathematical Discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view. Participating in mathematical discourse practices can be understood in general as talking and acting in the ways that mathematically competent people talk and act when talking about mathematics (Moschkovich, 2003, p. 3-326).

Participating in mathematical discussion is an important aspect of mathematical discourse in the classroom (Moschkovich, 1999). Students participate in mathematical discussions according to Brenner (1994) by "making conjectures, presenting explanations, constructing arguments, etc. about mathematical objects, with mathematical content, and towards a mathematical point" (as cited in Moschkovich, 1999, p. 12). Specifically, we use positioning theory (as outlined by Herbal-Eisenmann, Wagner, Johnson, Suh, & Figueras, 2015) as a lens to narrow on the interactions and relationships occurring in single-sex classrooms. In this study, "positioning" correlates to the discursive processes where individuals are in conversations "as observably and subjectively coherent participants in jointly produced story-lines" (Davies & Harre, 1990, p. 48) and positioning as a metaphor represents relationships (Harre & van Langenhove, 1999; as cited in Herbel-Eisenmann & Wagner, 2010). Positioning theory guides this study as a way to narrow in on the interactions and relationships that occur in single-sex classrooms. Drawing on these theoretical frames informing our research questions, we recognize classrooms are spaces which encompass students and teachers of all backgrounds, a variety of Discourses, and numerous forms of positioning.

Methods

To gain comprehensive and different perspectives of how girls construct mathematical Discourses, a holistic case study design was implemented to investigate mathematical Discourses girls construct in a middle grades single-sex classroom. The case includes all data gathered involving mathematical Discourses constructed in a middle grades single-sex classroom.

Data for this study were drawn from audio-visual recordings in an all-girls middle grades mathematics classroom. The teacher was a middle-aged white male. Data were collected and analyzed based on interactions and mathematical Discourses which took place in the classroom setting. The goal of the collected data was to provide a descriptive view of the classroom space and student experiences.

Participants

In selecting participants, we looked for the extent to which students exhibited willingness to interact with other students, engage in conversation with the teacher and their peers, and write and participate in their mathematics work. In this paper, we focus only on students in a single-sex classroom. Below we provide a description of each single-sex participant and their rationale for selection.

Jasmine. Jasmine often engaged in conversation with her teacher by responding to questions posed by the teacher and speaking when selected to share her mathematical thinking. Jasmine raised her hand to verbally ask the teacher questions about her mathematics work both publicly and privately. Additionally, Jasmine volunteered to demonstrate her written mathematical work on the board for her classmates and teacher to observe. Jasmine often wrote and participated in her mathematics work. Jasmine's engagement in all forms of mathematical Discourse (Moschkovich, 2003) throughout the entirety of the lesson was the importance for her selection and she represented one end of the participation spectrum. Jasmine participated in mathematical discussion (Moschkovich, 1999) and was an active participant in the classroom. Jasmine engaged in written, spoken, acted, listening, and interacted (Gee, 1999) mathematical Discourses (Moschkovich, 2003).

Zara. Zara was reserved verbally in the beginning and did not engage in conversation with the teacher. Zara did write and participate in her mathematics work; however, this was the norm for all students in this particular classroom. During one particular lesson Zara was called on unexpectedly by the teacher and she was not prepared to respond to his mathematical question. A shift occurred after this interaction, in which Zara became more vocal and interactive with her teacher. The transition of Zara's engagement was the importance for her selection. Although

Zara engaged in conversation with her teacher after being called on, Zara was not an active participant in the classroom.

Analysis of Data

For the purposes of this paper, we analyzed a case from a middle grades single-sex mathematics classroom. In analyzing the constructed discourses, we focus on mathematical Discourses constructed by girls and the interactions and actions which occur around the construction of those mathematical Discourses. Further, we look for different positions and/or storylines occurring in interactions. We labeled and identified the types of positionings, storylines, and/or communication acts that occurred (Herbel-Eisenmann et al., 2015).

We collected video recordings of three different lessons which we transcribed to include the participants' interactions, actions, gestures, writing, and talk with other students and the teacher. We coded all transcriptions for Jasmine and developed a list of codes. We then used these codes to code Zara's transcriptions. Through coding the transcriptions for Zara, additional codes emerged. We used the updated list of codes to recode Jasmine's transcriptions. We generated a separate list of codes for the teacher. We transcribed all talk, actions, interactions, and gestures of the teacher in each lesson. We only coded for the teacher when he was interacting, talking, or gesturing towards the specific participants in our study. After coding all participant transcriptions, we looked for common themes that emerged. Examples of emergent themes include students being positioned and positioning themselves as active academic students, and students positioning themselves as passive students.

Findings

Jasmine. Jasmine was eager to share her mathematical work and contribute to mathematical discussions publicly for her classmates and teacher. For instance, Jasmine volunteered at every opportunity to demonstrate and describe her mathematical work for the class to observe. The teacher asked for volunteers to publicly share work during two different lessons, and during both Jasmine volunteered, and was selected to show her work. Additionally, Jasmine was attentive to her teacher throughout the lessons. She often gestured she was paying attention by taking notes and looking up to the board when the teacher was talking and writing. Jasmine responded both chorally and individually to questions the teacher asked throughout the lessons, and she was willing to ask her own questions when unsure of the mathematics or needed to clarify her understanding. Jasmine demonstrated her attentiveness to her teacher by gesturing in agreement or disagreement to questions he posed. When the teacher asked students to raise their hand once they figured out the solution to an answer, Jasmine consistently raised her hand. Jasmine also volunteered to respond when the teacher asked for students to share their answers. Jasmine was active in a variety of ways in class and frequently engaged in constructing a mathematical Discourse (Moschkovich, 2003).

Jasmine's actions made apparent that she was actively willing to share her mathematical work and to engage in mathematical conversation with her teacher. For instance, when the teacher asked the class "What is a complementary event?", Jasmine raised her hand and was selected to respond. The following conversation took place between Jasmine and the teacher:

Jasmine: The opposite? *Talking for the whole class to hear, while also moving things around in her binder, and sitting at her desk.*

Teacher: The opposite of what? *Walks to stand in the center front of classroom, hands in pockets.* You are right on track.

Jasmine: Of what you don't have. *Puts notebook on the floor next to her desk and arranging her notes on her desk while talking.*

Teacher: Like give me an example. *Leaning against the board and standing at the front of the classroom.*

Jasmine: Like if you have, like, a chance of grabbing 1/4th the opposite of 3/4ths so that like if I grab one marble and it's red, orange, or blue. Whatever. But um, if I find the probability of that 1/4th but what I am not worried about is the compliment. *Hands tucked under legs and leaning forward.*

Teacher: Right. So the probability of the complement event is the opposite or what is not currently. So like you were saying, let's say I have a spinner A, B, C, and D. The probability of, it has 4 sections, the probability of spinning an A would be 1/4th. The probability of not spinning an A would be 3/4ths right?

Jasmine: *Nods her head in agreement.*

The teacher continued to elicit Jasmine's thinking by posing additional questions for Jasmine to answer. Additionally, Jasmine's teacher often selected her to share her answer even though many students may be attempting to volunteer a response. Jasmine's teacher also used Jasmine's responses at times as positive contrasts to responses from other students in the class, so Jasmine's teacher thus positioned Jasmine as a capable mathematics student. Through both

Jasmine's and her teacher's discourses, positions, and storylines, Jasmine is occupying a role of a capable mathematics student with status (Cohen & Lotan, 1995) in this classroom.

Zara. Although Zara often gestured she was paying attention to the teacher by taking notes and looking to the board when the teacher was talking and writing, she rarely responded to questions posed by the teacher and did not raise her hand to volunteer to share her mathematical work. When her teacher posed questions for students to respond verbally in chorus, Zara often looked down and wrote in her notebook rather than speaking. Zara does not respond to many opportunities to actively engage in public mathematical Discourses.

Zara's teacher attempted interacting with Zara, in part, by calling on her to share responses when Zara had not volunteered to do so. In the exchange below, the teacher posed a question to Zara without prior warning:

> Zara: *Zara leans over her desk and looks at her notebook.* Ummm… *Zara continues to look through her notebook flipping the pages back and forth.*
>
> Teacher: When I combine a complimentary event and its probability what should that equal?
>
> Zara: *Looking back down at her notebook, flips the pages of her notebook, sits on her hands, and changes her body position in her seat.* I don't know.
>
> Teacher: Yes you do. It's in there. Keep looking.
>
> Zara: *Looks back down at her notebook, flips the pages of her notebook again.* You get one-whole.
>
> Teacher: One-whole. *Nods in agreement with Zara.* So just like **Jasmine** was saying, 1/4th and 3/4ths, that is one-whole.

As this exchange ends, the teacher gives credit to Jasmine for Zara's response and positions Zara as a student who had not been attentive to the previous instruction. After this interaction, Zara's engagement started to shift and she became more vocal publicly in the classroom.

Following the previous exchange, Zara raised her hand several times to be called on and other times she spontaneously shouted out when she had questions. Zara pushed back on her perceived positioning by the teacher, a second order positioning (Harré & van Langenhove, 1991). For instance, when the teacher addressed the question of another student, Zara publicly shouted the following without raising her hand:

> Zara: Wait so how, I don't get that either. *Shouts this out to teacher, but publicly for the entire class to hear.*

Teacher: So this little thingy means everything that is not 7. So everything that is not 7. So, all these are not 7. Or I can add all these up. And then all these are not 7 so I could add up those. And there are 30 of those. Because I know that 6 of them are 7.

Zara: *Looking at teacher as he talks.* Oh. So you just take 7 and just get rid of all the 7's out of the whole box? Okay. *Then looks back down at notebook.*

Zara's active engagement picked up following the earlier teacher exchange as Zara continued to frequently shout out spontaneous questions. Additionally, Zara began to raise her hand to indicate her willingness to respond to teacher questions. For one such teacher question, Zara was initially the sole volunteer, but her teacher never called on her to respond. Thus, although Zara began as a more passive student, her class engagement transitioned during the class. However, her teacher passed up opportunities, such as selecting her from volunteering students, to position her as a capable student with status (Cohen & Lotan, 1995).

Discussion and Implications

In seeking to contribute to our understandings of the affordances and drawbacks of single-sex classroom spaces, our findings suggest that girls in middle grades mathematics single-sex classrooms construct mathematical Discourse through engaging in written, spoken, acted, listening, and interacted (Gee, 1999) mathematical Discourses (Moschkovich, 2003). Some students were positioned by the teacher and positioned themselves as active academic students, while others as passive students.

Findings from this study are similar to other single-sex mathematics education studies (Che et al., 2012; Carter et al., 2014; Kombe, Che, Carter, & Bridges, 2016) which suggest further exploration of these classroom spaces, and in some cases no differences in evidence to support single-sex education versus coeducation. Just as in any classroom, mathematical Discourses varied and were constructed in a different ways. Findings from this study suggest these classroom settings do not seem to influence the ways students are constructing mathematical Discourse.

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INVESTIGATING A STEM CIRCLE APPROACH WITH MULTILINGUAL STUDENTS AND FAMILIES

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STEM Circles allow small groups of students to tackle an open-ended problem that has multiple possible approaches and no immediate solution, while allowing students to explore concepts before mastering specific vocabulary and principles. STEM Circles mesh well with research on teaching mathematics to emergent multilingual students because of their focus on inquiry-based interaction and STEM literacy. We examined the potential for our STEM Circle approach to incorporate content-area vocabulary while helping students learn interdisciplinary science, technology, engineering, and mathematics content. Are STEM Circles also effective for a family engagement program with multilingual students and their parents?

During the fall of 2016, 4.9 million public school students were classified as English language learners by the National Center for Education Statistics (McFarland et al., 2019). As multilingual students become a larger proportion of the U.S. student body, multilingualism in the content areas has received increasing attention (Buxton & Lee, 2014). Emergent multilingual students possess knowledge of multiple languages and language registers as well as meaning-making modalities that facilitate their participation in STEM contexts and activities—that is, the integration of science, technology, engineering, and mathematics (National Academies of Science, Engineering, & Medicine, 2018), including a complex mathematics discourse which involves multisemiotic reasoning (Hansen-Thomas & Bright, 2019). Yet despite the contributions emergent multilingual students bring to mathematics classrooms, many teachers feel ill-prepared and inadequately trained to help students develop English language proficiency while teaching mathematics content (Hoffman & Zollman, 2016).

STEM Circles are one instructional approach mathematics teachers can use to help students develop English language proficiency and benefit all students in developing academic language, conceptual understanding, and meaningful skills. As a classroom activity, STEM Circles employ active learning and can serve challenging curriculum, multiple learning approaches, and an inclusive school environment (Suh, Hoffman, Hughes, & Zollman, in press).

This study explored the potential of STEM Circles for engaging elementary-level emergent multilingual students and their families in STEM exploration in an informal learning environment. The study was guided by the following research question: How do STEM Circles promote multilingual family engagement at the elementary level?

Literature Review

Understanding content area discourse is essential for learning and engagement with complex STEM topics (Reyes, 2008). Academic discourse becomes most accessible when connected to social language (Ryoo, 2015). Inquiry-based STEM instruction has proven effective with emergent multilingual students in numerous studies (Lee & Buxton, 2013; Mercuri & Mercuri, 2019; Stoddart, Pinal, Latzke, & Canaday, 2002).

Research suggests learning is constructed through active discovery (Bruner, 1966; Deering et al., 2016; Sansone, 2018) and is most effective when scaffolded within constructive sociocultural contexts (Vygotsky, 1978). A hands-on, problem-solving activity with reflection invites students, families, and educators to participate in academic and STEM Discourses (Gee & Hays, 2011). Providing hands-on classroom opportunities and expressing STEM academic concepts in multiple ways are positively associated with English language learning and also are recommended practices for mainstream classrooms regardless of students' language proficiency (Hoffman & Zollman, 2016).

STEM Circles are planned classroom tasks in which small groups of students tackle an open-ended problem that has multiple possible approaches and no immediate solution (Suh et al., in press). Our STEM Circles approach is derived from Math Circles (Kennedy & Smolinsky, 2016). STEM Circles' objectives and procedures mesh well with research-support practices for teaching science and mathematics to emergent multilingual student because of the Circle's focus on inquiry-based interaction and STEM literacy (Hoffman & Zollman, 2016; Moje, 2015; Zollman, 2012). STEM Circles follow a learning cycle that allows students to explore concepts before mastering specific vocabulary and principles (Lawson, 2009).

Although all students can benefit from increased parental involvement, family engagement efforts are especially necessary for supporting multilingual students to connect their home and school knowledge given the linguistic and cultural differences which can exist between these two spaces (Shanahan & Echevarria, n.d.; Tarasawa & Waggoner, 2015). Strategic school-family-community partnerships have been linked to increased academic achievement and positive attitudes towards school, among other advantages (Philadelphia Education Research Consortium, 2016). Family engagement is critical for incorporating academic language into conversations in the home language and supporting students' academic English skills (Philadelphia Education Research Consortium, 2016; Shanahan, & Echevarria, n.d.). To successfully promote meaningful family engagement, STEM Circles must include culturally and linguistically appropriate practices.

Methodology

This study utilized a multiple case study design (Patton, 2002; Yin, 2014) to explore multilingual families' engagement with STEM education at a "multilingual STEM family night" held at three elementary schools in two different southeastern Indiana school districts. Students formally identified as English language learners (ELLs) range from almost 9% to 37% of each school's student population. The number of multilingual families is higher, though, as the school where 37% of students receive English language services also identifies over 51% of its students living in Spanish-speaking households. The two schools with the highest number of English learners are in a district which saw its ELL population increase over 500% from 2005 to 2015 and currently provides services to over 1,100 students speaking 35 languages. The vast majority of those emerging multilingual students (over 900) speak Spanish at home, with Arabic and Mandarin being the next most common languages. One of our participating schools serves students from 18 different home languages and from 31 countries of origin. All three of the participating schools receive federal Title I funds, and percentages of students receiving free and reduced lunch range from 33% to 79% (National Center for Education Statistics, 2019).

Over 150 students and their family members attended the three events. One event was held at each school. For our first event at the school with the smallest percentage of identified English language learners (9% of the student population), we advertised the event specifically to multilingual families and sent invitations only to parents of students identified as English language learners. For the second and third event though, we opened the event to all students and families at the school administrators' request. Flyers and invitations for all three events were provided in both English and Spanish.

Events at each school followed the same agenda with the same activities. As families entered, we provided food and assigned parents to sit with other parents and students to sit with other students. Similarly, school administrators were assigned to sit as a separate group. The limited number of attending teachers chose to help facilitate the activities instead of participate in the STEM activity. Each group received a packet of the same materials to complete the same task.

The students, in particular, enjoyed competing with the adult groups. The parents enjoyed working separately from their children (so the adults could be silly, make mistakes, etc.). In each instance, the student groups outperformed the adult groups. While we had interpreters on site, we deliberately assigned parents to sit with parents of different languages to model that science and mathematics can be accomplished with limited verbal communication.

The STEM activity we did was a variation of the "marshmallow challenge": the task was to build the tallest freestanding tower out of spaghetti, masking tape, and string, with a marshmallow on top. In order to adapt this task for a STEM Circle activity, we only outlined minimal rules and prompted participants to determine what questions to ask within their groups.

After the activity, all participants joined a large-group reflection discussion in which we talked about what science and mathematics are and are not. For example, solving a real-world problem such as using geometry to build a tower, connects mathematics to science and to engineering. However, timed basic fact tests are not mathematics. We gave out a "take-home" bilingual sheet of advice for reducing math anxiety, supporting a growth mindset for learning, and expressing positive expectations to their children. We asked parents and guardians to place this take-home sheet on their refrigerators.

Data collection included pre- and post-event parent surveys, separate administrators' surveys, researchers' observational data, and video recording of the activities. The survey asked for the family's native language, their definition of STEM, their beliefs about their child's knowledge about STEM, their interests in knowing how to support their child's STEM learning, and their academic hopes and expectations for their child.

The researchers kept observational data, including notes about how parents engaged with each other within their groups during the activity and debrief conversation. From the data, the researchers created individual cases of family engagement at each school. The researchers then compared cases to establish generalizations across the three sites, triangulating findings from the observational, parent surveys, and administrators' survey data. The researchers kept observation notes from conversations with the administrators who were also surveyed based upon their experiences hosting STEM family engagement events and their perceptions of additional resources needed to expand STEM family engagement for multilingual students and families.

Results

This exploratory study engaged the research question, "How do STEM Circles promote multilingual family engagement at the elementary level?" In this paper we present only the cross-case analysis related to the dominant themes of administrative perceptions and support, and family investment and involvement. Event attendance, survey responses, and administrator feedback all indicate that multilingual parents see a need for strong STEM education and were interested and engaged in supporting their children's STEM learning.

Survey item responses were coded through thematic analysis (Braun & Clarke, 2006), which is beneficial for survey data analysis as it allows researchers to examine emergent themes beyond those present in the survey questions (Tanaka, Parkinson, Settel, & Tahiroğlu, 2012). Responses to each open-ended item were combined, and the researchers independently coded for broad, inductive themes and then more specific themes within the codes. The unit of analysis was each school. We report below on survey data from parents and school administrators. Although we would have liked to collect data from teachers as well, teachers typically do not attend evening events, and the few teachers present at our events were already engaged in facilitating the events with us.

Administrators at all sites identified STEM learning and family involvement as the two largest needs related to facilitating STEM family engagement. None of the schools offered students sustained engagement or learning in a STEM environment. Although students received instruction in both mathematics and science as a part of their general curriculum, they did not have opportunities to engage in interdisciplinary STEM learning. Administrators at the schools described their interest in increasing students' exposure to STEM. Although two of the surveyed administrators had STEM activities built into their students' weekly schedule, only one surveyed administrator described hosting a STEM engagement night at her school.

Additionally, administrators at the schools described parental willingness to attend as essential for family engagement and student learning. Surveyed administrators also noted the importance of family support for STEM learning. These administrators concurred, noting how school STEM events are essential to family engagement. As one administrator noted, "If the families can get on board with STEM activities and the mindset, then the kids will be more willing to try" (October 4, 2019). However, administrators held a widely shared belief that parents do not engage if they do not believe an event is worthwhile or if they do not understand

STEM, suggesting that lack of parental knowledge of STEM might prevent family engagement in STEM Circles and STEM learning.

This study identified benefits in offering STEM activities beyond the classroom that parents can attend. Parents reported limited previous understanding of STEM as a content area. Many first learned the STEM acronym from the event flyer but were unable to articulate how their students engaged in STEM learning. Despite their emergent understanding of STEM, attending family members described themselves as committed to supporting their children's education and enthusiastically participated in the activity themselves to understand the integration and applications of STEM content.

The post-event survey reported the parents seeing STEM education as more hands-on, enjoyable, and problem-based than expected. For example, one parent reported, "I think my children have viewed it differently because I think they view it funner." Additional parent feedback included mentioning that the learning process was as important as the "answer." Parents also valued communicating in a team, allowing mistakes, and persevering as important aspects of learning STEM. No parents reported having previous experience in school learning that involved communication, problem solving, perseverance, or modelling. They liked the various role models for the students in the room, in terms of gender and ethnic diversity of presenters. Finally, they appreciated the "take-home" bilingual sheet of advice for reducing math anxiety, supporting a growth mindset for learning, and expressing positive expectations to their children.

Our STEM Circle activity allowed parents to be presented with a non-standard problem that they could tackle. Rather than interpreting procedures or focusing on word walls, parents--and their students--could directly and immediately engage with real-world STEM content through hands-on activities.

Discussion

Engaging emergent multilingual students (and parents) in mathematics can be daunting—particularly when some students are still developing English language proficiency while teachers are focusing on teaching the mathematics concepts, processes, and content. We want to assure parents and educators that students' cognitive "sense-making"—in any language—develops their STEM literacy skills as well as their academic identities. We modelled a STEM Circle approach as one active-learning method to engage families, develop language skills among emergent multilingual students, and help all students persevere in solving problems and constructing viable arguments (Suh et al., in press).

Although this study was limited by self-selection bias of parents who chose to attend, we still feel that family STEM Circles are a nonthreatening way to demonstrate for school faculty (and parents) the research on how students can learn mathematics effectively. STEM Circles provide a model of the “you do, we do, I do” approach that differs from traditional teacher-directed instruction. Based on these case studies, we observed that strategies, such as STEM Circles for developing English language proficiency, are effective for educators to support all students in developing academic language, conceptual understanding, and meaningful skills--as well as develop family engagement. Our limited results suggest that STEM Circles may also lead to family engagement discussions about developing academic English language proficiency along with mathematical conceptual understanding and meaningful skills. These are areas worthy of further research.

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REDESIGN AND IMPLEMENTATION OF A LIBERAL ARTS COLLEGE MATHEMATICS COURSE

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With the increasing push to graduate students in a timely manner, mathematics departments have been under pressure to create and maintain inclusive mathematics classes that do not create a "bottleneck" for students. At the University of Nevada, Las Vegas, the Liberal Arts Mathematics course was redeveloped to be more inclusive by tailoring the course content and structure. Specifics for the course and the effects on pass rates before and after implementation will be examined.

The University of Nevada, Las Vegas (UNLV) is an urban research institution in the heart of Las Vegas, Nevada. Fall 2018 boasted an enrollment of over 30,000 students, with about 60% minority – ranking it the 4th most diverse campus in the nation by the U.S. News and World Reports (Campus Ethnic Diversity, 2019). The Department of Mathematical Sciences (DMS) offers a full range of mathematics courses, one of which is titled "Fundamentals of College Mathematics" (FCM) and is intended for students enrolled in the Arts and Humanities. UNLV has a general education core and requires a minimum of three credits of Mathematics for all students, of which FCM is the "lowest bar" or the course that requires the lowest entrance requirement while still fulfilling the core. Up until its revision in Fall 2017, the course was a "catch all" of unrelated mathematical concepts that was largely unpopular with the students and departments across campus. Between Fall 2016 and Spring 2019 there were on average 650 students enrolled in the course per term, grouped in sections of 25 to 45 students. The course was (and is) taught mostly by part time instructors and graduate students. To enter the course, a student needed to demonstrate a minimum set of mathematical competences, as determined by an appropriate SAT or ACT score, or the completion of a remedial mathematics course in elementary algebra.

The largest driving factor for the redevelopment of FCM was the fact that it was so unpopular, as reported to the DMS administration during meetings with other department chairs across campus. In addition, in 2012 there was a change in the state funding formula (New Model, 2019) and departments were left scrambling to assist those students not progressing towards graduation. The dissatisfaction felt about the course across campus was now coupled with rumblings that math was often the cause for delays in graduation attainment. At that point, individual departments began to discuss creating their own math requirement, or perhaps even eliminating the math requirement altogether. As such, it was well past time for the DMS to remedy these problems with FCM.

Development of the Course

In order to develop a course that the university could stand behind, it was essential to get faculty and administration input before any changes were even considered. All faculty and staff were provided the opportunity to complete an online survey, created by the author and disseminated through the university's survey tool. The survey was aimed at determining their satisfaction with the current FCM course and providing input on what they would like for it to contain. Over 100 faculty and staff across all disciplines completed the survey. To support the claim that FCM was largely unpopular, 41% of respondents were either "somewhat" or "extremely" dissatisfied with FCM, and only 16% were "somewhat" or "extremely" satisfied. When asked about the satisfaction level of their students, respondents indicated a dissatisfaction level of 75% as reported to them by their students. Written responses indicated that students felt FCM included topics they would "never need." In addition, administrators expressed frustration with how mathematics (in general) was a block for student progression to graduation – if students could not enter into or succeed in FCM in a timely manner, they believed it resulted in the student not completing their degree program.

This is supported in research on remedial mathematics education. According to a statistical report by the U.S. Department of Education, remedial students graduated at a rate of 30-55%, whereas nonremedial student's attainment rates were 67% (Chen, 2016). In the same study, remedial students had a higher probability (11 to 12% higher) of leaving college by their second year as compared to nonremedial students (Chen, 2016). While FCM is *not* remedial, the process of qualifying for and then subsequently completing the course proved just as difficult for our student body. One survey respondent went so far as to say, "the math requirement is the single biggest obstacle to graduation for our students." The survey and supporting research indicated the importance for a redeveloped course that maintains mathematical integrity but is easier to enter and successfully complete.

Because FCM is the "low bar" for the general education core, it is most often populated by students who do not need to go further in the Mathematical Sciences, such as Fine Arts, Health Science, Hotel, Liberal Arts and Urban Affairs. These departments were specifically targeted and interviewed. While survey respondents indicated desired objectives for the course, it was hoped that interviews with the main audience for FCM would help solidify which of these objectives were selected (see the Implementation Section for a list of selected objectives). During these interviews it also became apparent that content was not the only issue of importance for the university community – the teaching style and methods of assessment were all important to FCM's redesign.

Development of Course Structure and Objectives

Over 86% of survey respondents indicated they "strongly agree" that the mathematics should be embedded in other, real world problems. Given survey and interview preferences for connected topics and real-world applications, it was decided that FCM would expose students to overarching ideas. The course was designed around these ideas (or themes), and the concepts and skills necessary to study each theme were developed within that framework. Though there were initially four themes, the last (math as it relates to the fine arts) was optional for time consideration and has since been omitted. This structure was an effort to make apparent the direct application of mathematical concepts to real-life situations. The three themes and an abbreviated list of their associated topics are as outlined in Table 1.

Table 1

Themes and Associated Objectives

Theme 1: Using Mathematics to Reason and Answer Questions Standardized units, unit conversion, unit analysis, four step problem solving process, outcome and event, theoretical probability, relative frequency probability, calculating probabilities, law of large numbers, fallacies or deceptive arguments, sets and elements, Venn diagrams, inductive and deductive reasoning, validity Theme 2: Financial and Practical Numeracy Percentages, significant digits, rounding, relative error, budgeting, managing expenses, simple and compound interest, APR, APY, savings, investments, loans and mortgages, credit cards Theme 3: Mathematical and Statistical Modeling Statistical studies, frequency tables, bar graphs, pie charts, histograms and line charts, correlation, measures of central tendency, distributions, symmetry, skewness and variation, quartiles, standard deviation, normal distribution, 68-95-99.7 rule, linear and exponential models, half-life and doubling time.

While some of the topics from the original FCM remained (statistics, consumer math and financial management), some were disregarded (parts of set theory, and geometry). Moreover, the course "felt" very different because topics were used to answer overarching questions within each theme. For example, in Theme 2 students are presented with a "life scenario" and are asked to use mathematics to determine whether they can afford to buy a home, and if so where should it be and how much should it cost. Monthly bills, financing, auto ownership, whether to marry and have children, how to care for children… all lifestyle choices are considered and determined using the skills presented in Theme 2.

Choosing Specifics – Pedagogical Strategies and Assessment

From the survey it was clear that in addition to content, perhaps more concerning was teaching style and assessments. In the new FCM course, learning with understanding is at the forefront of the classroom environment. Survey responses indicated at a rate of 95% that "it is more important for students to have problem solving skills as compared to being able to memorize or use specific math facts or formulas." This focus away from rote procedures and memorization is also supported in research. When a student merely memorizes facts or procedures without understanding, they are often not sure when or how to use what they know (Bransford, Brown, & Cocking, 1999). According to the NCTM, procedures that used to be important to mathematical learning are now easily computed with technology, so more attention can be paid to conceptual understanding (NCTM, 2000). In fact, the previous FCM did not even *allow* for the use of any technological tools – a policy that was quickly overturned for the new FCM course which now allows for the use of a scientific calculator and statistical software.

In order to create an environment focused on investigative learning, each theme contained one major project and many "mini projects." According to the NCTM, classroom discourse and social interaction is an effective tool to help students learn and make connections (NCTM, 2000). Allowing students to talk about their strategies provides an opportunity for them to develop procedural fluency and conceptual understanding (NCTM, 2000). As such, all projects in the course were collaborative in nature. Instead of being exclusively lectured to, students were provided with the opportunity to work in groups and reason through the material. Interestingly enough, the investigative approach with little to no lecture was a struggle for students and instructors alike – it seems lecturing is so ingrained in our community that its complete abandonment created distress and fear. After the first few weeks of classes, many instructors had to implement *some* type of lecture to put their students at ease. Currently there is about a 2:1 ratio of lecturing to investigative learning. In my opinion this is not ideal, and it is hoped the culture will slowly shift over time. The effects of time spent lecturing is further discussed in the implementation section.

Another complaint addressed in the survey was the apparent disconnect between the mathematics being learned and the mathematics being assessed. Some comments were, "students are not prepared for the final exam," and "the homework builds a false sense of understanding." In the redesigned FCM course, assessments are based on a mixture of procedural questions, open-ended questions and performance tasks. According to the NCTM, this mixture of assessment types is necessary, and they further indicate that assessments should be relevant to the information students need to know (NCTM, 2000). The previous FCM course had an overreliance on formal paper-and-pencil tests driven by demonstrating procedures. In the new FCM course students are provided with multiple approaches to show what they are learning. In addition, all sections of the course provide students with a set of learning objectives and specific guidelines for what will be assessed, so there is now clear communication for what is expected.

Implementation of the Course

The redesigned FCM was first offered to students Fall 2017. The previous two terms (Fall 2016 and Spring 2017) there were 1,328 students total for the academic year with an average of 16.5 sections per term containing 40.4 students. From Fall 2017 through Spring 2019 there was an average of 1,305 students per academic year with an average of 19 sections per term containing 34.4 students. Student composition and the pool of instructors used throughout the implementation has been consistent. Of the survey respondents, 75% supported a maximum of 30 students per class. As such, the new FCM class saw a slight decrease in class size, an argument made to administration and thankfully supported. There was one coordinator who oversaw implementation, and all instructors attended regular meetings throughout the term to discuss how things were going, support the instructors in the new learning environment, and make appropriate changes. It should be noted that there was originally an online course. This was offered Fall 2017, but since it could not make use of the collaborative nature of the projects, it was a reflection of the old FCM. Eventually it was dropped from the schedule of offerings. Data on student performance was collected for two full semesters prior to implementation as well as each full semester the course had been offered.

Figure 1 displays the percentage of students earning grades A-F, along with the percentage of students who withdrew from the course (W) and audited the course (AD). According to Figure 1, for those students who chose to withdraw or audit, there is only a slight difference between the old and new FCM. At first one would expect that if the level of satisfaction went up, the number of drops and audits would decrease. However, students seem to persevere at the same rate, indicating that perhaps the choice of withdraw or audit has more to do with life circumstances then it does course satisfaction.

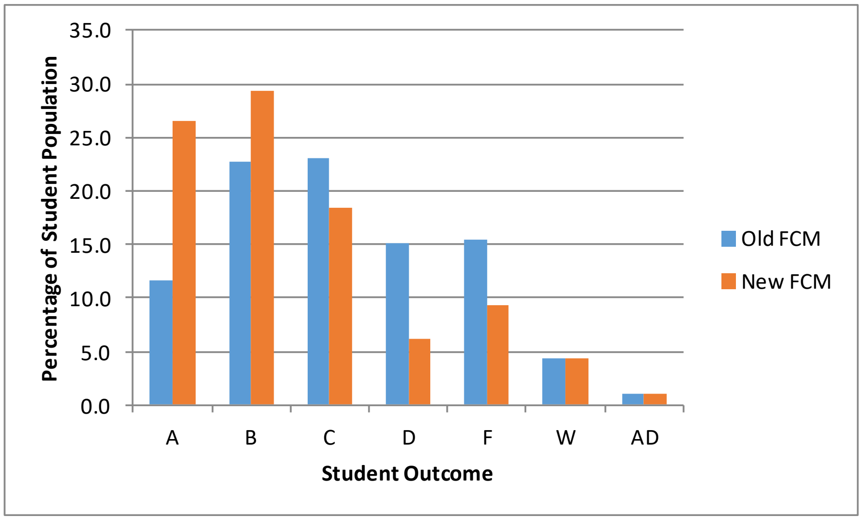

Figure 1. Comparison of Student Grades for Previous and Newly Designed FCM

A chi-square test of association between class-type (row factors of Old FCM and New FCM) and result (column factor of Pass and Repeat, pass being grades A-D and repeat being a grade of F, W or AD) yields a corresponding *p*-value of 2.23×10^{-7}, which indicates the data strongly supports that there is an association between the class-type and result. That is, the newly redesigned FCM has a significantly higher pass rate than the old FCM.

Table 2 indicates the pass and repeat rates for both the new and old FCM. Passing is again defined as a grade of D or higher. Because the course is terminal, and is not required as a perquisite for any other course, a grade of D typically satisfies the requirement for a student's degree program. Repeat is defined as F, W and AD, each of which implies the student must repeat the course to receive credit.

Table 2

Comparison of Pass Rates for new and old FCM

	Pass Rate		Repeat Rate	
	Count	Percent	Count	Percent
Old FCM	963	77.6	278	22.4
New FCM	2095	84.5	383	15.5

It should be noted that one could consider other thresholds of pass and repeat. There are perhaps some departments that require a minimum GPA or minimum standard for all courses. However, if we consider passing as C or higher, and repeat as D, F, W or AD, the new FCM still has a significantly higher pass rate of 78.0% as compared to 61.4% for the old FCM.

Equally important to student performance is student satisfaction. Interestingly enough, each term the new course is offered, students participate in and analyze data on their own performance and satisfaction. While UNLV requires formal student evaluations at the end of each term, it is personnel related and not part of our analysis. This survey was made by the author, in coordination with the instructor team, and disseminated through UNLV's survey tool. Instead, we analyzed the student surveys to determine satisfaction (unfortunately these surveys were not part of the old FCM course). In the survey, they were asked about their level of satisfaction with the projects and mini projects, as well as their overall evaluation of the course. This anonymous student survey is done online towards the end of each term, and for all terms there was a response rate of 65%. While participation is required, student information is self-reported and non-identifiable. Table 3 summarizes the data reported by students from Fall 2017 through Spring 2019 for the indicated questions.

Table 3

Percent of level of student satisfaction for the new FCM

	+3	+2	+1	0	–1	–2	–3
Level of satisfaction with the mini projects	23.6	35.1	16.8	12.2	5.5	2.8	4.0
Level of satisfaction with the major (theme) projects	22.9	32.8	16.3	13.0	7.1	3.2	4.9
Level of satisfaction with the entire course, in general	29.9	28.3	13.7	10.1	7.8	5.5	4.6

Note. +3 Extremely satisfied, +2 Moderately satisfied, +1 Slightly satisfied, 0 Neither satisfied nor dissatisfied, –1 Slightly dissatisfied, –2 Moderately dissatisfied, –3 Extremely dissatisfied

As indicated in Table 3, over 70% of all students who responded indicated they were satisfied (slightly, moderately or extremely) in each of the above items (mini projects, theme projects and the course in general) and less than 20% indicated dissatisfaction (slightly, moderately or extremely). While we strive to have higher satisfaction levels, we have to appreciate the context – these students are typically not drawn to mathematics, and for over two-thirds of them to be satisfied with their math class is noteworthy.

In addition, as indicated previously, there was much resistance to the "little to no" lecture format that were considered to be best practices for this audience. Despite our best efforts, students are self-reporting that their instructors are lecturing quite often. Fall 2017 was our best effort with only 13.9% of students reporting their instructors were “always” lecturing. Consolidating all terms, 68.7% of students report instructors lecturing either “most” or “all” of the time. Of those students, 77.2% were satisfied (slightly, moderately or extremely) and 6.8% were dissatisfied (slightly, moderately or extremely). In contrast, for the 31.1% of students that self-report their instructors lecturing “half the time” or even less, only 52.5% were satisfied and 32.9% were dissatisfied. As much as we would like to spend much of our class time investigating and problem solving, we will have to strive to create an environment where this creates less apprehension for the student.

Re-Redesign of the Course

According to the NCTM, it is crucial to constantly reflect on and refine instructional practice (NCTM, 2000). As such, the new FCM course is always undergoing revisions and changes, particularly as it relates to how the students are connecting to the content. In addition, there are outside influences that affect how we structure our course offerings. One of these influences includes recent mandates by the state to eliminate remedial coursework and instead offer co-requisite courses. This implies that students who do not qualify for entrance into FCM would be provided with a class that includes “real time” prerequisite material. In addition, there are several degree programs at UNLV that are predominately offered online, and these departments are requesting a re-instatement of the online version of FCM, which had been discontinued. These outside influences will eventually play a role, but are only a part of what we as instructors need to continue to do to ensure the quality and effectiveness of our courses.

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USING NUMBER TO SUPPORT MIDDLE-GRADES STUDENTS' GENERALIZING

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One bridge between arithmetic and algebra is making and justifying generalizations. This research study focuses on how middle-grades students' concept of number supports generalizations of linear patterns. Students whose reasoning about number was limited to constructing composite units (i.e., units of units) in activity were limited to factual algebraic thinking. Students who assimilated with composite units, which is more sophisticated than constructing composite units in activity, were supported in factual and oral algebraic thinking. Analysis uses the students' composite units to model their generalizations.

Literature Review

The study of algebra comprises a large portion of middle and high school mathematics. Cai and Knuth (2011) called it a "gatekeeper" (p. vii) to higher mathematics and urge researchers to develop a more thorough understanding of how student thinking transitions from arithmetic to algebra. Russell, Schifter, and Bastable (2011) identified one such aspect of this transition as making generalizations. This makes generalizing one manner by which middle-grades students may begin to reason algebraically.

Radford's (2011) research echoes this idea by highlighting the types of non-symbolic reasoning that are and are not algebraic. These are commonality, generalizing, factual algebraic thinking, and oral algebra. Identifying a *commonality* and *generalizing* the commonality to the next term are not algebraic in nature because they lack indeterminacy. *Factual algebraic thinking* involves extending a pattern by applying a rule to a larger, explicitly calculated term. For example, calculating the first three terms of a pattern and then the 100th term. This requires students to reason abstractly about unknowns, making it the first instance of algebraic reasoning (Radford, 2011). Finally, *oral algebra*, or verbalizing a pattern in reference to an unknown term, makes an unknown explicit, thereby making it the most sophisticated (Radford, 2011).

Symbolic representation of a pattern is more sophisticated than oral algebra. Once symbols are introduced into algebraic reasoning, many misconceptions about variable emerge. MacGregor and Stacey (1997) categorized middle-grades students' misconceptions. Their categories include thinking of variables as an alpha-numeric code and using different letters to represent related unknowns. The present study examines how middle-grades students' concept of number supports their non-symbolic and symbolic generalizations, and to what extent variable misconceptions are applied when students introduce symbolic notation. Building on existing literature, this research study will interpret students' generalizing and conceptions of variable in terms of their concepts of number.

Conceptual Framework

Students' concept of number will be framed using the number sequences, which is a neo-Piagetian hierarchy of concepts of number that were developed in research with elementary children (Steffe & Olive, 2010). They include the initial number sequence (INS), tacitly nested number sequence (TNS), explicitly nested number sequence (ENS), and generalized number sequence (GNS). The advanced tacitly nested number sequence (aTNS) was added to the hierarchy by Ulrich (2016b) in her research with middle-grades students, and it falls between the TNS and ENS. In general, "children use their number sequences to provide meaning for number words" (Steffe & Olive, 2010, p. 27), and the number sequences are based on units coordination, among other mental constructs. This research will focus on TNS, aTNS, and ENS students.

Tacitly Nested Number Sequence (TNS)

TNS students construct two levels of units, or composite units, in activity, meaning that they immediately interpret number words, such as seven, as seven individual units of one. Then, in mental activity, TNS students can construct a *composite unit* of seven, implying they chunk the seven units into one unit of seven (Steffe & Olive, 2010). This supports students in conceptualizing seven as a composite unit that can be transposed to represent a counting sequence from 23 through 29, for instance.

Advanced Tacitly Nested Number Sequence (aTNS)

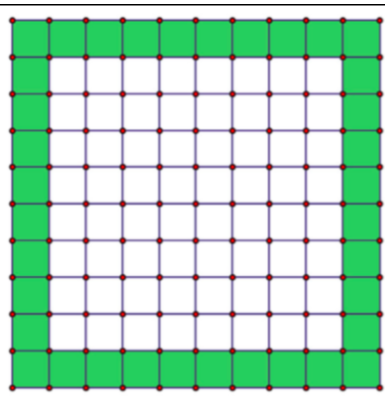
Students construct an aTNS when they *assimilate* with composite units (Ulrich, 2016b), meaning they immediately perceive of numbers such as seven as a single unit containing seven units (Ulrich, 2016a). Assimilating with composite units implies aTNS students can construct a third level of units in activity, meaning aTNS students can construct 21 as three units, each of which contain seven units, for instance. The third level of units decays following activity, implying that only the composite unit is material for reflection (Ulrich, 2016a). However, this is more sophisticated than TNS students, for whom the composite unit decays following activity.

Explicitly Nested Number Sequence (ENS)

ENS students also assimilate with composite units but can additionally disembed. *Disembedding* is the ability to imagine a unit being removed from the composite unit without destroying either of the units (Steffe & Olive, 2010). For example, if asked, "2 plus what makes 7?" ENS students can conceive of a composite unit of seven as containing a composite unit of two, and a composite unit of unknown quantity. Then, they can mentally disembed the composite unit of two from seven and compare the two and seven to find that the unknown is five. For students who do not disembed (i.e., TNS and aTNS students), mentally removing a unit of two from seven would destroy at least one of the composites, leaving them with no material for reflection.

Units Coordination in Algebra

Hackenberg (2013) has modeled algebraic reasoning based on students' units coordination. On the border problem (Figure 1), a student who constructed composite units in activity, as TNS students do, found the number of squares on the border of a 10-by-10 grid by adding $10 + 10 + 8 + 8$, but did not verbalize the relationship between 10 and 8. Hackenberg (2013) attributed this to the students' not disembedding, which limited him from mentally removing a composite unit of 8 from a unit of 10 and reflecting on the relationship between the two quantities. Although this student did not represent his result symbolically, Hackenberg hypothesized that he may have written $x + x + y + y$ because not disembedding would preclude him from relating x and $x - 2$.



<u>Interviewer Question</u>: Without counting one by one, and without writing anything down, can you describe a method for finding the number of squares on the border of this 10-by-10 grid?

<u>Follow-up Questions:</u> Can you use that method to find the number of squares on the border of a 6-by-6 grid? A 100-by-100 grid?

How would you explain to your math teacher your method for finding the number of squares on the border of *any* square grid?

Can you write an expression to represent the number of squares on the border of an n-by-n grid?

Figure 1. The border problem (problem 1; modified from Hackenberg, 2013).

Hackenberg, Jones, Eker, and Creager (2017) posit that difficulty related to operating on unknowns can be related to units coordination. Specifically, to conceive of variables as unknowns is supported by composite units. The present research study examines how middle-grades students generalize unknown quantities, symbolically and non-symbolically, and uses the number sequences to model this reasoning. The number sequences are used because they are based on students' units coordination but also distinguish between two groups of students who assimilate with composite units – aTNS and ENS students. This study specifically asks, what are the similarities and differences in TNS, aTNS, and ENS students' non-symbolic and symbolic generalizations?

Methods

This study examined the generalizing of 14 students in grades six through nine, all within one school system in the United States. Students participated in two clinical interviews that lasted approximately 45 minutes each. First, the interviews examined students' levels of units coordination and attributed a number sequence to each student. Questions and methods of analysis were drawn from Ulrich and Wilkins (2017). Second, the interviews elicited generalizations using 3 problems (see Figure 1 for problem 1). On problems 2 and 3, students were shown a pattern of blocks in which the number of blocks was two more and six more than the figure number, respectively. Students were asked to extend, verbalize, and symbolically represent the patterns.

Results

Table 1 shows the results of students' generalizing. Greyed cells indicate 50% or more of students in that group demonstrated the specified reasoning. All TNS students demonstrated factual algebraic thinking, and one demonstrated oral algebra on one problem. aTNS students demonstrated factual and oral algebraic thinking, but no more than half wrote symbolic expressions on any problem. ENS students demonstrated factual algebra, oral algebra and symbolic representation. On problem 1, students used any of three methods. These methods were algebraically equivalent to $2n + 2(n - 2)$, $n + 2(n - 1) + (n - 2)$, and $4n - 4$.

Table 1

Numbers of Students who Generalized Relationships on Problems 1, 2, and 3

	Factual Algebraic Thinking			Oral Algebra			Symbolic Representation		
	1	2	3	1	2	3	1	2	3
TNS	1/2	2/2	2/2	1/2	0/2	0/2	0/2	0/2	0/2
aTNS	6/6	5/5	6/6	6/6	5/5	6/6	2/6	2/5	3/6
ENS	6/6	4/4	5/5	6/6	4/4	5/5	6/6	4/4	5/5

Note. One aTNS and two ENS students did not attempt problem 2 due to time limitations. One of the two ENS students who did not attempt problem 2 also did not attempt problem 3 due to time limitations.

TNS Students' Generalizing Behaviors

Tabitha demonstrated factual algebraic thinking on all problems and oral algebra on one. She calculated the number of squares on the border of a 100-by-100 grid, indicating factual algebraic thinking, and she used a method equivalent to $2n + 2(n - 2)$. She then verbalized that method for finding the number of squares on the border of *any* grid, indicating oral algebra. She said:

> You could still subtract the top part by 2 and you could get the side length. … And then you would add the top and bottom part together, and once you get that you would add it by the side part and then after you do that you would do the second side.

Tabitha refers to the horizontal side length as the "top part" and says that to find the length of the vertical side she would subtract two. Then, she would sum the horizontal and vertical sides.

Tabitha's generalizing was the more sophisticated of the two TNS students, although she was still unable to engage in oral algebra on problems 2 and 3. When attempting to verbalize, both TNS students applied recursive reasoning on problems 2 and 3, explaining that to find the number of blocks in any figure requires the number of blocks in the preceding figure. Despite calculating numerical examples explicitly, neither student could verbalize the explicit pattern.

aTNS Students' Generalizing Behaviors

All aTNS students demonstrated factual and oral algebraic thinking on all problems. About half represented each problem symbolically (Table 1). On the border problem, four aTNS students verbalized $2n + 2(n - 2)$ and one verbalized $n + 2(n - 1) + (n - 2)$, but none represented these methods symbolically. Three aTNS students verbalized $4n - 4$, and two represented it symbolically.

A poignant difficulty for aTNS students in symbolic representation was relating the given side, *n*, to the given side minus the corners. Alex verbalized and attempted to symbolically represent $2n + 2(n - 2)$ but when he could not, he generated $4n - 4$ instead. Alex reasoned:

> Can it be, like, $n + n + n + n$? No, because of this [points to a corner]. … You would be adding 1 extra, I think. Yeah. I could do $n + n$ [slides his finger across the horizontal sides], but then these [points to a vertical side] would have to be different. So I could express that better. [Writes $n + n + b + b$.] … I want to say use a different variable, but it has to be a certain amount, number, in here, too. If you do $n + n$, you might be able to subtract, um, 4? I want to say $n + n - 4$, and then you could just do the sides together. $n + n - 4 + n + n$.

Alex expressed awareness that the sides are related when he stated "it has to be a certain amount," meaning b has to have a certain relationship to n. Alex consistently subtracted two from the given side in each earlier numerical example and verbalized this relationship, but did not represent it symbolically. To resolve this perturbation, Alex used an unrelated variable, b, to represent the vertical sides because b is the second letter of the alphabet. Then, because he recognized b does not reflect the relationship between the sides, he generated a new method, $4n - 4$. Alyssa and Andy made similar adjustments to their expressions. Andy wrote $n + n + x + x$, but did not show awareness of the relationship between *n* and *x*, despite verbalizing is as "subtract two" earlier. Alyssa wrote $n + n + L + L$. She selected *L* because *L* is two letters before *N* in the alphabet. Of the four aTNS students who attempted to represent $2n + 2(n - 2)$, all were unsuccessful, and three used a variable that was unrelated to the side length, n.

ENS Students' Generalizing Behaviors

All ENS students demonstrated factual algebraic thinking, oral algebra, and symbolic representation on all problems attempted (Table 1). On the border problem, three ENS students verbalized $2n + 2(n - 2)$, and two represented it symbolically. Two verbalized and symbolically represented $n + 2(n - 1) + (n - 2)$. Three verbalized and symbolically represented $4n - 4$. A total of eight ENS students verbalized patterns because two verbalized the first and third methods. A total of seven represented patterns symbolically because one student who verbalized methods one and three also represented both symbolically.

Discussion

Both TNS students demonstrated factual algebraic thinking, which is the earliest form of non-symbolic algebraic reasoning because it requires "indeterminacy and analyticity" (Radford, 2011, p. 311). This was supported by their construction of composite units in activity. On the border problem, for instance, Tabitha compared the two sides, but because TNS students must construct this comparison of composites in activity, Tabitha did not maintain the relationship between the individual side lengths. This analysis is consistent with Hackenberg's (2013), which found that on the border problem, the side lengths of the grid were absorbed by the sum for students who construct composite units in activity. Thus, TNS students reasoned algebraically, although with limitations created by only constructing composite units in activity.

Oral algebra and symbolic representation require students to make explicit the indeterminacy (Radford, 2011), thereby necessitating that students reflect and operate on unknowns.

Hackenberg et al. (2017) found that unknowns constitute composite units. This provides a rationale for the difficulty TNS students had; not assimilating with composite units prohibited their reflection and operation on unknowns, which is characteristic of the explicit indeterminacy of oral algebra. In this study, only one TNS student demonstrated oral algebra on one problem. Hackenberg (2013) similarly found that students who assimilate with one level of units had difficulty verbalizing on the border problem, which she attributed to their not disembedding. Since TNS students do not disembed, this is a plausible explanation; however, aTNS students do not disembed either, and in contrast to the TNS students in this study, all aTNS students verbalized on the border problem making it necessary to conclude that an additional mental construct supports oral algebra. That construct is concluded to be an assimilatory composite unit, which is not available for TNS students but is for aTNS students.

To further support this conclusion, consider the oral algebra of aTNS students, who did not rely on recursive reasoning on problems 2 and 3. Rather, they described how an explicit relationship applied to any figure, making explicit the indeterminate relationship between the unknown figure number and the unknown number of blocks. Because an unknown constitutes a composite unit (Hackenberg et al., 2017), their oral algebra is attributed to an assimilatory composite unit. aTNS students assimilated the figure number as a composite unit containing an unknown number of ones and operated on the unknown in activity. This operation was marked by oral algebra.

Similar to aTNS students, ENS students assimilate with composite units, which rationalizes their success in oral algebra. However, ENS students had success writing 100% of equations whereas aTNS students had less than 50%; this difference is attributed to disembedding. Evidence is that on the border problem, aTNS students only symbolically represented the method $4n-4$. To write $4n-4$ the student must only conceptualize one unknown quantity. Thus, aTNS and ENS students alike wrote $4n-4$ by operating on composite units in activity. Comparatively, no aTNS student wrote $2n+2(n-2)$ or $n+2(n-1)+(n-2)$, both of which require simultaneous conceptualization of the relationship between two related sides. Only ENS students maintained this relationship. Hackenberg (2013) attributes the simultaneous conceptualization of *n* and *n-2* to disembedding, allowing for the conclusion that aTNS students' equation writing, some of which primitively included *n* and *n-2* as unrelated variables or an alpha-numeric code, was limited because they could not disembed. This builds on MacGregor and Stacey's (1997) categories of variable, by suggesting a theoretical rationale for the use of unrelated variables and variables as alpha-numeric codes.

This research asked how the symbolic and non-symbolic algebraic reasoning of TNS, aTNS, and ENS students compare. Hackenberg (2013) identified disembedding as a mental construct that supports symbolic representations. This research examined levels of non-symbolic generalizing within the frame of number, which points to assimilatory composite units as a supporting mental structure, in addition to disembedding. This research also identifies a potential rationale for students' application of two of Stacey and MacGregor's (1997) concepts of variable – using unrelated variables to represent related unknown quantities and variables as alpha-numeric codes. This rationale can be used to inform instructional decisions when students apply similar variable conceptions. Additional ways of leveraging students' composite units to support algebraic reasoning requires further consideration.

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Increasing the Odds for Validity and Assessment

CHALLENGES ASSESSING STATISTICS ATTITUDES: OPPORTUNITIES AND COSTS

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Eccles and colleagues' Expectancy-value theory (EVT; 1983) has been widely-used in education research, and the cost component of this framework has recently been the subject of increased research. While advances in measuring the cost component have been made (e.g., Flake et al., 2015), an on-going instrument development project for measuring attitudes in statistics education has encountered difficulties that motivate a deeper look at this construct. An overview of cost in EVT, the current state of measuring it in statistics education, and plans for a new study are described. The first project-specific data are to be collected in Spring 2020.

Introduction

Affective constructs have long been of interest to statistics educators and instruments have been available for measuring constructs such as *attitudes* (e.g., Wise, 1985) and *anxiety* (e.g., Cruise, Cash, & Bolton, 1985) for decades. Following increased calls for attention to affective constructs as outcomes in statistics education (e.g., Gal, Ginsburg, & Schau, 1997), a bevy of instruments assessing a multitude of germane constructs have been developed (Ramirez, Schau, & Emmioğlu, 2012). However, there have increasingly been critiques of existing instruments (e.g., Whitaker, Unfried, & Bond, 2019b), and there have also been attempts to more carefully define constructs such as anxiety that have heretofore not been properly distinguished from other affective constructs (e.g., Chew & Dillon, 2014). This is the context in which a new instrument for measuring student motivation for learning statistics is being developed: the Student Survey of Motivational Attitudes toward Statistics (S-SOMAS). This instrument is being developed using expectancy-value theory (EVT; Eccles et al., 1983) as the framework guiding the development process (Whitaker, Unfried, & Batakci, 2018). However, during the development process, the EVT construct *cost* seemed to be more difficult to develop appropriate items for than other constructs. This observation is consistent with documented difficulties with measuring cost (Flake, Barron, Hulleman, McCoach, & Welsh, 2015). To that end, a more focused examination of measuring cost in the context of statistics education is proposed.

This work is in the preliminary stages, and the first project-specific data is expected to be collected in spring 2020. This manuscript provides an overview of cost in the EVT framework and a review of the current state of measuring cost and cost-related constructs in statistics education. Then a few specific challenges to measuring cost encountered during the development of the S-SOMAS that have not yet been addressed by the current literature on cost (e.g., Flake et al., 2015; Jiang, Rosenzweig, & Gaspard, 2018; Wigfield, Rosenzweig, & Eccles, 2017) are explained. Finally, a brief summary of preliminary S-SOMAS pilot results (Unfried, Kerby, & Coffin, 2018) germane to cost is given along with a sketch of plans for using non-Likert-type items (e.g., Cacioppo, Berntson, Norris, & Gollan, 2012) to measure cost, with preliminary data collection to begin in spring 2020 from students enrolled in an introductory statistics course in Canada.

Literature Review

Cost in Expectancy Value Theory

Contemporary expectancy-value theory (EVT) is a theory of achievement motivation that stems from the work by Eccles and her colleagues (e.g., Eccles et al., 1983). In EVT, the choice of task, performance on the task, and persistence on the task are affected by one's expectancies and values; expectancies and values are constructs through which all other potential variables and constructs are mediated (Eccles & Wigfield, 2002). EVT draws on social exchange theory which defines cost as "any factors that operate to inhibit or deter the performance of a sequence of behavior" (Thibaut & Kelley, 1959, p. 12). Cost is viewed as "especially important" to the choices made by students (Wigfield et al., 2017, p. 124). However, cost has also been described as a "forgotten component of expectancy-value theory" (Flake et al., 2015, p. 232) due to the limited way in which it has historically been measured, though there has been an increase in research recently (Wigfield et al., 2017). In arguing that cost has not been adequately assessed in EVT research, Flake et al. (2015) clarified previous attempts to measure the construct, clarified and expanded the dimensions of cost, and developed an instrument that measures each of the four constructs using Likert-type items.

In the original conceptualization of Eccles and colleagues' EVT framework (1983), three dimensions were ascribed to cost: effort, loss of valued alternatives, and psychological cost of failure. There are other types of costs – such as economic or social costs – but these are still less-studied in relation to EVT (Wigfield et al., 2017). Eccles and colleagues hypothesized that interactions among the dimensions of cost and other EVT constructs were important for determining the value of a task. Moreover, the ratio of costs to benefits was posited as being related to achievement behaviors rather cost in isolation (Eccles et al., 1983). Based on the growing cost literature and focus groups about motivation conducted with college students, Flake et al. (2015) identified and defined four dimensions for cost: the emotional cost and loss of valued alternatives cost (previously identified by Eccles and colleagues (1983)) and two effort dimensions (task effort cost and outside effort cost).

To operationalize this notion of the negative appraisal, many of the items on their instrument include the phrase *too much*, as in "This class is too much work" (Flake et al., 2015, p. 242). By defining cost only in terms of negative appraisals, Flake et al. distinguished the dimensions of cost from other related constructs such as difficulty or general effort. As a demonstration of the efficacy of these definitions, Flake et al. also developed a 19-item instrument that measures the four cost dimensions. Flake et al. (2015) collected data from undergraduate students in introductory calculus classes using the final version of their instrument and observed correlations consistent with the EVT framework: the cost dimensions were strongly, positively correlated among themselves and were moderately negatively correlated with measures of expectancy, values, interest, and achievement.

Affective Constructs in Statistics Education

The Survey of Attitudes Toward Statistics (SATS; Schau, 1992, 2003) and Statistics Anxiety Rating Scale (STARS; Cruise et al., 1985) instruments are among the most widely used instruments measuring affective constructs in statistics education (Chew & Dillon, 2014; Ramirez et al., 2012). There is more validity evidence available supporting the use of the SATS and STARS instruments than for other instruments assessing similar constructs (Nolan, Beran, & Hecker, 2012; Onwuegbuzie & Wilson, 2003), and this is one likely reason why these instruments have been more widely-used than others. However, this widespread use of the SATS and STARS combined with imprecise construct definitions guiding their initial development (Chew & Dillon, 2014; Ramirez et al., 2012) has resulted in construct ambiguity in practice, though there have been attempts to clarify the construct definitions (e.g., Chew & Dillon, 2014; Onwuegbuzie, Da Ros, & Ryan, 1997). For example, the STARS instrument includes six subscales; of these, three subscales measure anxiety constructs and three measure attitude constructs (Chew & Dillon, 2014). Owing in part to documented challenges to the use of the SATS instruments in new research contexts (e.g., Whitaker et al., 2019b), work has begun on a new family of instruments for measuring statistics attitudes. To support the development of the student version of the Survey of Motivational Attitudes toward Statistics (S-SOMAS), a theoretical framework based on EVT has been developed (Whitaker et al., 2018). This theoretical framework is guiding the development of the S-SOMAS instrument a priori, in contrast to the SATS instruments which were aligned to EVT a posteriori (Ramirez et al., 2012).

Current Work

This current work on measuring cost has been informed by the work of Flake et al. (2015), but there are four areas that have been identified as areas for further study. These areas relate to both how cost has been defined for operationalizing in instruments and in specific ways that current instruments, though indicative of remarkable advances in the understanding of the cost construct, are not aligned with ways in which the S-SOMAS instrument is expected to be used. These four areas are reviewed, and then current efforts to address them are reviewed. There are two research questions guiding this work: (1) How can the EVT cost construct be measured in the context of learning statistics? and (2) How can non-Likert-type items be used in the measurement of cost? Note that much of this work is preliminary: while some data have been collected, they have not been focused specifically on cost. New items are currently being written with a planned pilot in early 2020.

Challenges to using existing cost scales

First, Flake et al.'s (2015) cost scale measuring four dimensions explicitly positions the respondents as students who are enrolled in a course: each of the 19 items uses the phrase *this class* and asks students to respond to statements about the class. In the development of the S-SOMAS instrument, one goal is to develop an instrument that can be used with respondents who are not enrolled in courses to facilitate longitudinal research (even though most respondents are expected to be students enrolled in a course). While the term *this class* might be replaced with another phrase such as *learning statistics*, it is not clear that a simple substitution would perform well. Flake et al. viewed the class as the experience to be evaluated for cost and their focus groups and subsequent instrument development used this assumption. Moreover, students are asked to respond to statements that may not be appropriate early in a semester. For example, items such as "This class takes up too much time" or "this class is too exhausting" (Flake et al., 2015, p. 241) may not be meaningful to students on or before the first day of class. It is anticipated that the S-SOMAS instrument might be administered several times in a semester, including on the first day of classes. Many items that were included on Flake et al.'s (2015) cost scale are not suitable for the intended uses of the S-SOMAS instrument.

While Flake et al. (2015) suggested that their definitions of the cost dimensions distinguish them from related constructs (e.g., difficulty or general effort), the data that they collected did not include items or scales measuring such closely-related constructs. More research about the extent to which these cost dimensions are empirically distinguishable from other closely-related constructs is needed. Additionally, the original EVT framework refers to one's value of a task being affected by the ratio of costs to benefits (Eccles et al., 1983), and in qualitative work Flake (2012) found that effort can be perceived by students as positive or negative in their most motivating and least motivating classes, respectively. Contemporary work on cost has focused on its role as a construct that is negatively correlated with others, and instrument development work for cost has used definitions of cost frame all costs as negative (e.g., Barron & Hulleman, 2015; Flake et al., 2015; Jiang et al., 2018). This ignores any potential positive costs (benefits), which may not be accounted for by other EVT components such as values.

S-SOMAS Pilot

As part of the larger development of the S-SOMAS instrument, seven items measuring cost were written and reviewed by subject matter experts. These items are part of a larger pool of 92 items assessing several EVT constructs administered in the pilot survey. Unfried, Kerby, and Coffin (2018) report results of an exploratory factor analysis (EFA) for this initial pilot survey. The EFA used the varimax rotation and parallel analysis to identify the number of factors (Unfried et al., 2018). The cost items were administered with items assessing academic self-concept, statistics self-concept, expectancies, difficulty, and attainment value; 134 undergraduate statistics students responded to this form (Unfried et al., 2018). Data collection has continued since Unfried et al.'s work, and more detailed work using the larger dataset will be presented.

The cost items used on the S-SOMAS pilot were not written using the same definition requiring a negative appraisal nor an emphasis on "too much" as in Flake et al. (2015). While the sample of 134 is somewhat small for the identified five-factor solution, a few preliminary findings were noted: it was difficult to empirically distinguish the cost items from items written to measure difficulty (a recognized similar construct) and attainment value (Unfried et al., 2018). A similarity between the cost construct and an attainment value construct has not been suggested to the extent of other noted construct similarities. As more data are collected and analyzed, we will examine these factor loadings and consider revising or writing new items.

Application of Other Measurement Types

Existing scales for measuring cost constructs have used Likert-type items (e.g., Flake et al., 2015; Jiang et al., 2018; Unfried et al., 2018; Whitaker, Unfried, & Bond, 2019a). Bipolar response scales (e.g., Likert-type items) imply a reciprocal relationship between the poles: as disagreement with an item becomes stronger, agreement with the item necessarily becomes weaker (Cacioppo et al., 2012). However, as observed by Flake (2012) the perception of cost dimensions such as effort may be perceived negatively – or might be perceived positively. Furthermore, the because the understanding of cost is evolving and its exact relationship with other EVT constructs is unknown (e.g., Barron & Hulleman, 2015; Wigfield et al., 2017), it may not be prudent to focus solely on cost as a negative. To address this, a set of items is currently being developed to measure cost that will have respondents assess both their negative perception and positive perception of the task using two unipolar scales. This is consistent with the evaluative space model (Cacioppo et al., 2012) wherein items use a two-dimensional grid to indicate how negative and how positive respondents are toward an item. Once data is collected from these cost items, it will be possible to determine to what extent the reciprocal relationship implied using a bipolar scale is appropriate for cost. If a bivariate relationship is better-suited, this information may help clarify the relationships that cost has with other EVT constructs.

Bivariate items may be written in several ways. Respondents are presented with a statement or question to respond to and shown a grid that they will use to respond. In the pilot data collection, items will use a response grid modeled on those used by Audrezet, Olsen, and Tudoran (2016). Each axis will include the values 1-5, and the horizontal axis will include a positive label statement, and the vertical will contain a negative statement. A consensus for how to label the horizontal and vertical axes has not been established (e.g., Audrezet et al., 2016; Borriello, 2017), and so several other pairs will be used such as *How POSITIVE/NEGATIVE does this make you feel?* or *How SATISFIED/DISSATISFIED does this make you feel?* Examples of statements included in the data collection that accompany such a grid include: using statistical software (e.g., Minitab or R), interpreting a confidence interval, interpreting a graph, or spending a long time doing a statistical analysis for homework but feeling like you got it right. Several different types of statements will be included in the data collection.

Discussion & Conclusion

While affective constructs have long been studied in statistics education research, there is still a growing need to clarify constructs and determine how best to measure them. Cost has been particularly difficult to assess as evidenced by the historical lack of research on it. Flake et al. (2015) were able to develop an instrument that can be used to measure cost with students enrolled in a specific course, but to do so they defined cost solely in terms of negative appraisals of activities. Historical EVT research (e.g., Eccles et al., 1983) has usually identified the ratio of costs to benefits as having an effect on one's values and ultimately achievement-related behaviors rather than costs in isolation. This distinction may be particularly important in settings with both a high cost and a high benefit – perhaps, for example, in difficult, upper-level courses closely connected to one's field of study and future career. The use of Likert-type instruments to measure cost (e.g., Flake et al., 2015; Jiang et al., 2018) imposes a reciprocal relationship: as one's negative appraisals of an activity increase, the positive appraisals decrease by the same amount. It is plausible that this may not be the case for cost, and the proposed study will examine a variety of other item types, including bivariate items using a two-dimensional grid (Cacioppo et al., 2012). The results of this study should clarify the appropriateness of Likert-type items for measuring cost and suggest how the construct might be assessed in statistics education.

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CONSTRUCTING AND VALIDATING AN EARLY ALGEBRA ASSESSMENT

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This study examined the initial construction of an assessment of 480 4th and 5th graders' understanding of early algebra concepts learned during an intervention involving playing 3 Math Snacks Early Algebra games and engaging in associated lesson activities. Preliminary content and internal structure validity evidence is presented and Rasch analysis was used to support the validity argument. Future research is needed with other populations, with a longer intervention, and to examine validity evidence from response processes, relation to other variables, and consequences of testing to establish a more generalizable validity argument.

There is a long history of students struggling with algebra, even dating back to Slaught (1908). More recently, efforts have focused on how to introduce algebra concepts to students in the middle or elementary grades, otherwise referred to as *early algebra* (Carraher & Schliemann, 2007). Substantial gains have been made in understanding students' engagement in early algebra, so much so that some have described this body of work as having "matured" (Blanton et al., 2015). Conversely, recent calls for further research have not only been ambitious—longitudinal, experimental studies (e.g., Stephens, Ellis, Blanton, & Brizuela, 2017) are difficult to achieve and require substantial external funding—but in order to adequately measure impact and to handle the larger scale, a psychometrically rigorous measurement tool is desirable. Although many studies have been conducted on early algebra, a validated instrument designed to measure student knowledge has not been constructed. Moreover, a recent special issue on validity and measurement tools showed a wide variety of calls for more focus on constructing high quality quantitative instruments to aid the field in moving forward as ideas are scaled up (Bostic, 2017). This study aims to respond to some of these calls by investigating sources of validity evidence related to an assessment designed to measure 4th and 5th grade students' knowledge of early algebra concepts.

Conceptual Framework and Related Literature

There have been several theoretical developments about the big ideas that make-up the learning of algebra. For instance, Graham and colleagues drew on Usikin's (1988; cited in Graham, Cuoco, & Zimmermann, 2010) four conceptions of algebra as well as ideas from the National Council of Teachers of Mathematics (NCTM; 2000) when constructing what they referred to as key elements of reasoning and sense making as it related to algebraic concepts.

Moreover, the authors of the *Common Core State Standards for Mathematics* also drew heavily on these same ideas when constructing the high school algebra and function standards. However, these ideas were intended for use in formal learning of algebra, such as in an Algebra I course, as opposed to *early algebra*, which focuses more on the elementary grade levels. The most recent handbook chapter on early algebra (Stephens et al., 2017) draws on Blanton et al.'s five big ideas of early algebra: "(a) equivalence, expressions, equations, and inequalities; (b) generalized arithmetic; (c) functional thinking; (d) variable; and (e) proportional reasoning" (2015, p. 43). Blanton et al. (2015) implemented a year-long comprehensive early algebra program in a third grade classroom. Findings revealed that these five big ideas showed promise in developing early algebra concepts among young children. They created several items to aid in measuring understanding of the first four big ideas. Although Blanton et al. provided some validity evidence, they did not provide the level of evidence called for by Bostic (2018) or by the American Education Research Association, American Psychological Association, and National Council on Measurement in Education (AERA, APA, & NCME; 2014). This is not surprising, as Bostic has reported that "few articles describing an instrument's validity evidence go beyond reporting reliability or content evidence" (p. 58). This study sought to provide validity evidence for test content and internal structure as described by AERA et al. (2014). Due to space limitations, evidence of validity for response processes, relation to other variables, and consequences of testing are not reported.

Method

Participants and Setting

Twenty-eight elementary teachers of grades 4 and 5 were recruited from 10 of the 25 elementary schools in a medium-sized public school district in the Southwest border-region of the US to participate. From their classrooms, 480 students participated in the entire intervention. The district's total enrollment was about 25,000, had a free/reduced lunch rate of about 75%, and 75% of students were classified as Hispanic, 20% as Caucasian, and the remaining 5% as Other.

Context of the Use of the Math Snacks Early Algebra Assessment

The Math Snacks Early Algebra Assessment was used as a pre and post assessment for an early algebra intervention. The early algebra intervention involved 1) students playing two early algebra games, *Agrinautica* and *Curse Reverse*, and using an interactive tool, *Creature Caverns* (all available at http://mathsnacks.com), and 2) engaging in three 60–70 minute lessons connected with the games and interactive. Teachers attended professional learning on how to implement the lesson plans in combination with the games and interactive.

Math Snacks Early Algebra Assessment Development

Purpose. The Math Snacks Early Algebra assessment was designed to measure an individual student's understanding of early algebra as it relates to experience with playing *Agrinautica*, *Curse Reverse*, and *Creature Caverns*, as well as the associated lessons. The assessment was intended to measure learning acquired while playing *Agrinautica* that falls within the project's key concept of *Write and Interpret Expressions*. The assessment was also intended to measure learning acquired while playing *Curse Reverse* that falls within the project's key concepts of *Express Patterns and Relationships Between Quantities* and of *Write and Interpret Expressions*. Lastly, the assessment was intended to measure learning associated with *Creature Caverns,* related to the key concept of *Express Patterns and Relationships Between Quantities*. The key concepts are linked to Blanton et al.'s (2015) big ideas of early algebra (see Table 1).

Table 1.
Math Snacks Early Algebra assessment item alignment with early algebra concepts and original source

Item Number	Math Snacks Key Concept	Big Idea (Blanton et al., 2015)	Game/ Interactive	Source
1a, 1b	Write/Interpret Expressions	1, 4	Curse Reverse	Blanton et al. (2015)
2a, 2b, 2c	Express Patterns/Relationship	3	Curse Reverse	Foegen and Dougherty (2017)
3	Write/Interpret Expressions	1, 4	Curse Reverse	Blanton et al. (2015)
4	Express Patterns/Relationship	1, 3, 4	Curse Reverse	Foegen and Dougherty (2017)
5a, 5b	Write/Interpret Expressions	1	Agrinautica	Blanton et al. (2015)
6	Write/Interpret Expressions	1	Agrinautica	In-House
7	Write/Interpret Expressions	1, 4	Agrinautica, Curse Reverse	In-House
8E1, 8E2, 8E3, 8b	Write/Interpret Expressions	1	Agrinautica	In-House
9a, 9b, 9c	Express Patterns/Relationship	1, 3, 4	Curse Reverse	Foegen and Dougherty (2017)
10a, 10b, 10c	Express Patterns/Relationship	1, 3, 4	Creature Caves	Rivera (2010)

Note: Blanton et al.'s (2015) big ideas are numbered as 1) equivalence, expressions, equations, inequalities; 3) functional thinking; 4) variable

Item selection and construction. As suggested by AERA et al. (2014), a main goal was to maintain a close connection to our purpose. Therefore, our bank of assessment items had to be adapted to more closely relate to the intervention. Ultimately, three items were modified from Blanton et al. (2015), three from Foegen and Dougherty (2017), one from Rivera (2010), and three were constructed by project staff. This resulted in 10 items including a total of 22 sub-parts. Item 7 was multiple choice and the others were open-response.

Item revision. After initial construction, the assessment was field tested with 45 upcoming 6th graders during a summer camp to aid in development of the assessment and games. After a round of revisions based on qualitative inspection of student responses, the assessment was sent out for expert review. After a round of revision from expert review, we conducted task-based clinical interviews with three pairs of 5th graders. After final revisions, the assessment included 10 items and 21 sub-items (see Table 1).

Inter-Rater Reliability

First, rubrics were created to dichotomously score sub-items. All assessments were de-identified. Eight raters attended a training session and independently coded two assessments. After resolving differences, rubrics were revised. Then, all raters were given the same set of 18 randomly selected assessments to independently code. The average proportion of agreement ranged from 91.7% to 98.2%. Moreover, of the 18*21=378 sub-items, only 19 had less than 75% agreement, and 319 had 100% agreement, providing evidence of interrater reliability.

Analysis

To assess test content validity, we examined the literature for items previously used to assess early algebra concepts, aligned the items to current theories on the important concepts to be learned in early algebra, and submitted the assessment for expert review. All 3 expert reviewers reported that the test content was aligned with the purpose of the instrument, the intended target content of early algebra, and with the opportunities to learn the content within the intervention.

To analyze internal structure validity, we first conducted a Rasch PCA and examined person and item fit statistics to examine unidimensionality. We then used results from a Rasch model (Bond & Fox, 2015), conducted using the eRm package (Mair & Hatzinger, 2007) in R, to examine person fit statistics and a person-item map. Infit and outfit mean square values should be less than 1.3 (Fox & Jones, 1998), and outfit and infit *t* values should be within ± 2.0 (Bond & Fox, 2015). For items outside of these ranges, we examined theoretical Item Characteristic Curves (ICC) in relation to actual performance to identify items varying substantially from the model (Bond & Fox, 2015). Last, Cronbach's alpha was used to examine internal reliability and a differential item function (DIF) analysis was conducted using the Mantel-Haenszel method for grade level and general logistic regression method for comparing by teacher and school to examine consistency across subgroups, using the difR package (Magis, Béland, & Raiche, 2018).

Results

Validity Evidence for Internal Structure

Examining unidimensionality. Because we assumed that the instrument was unidimensional, we first examined infit statistics and conducted a PCA on the standardized

Rasch residuals. Infit mean square values were within suggested limits, ranging from 0.808 to 1.171, implying that the model's predicted variability matched closely with the observed variability—an indicator of unidimensionality. Although several infit *t*-statistics were larger than expected values, it could be due to the large sample size (Bond & Fox, 2015).

The PCA revealed an eigenvalue greater than 2.0 for the first dimension, providing evidence of multidimensionality (Linacre, 2019). Further analysis indicated that item 10 had much larger factor loadings than the others. After reviewing the content of item 10, we removed it because the content was quite different from other items and had been designed to align with *Creature Caverns*—which was not fully developed before the assessment was finalized.

After removing item 10, a new Rasch PCA was conducted. Results indicated that no dimension's eigenvalue was greater than 2.0, and therefore, unidimensionality could be assumed. Follow-up examination of infit mean square values (see Table 2) showed all items were within bounds, and *t*-statistics indicated, as before, that some items were larger than expected—a possible symptom of the larger sample size. When combined with the Rasch PCA analysis, we concluded that the instrument was unidimensional. The remaining sections incorporate the 9 items, collectively containing 18 sub-parts, found to make up a unidimensional instrument.

Examining person fit statistics and person-item map. Rasch person fit statistics were found to largely be within an acceptable range, with mean square values near 1.0 (M_{outfit} = 1.037, M_{infit} = 0.957) and *z*-statistics near 0 ($M_{\text{z outfit}}$ = 0.026, $M_{\text{z infit}}$ = -0.163). Moreover, the vast majority of person infit *t*-statistics were within the expected bounds of ± 2.0. Furthermore, the person-item map (see Figure 1) revealed spread across the latent dimension, with the majority of students having ability scores less than 1.0. Moreover, items were found to have clustered well with difficulty expectations. For instance, items 8E1, 8E2, and 8E3 asked students to write expressions of increasing complexity. However, none of these three items required anything beyond *writing* the expression. Item 8b was expected to be more difficult because it required students to evaluate one of their expressions, and it was observed to be nearly twice as difficult as the simplest expression (8E1). Furthermore, items requiring the use of variable (items 1, 3, 4, 7, 9) were found among the most difficult, as expected.

Outfit statistics were out of bounds (see Table 2) for items 7, 8b, and 1a. Therefore, we examined the theoretical ICCs in relation to actual performance to locate potential erratic responses. For item 7, although the model appeared to over-predict a bit more frequently than it under-predicted, the mismatch between theoretical and empirical was quite small. This seems to imply no cause for concern (Bond & Fox, 2015). Similarly, the comparison of theoretical to empirical ICC for items 8b and 1a did not relay concern (Bond & Fox, 2015).

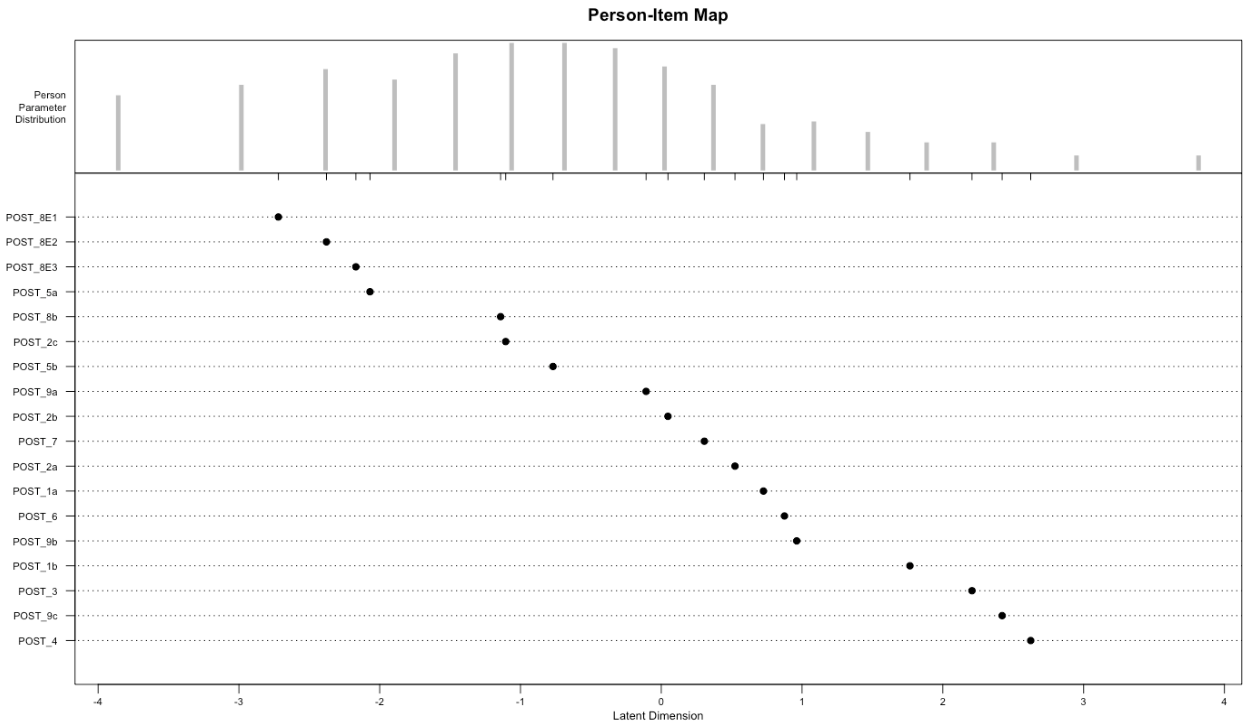

Figure 1. Person-item map of Rasch model, not including item 10

Cronbach's alpha revealed appropriate levels of internal reliability (α = 0.85, CI: 0.83, 0.87). DIF analyses revealed no DIF by teacher or school, but a DIF for three items when compared by grade level. Item 6 and 9b observed high effect sizes (Δ_{MH} = 7.49 and -1.92, respectively) and item 9a observed a moderate effect size (Δ_{MH} = -1.45). However, these differences are expected due to differences in grade level content standards.

Table 2.
Item statistics

	Item Difficulty	Standard Error	Lower CI	Upper CI	Outfit		Infit	
					Mean Square	*t* -statistic	Mean Square	*t-statistic*
8E1	-2.720	0.134	-2.981	-2.458	0.707	-1.370	0.834	-2.33
8E2	-2.378	0.126	-2.624	-2.132	1.01	0.110	0.868	-2.03
8E3	-2.169	0.122	-2.407	-1.931	0.754	-1.540	0.826	-2.9
5a	-2.069	0.120	-2.304	-1.834	1.079	0.530	0.987	-0.19
8b	-1.141	0.110	-1.357	-0.926	1.5	4.070	1.119	2.32
2c	-1.105	0.110	-1.320	-0.890	1.262	2.3	0.951	-0.99
5b	-0.769	0.109	-0.983	-0.556	1.004	0.07	0.955	-0.93
9a	-0.108	0.112	-0.327	0.111	0.817	-1.67	0.865	-2.71
2b	0.048	0.114	-0.175	0.270	0.757	-2.15	0.854	-2.82
7	0.306	0.117	0.077	0.535	1.816	4.88	1.105	1.76
2a	0.524	0.121	0.287	0.760	1.025	0.21	1.047	0.76
1a	0.726	0.125	0.481	0.970	1.373	2.04	0.97	-0.43
6	0.875	0.128	0.624	1.126	1.273	1.44	1.112	1.52
9b	0.962	0.130	0.707	1.218	0.776	-1.21	0.853	-2.05
1b	1.766	0.157	1.458	2.075	0.789	-0.69	0.809	-1.96
3	2.207	0.178	1.858	2.556	1.205	0.65	0.792	-1.78
9c	2.421	0.190	2.048	2.794	0.704	0.906	-0.68	-0.68
4	2.624	0.203	2.226	3.022	0.822	-0.27	0.826	-1.22

Discussion

The learning of early algebra concepts is an important precursor to formal algebra concepts introduced in secondary grade levels (Stephens et al., 2017). However, in order to adequately respond to calls for longitudinal, experimental, and large-scale studies that have the power to provide evidence of scalable programs (e.g., Stephens et al., 2017), a quantitative measure of early algebra concepts that has strong validity evidence is needed. This study provides validity evidence of content and internal structure for the Math Snacks Early Algebra assessment instrument.

Although results provided positive validity evidence, evidence for response processes, relation to other variables, and consequences of testing are needed in order to provide a strong validity argument for the assessment's use in research. Further, the assessment is limited in scope by an explicit purpose to measure learning achieved by 4th and 5th students playing *Agrinautica* and *Curse Reverse* in combination with the developed lessons that made up the Math Snacks Early Algebra intervention. Moreover, validity evidence is limited by student demographics and the location of the school district. One potential change in validity evidence that is reasonable to expect, and needs to be researched, is that item difficulty levels could be reduced from students spending longer playing the games, thereby possibly making it necessary to add items to ensure adequate coverage of the scope of latent ability scores. Additionally, since the item associated with *Creature Caves* was removed, it would be worthwhile for future research to re-examine how to measure student learning of early algebra as it relates to this interactive.

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MEASURING WHAT WE INTEND: A VALIDATION ARGUMENT FOR THE GRADE 5 PROBLEM-SOLVING MEASURE (PSM5)

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The purpose of this proceeding is to share validity evidence for the Problem-solving Measure for grade 5 (PSM5). The PSM5 is one test in the PSM series, which is designed for grades 3-8. PSMs are intended to measure students' problem-solving performance related to the Common Core State Standards for Mathematics (i.e., content and practices). In addition to sharing validity evidence connected to the PSM5, we discuss implications for its use in current research and practice.

Introduction

Problem solving is found in both the Standards for Mathematics Content and Standards for Mathematical Practice (Common Core State Standards Initiative [CCSSI], 2010). There is no doubt about its importance as part of classroom instruction (National Council of Teachers of Mathematics, 2000). Because it is an important part of instruction, it should be assessed in a way that provides students, teachers, and other school personnel with valuable information. Unfortunately, there continues to be few quantitative measures of problem solving that align with mathematics standards (Bostic, Krupa, & Shih, 2019; Bostic, Sondergeld, Folger, & Kruse, 2017). The purpose of this manuscript is to provide a validation argument for a new test within a series of Problem-solving Measures (PSMs). The PSMs are designed for grades 3-8 students learning mathematics. The test in the present study is meant for grade 5 students; hence, it is called the PSM5.

Relevant Literature

Problems and Problem Solving

There are entwined, mutually beneficial frameworks intended to frame the purpose and intent of the PSM5 and its items, specifically problem solving and problems. First, problems were defined using two frameworks. The first framework was Schoenfeld's (2011) notion that problems are tasks for a problem solver such that (a) it is unclear whether there is a solution, (b) it is unknown how many solutions exist, and (c) the pathway to the solution is unclear. The second framework for problems stems from work conducted by Verschaffel and colleagues

(1999). Problems are (a) open, (b) complex, and (c) realistic tasks for an individual. Open tasks can be solved using multiple developmentally-appropriate strategies. Complex tasks are not readily solvable by a respondent and require productive thinking. Realistic tasks may draw upon real-life experiences, experiential knowledge, and/or believable events. These two frameworks for problems are synergistic and provided PSM5 developers a roadmap for what should be included in tasks.

The framework for problem solving that guides PSM development is a process of "several iterative cycles of expressing, testing and revising mathematical interpretations – and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics" (Lesh & Zawojewski, 2007, p. 782). Such a problem-solving perspective requires tasks that encourage students to engage in productive, reflective, goal-oriented problem solving (Schoenfeld, 2011; Yee & Bostic, 2014). Problem solving takes substantially more cognitive effort compared to solving routine tasks (Polya, 1945/2004).

Validity and Validity Arguments

Validation is an important part of the assessment development process and while it, "may not be easy…it is generally possible to do a reasonably good job of [it] with a manageable level of effort" (Kane, 2016, p. 79). Validation, broadly speaking, involves the process of gathering evidence and constructing an argument that connects an instrument's outcomes and/or interpretations from it to its designed purpose (Kane, 2012). Validity is "the degree to which evidence and theory support the interpretations of test scores for proposed uses of tests" (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education [AERA, APA, & NCME], 2014, p. 11). Second, this research draws upon the *Standards* (AERA et al., 2014), which describe five sources of validity as necessary facets for assessment development: test content, response process, internal structure, relations to other variables, and consequences from testing. Claims from the PSM5 are associated with the definitions of each source. Third, a validation argument typically follows a specific format (e.g., Kane, 2016; Pellegrino, Dibello, & Goldman, 2016; Wilson & Wilmot, 2019) to convey validity evidence. A validation argument serves to inform readers of the validity evidence and why it justifiably grounds the implications and results from an instrument. To that end, the research question for the present study was: What is validity evidence associated with the PSM5? This study builds upon prior PSM work and its authors seek to develop a validity argument for the PSM5 using this evidence.

Method

This study draws upon a design science approach (Middleton, Gorard, Taylor, & Bannan-Ritland, 2003) and connects with recent literature that validation is a methodology within mathematics education research (Jacobsen & Borowski, 2019). Design science research is valuable for creating products that can be evaluated, refined, and re-evaluated. Jacobsen and Borowski argued that validation work serves as a methodology unto itself because there are specific characteristics of such work. For the purposes of this study, the *Standards* (AERA et al., 2014) were chosen as a mechanism to convey the validity argument for this manuscript. This approach for the validity argument was used for previous research examining the PSMs.

The *Standards* (AERA et al., 2014) advocate for assessment developers to gather evidence for the five sources; however, the quality of evidence rather than the quantity of evidence is more important. Past research that has drawn solely upon test content and internal consistency evidence does not provide a sufficiently robust validity argument such that others might trust that the results and interpretations are valid (Bostic, 2017).

Instrument and Participants

There were two groups of participants involved in this study. All names are pseudonyms. The study was approved by the Institutional Review Board. The first group was fifth-grade students. Fifth-grade students participated in think-aloud interviews, consequences from testing/bias interviews, and actual testing of the PSM5. Students were purposefully selected from rural, suburban, and urban districts within the Midwest USA. Seventy-three students in total participated in think alouds and 335 students participated in PSM5 test administration. The second group of participants were fifth-grade teachers, mathematics teacher educators whose focus is elementary grade levels, and mathematicians who have expertise is teaching mathematics content for elementary teachers. All adult participants for the expert panel communicated having sufficient understanding of the Common Core State Standards (CCSS) and agreed to review the PSM5 for content and potential bias.

The PSM5 that students completed May 2019 contained 18 items meant to measure students' problem-solving performance within the context of CCSS for Mathematics Content (SMC) and Practices (SMPs) as seen in Figure 1. There are at least three items for each of the five mathematical domains found in the fifth-grade SMCs (i.e., Operations and Algebraic Thinking, Number and Base Ten, Number and Fractions, Geometry, and Measurement and Data). A sample PSM5 item reads:

> The State Nut Company buys 22 pounds of pecans, 30 pounds of walnuts, 30 pounds of peanuts, 25 pounds hazelnuts, and 30 pounds of almonds. They sell mixed-nuts in 2.5-pound containers, which contain exactly 0.5 pounds of each nut type. How many containers will they make?

Items have been previously reviewed by an expert panel and those results were reported in Bostic, Matney, Sondergeld, and Stone (2018).

Item #	Item Description	Primary Standard	Secondary Standard (if applicable)
1	Allowance	5.OA.1	
2	Cherries	5.NF.2	
3	Keiko's House	5.G.3	
4	Pencil	5.OA.1	
5	Candy	5.NBT.6	
6	Goody Bag	5.NF.2	
7	Package	5.MD.5C	
8	Water Station	5.G.2	
9	Nut	5.NBT.7	
10	Milk	5.NF.6	
11	Sandbox	5.MD.5B	
12	Mall	5.G.2	
13	Road trip	6.RP.3B	5.OA.3
14	Fudge	5.NBT.7	
15	Catch	5.G.3	
16	Hamburger	5.OA.3	
17	Fishing Company	5.NBT.7	
18	Cake	5.NV.6C	

```
MEASURE    PERSON - MAP - ITEM
      <higher ability>|<more difficult>
                      |
 3                    +
                      |  Q14
                      |
                 .    |
                 .    |T
                      |
                      |
 2              .#    +
                      |
                     T|
                ##    |
                      |  Q9
                      |S Q2
                .#    |
 1                    +
               .##    |  Q11    Q17
                      |  Q16
               .##    |
                      |
               .##   S|  Q5
                 .    |  Q1
 0          .#####    +M Q3
                 .    |  Q4
            .#####    |  Q18    Q7
                 .    |  Q13
             .####    |
                 .    |  Q10    Q6
            ######    |
-1               .    +
                 .   M|
            .#####    |S Q15
                 .    |
       .##########    |
                      |  Q8
                 .    |
-2        .#######    +
                 .    |
                 .    |
                 .    |T
          .#######   S|  Q12
                      |
                      |
-3               .    +
       <lower ability>|<less difficult>
EACH "#" IS 4: EACH "." IS 1 TO 3
```

Figure 1. Linking PSM5 items with mathematics content standards and variable map

Data Collection and Analysis

Table 1 provides an outline of data collected, analysis technique used, and how it connects to the validity evidence framework. Expert panel reports were gathered from multiple fifth-grade mathematics teachers who had more than three years teaching experience in that grade, mathematics teacher educators, and mathematicians. Their reports provided feedback on connections to mathematics content, mathematics practices (CCSSI, 2010), and potential areas of bias. Think alouds were conducted with fifth-grade students several months prior to test administration and immediately following test administration. The goals for early think alouds were to explore ways that students might respond to PSM5 items. Think alouds following test administration were conducted to discern students' feelings and affect after testing. These qualitative data were analyzed using thematic analysis, similar to past PSM analyses (see Bostic & Sondergeld, 2015; Bostic et al., 2017). Thematic analysis aims to generate a theme or central idea from evidence (Creswell, 2012; Hatch, 2002). Quantitative data collection for relations to other variable evidence included collecting demographic evidence about the 335 respondents. Students' responses to the items were analyzed using Rasch modeling to interpret students' and items' qualities. Finally, bias was investigated using independent samples *t*-tests and Rasch (Rasch, 1960/1980) techniques to explore whether there were any differences in students' performance.

Table 1

Connections between validity evidence, data collection, and data analysis

Validity Evidence Source	**Data collected**	**Data analysis technique**
Test Content	Expert panel reports from 4 grade-level teachers, 2 mathematics educators, and 2 mathematicians participated. (qualitative)	Thematic analysis (Creswell, 2012; Hatch, 2002)
Response processes	Think-aloud data with representative purposeful sampling of students (i.e., different ability levels, genders, and geographic context) (n=73; qualitative)	Thematic analysis (Creswell, 2012; Hatch, 2002)
Relations to other variables	Ability level, gender, and geographic contexts (quantitative)	Independent samples *t*-tests
Internal Structure	Test results from 335 respondents across 4 schools (quantitative)	Rasch modeling
Consequences from testing/bias	Expert panel reports, think alouds with purposeful, representative sample of students following test administration, teacher interviews following test administration, and analyzing relations to other variables evidence (mixed methods)	Thematic analysis (Creswell, 2012; Hatch, 2002) Independent samples *t*-tests

Results

The results from validity evidence analysis are presented in relation to the five sources. A variable map is provided in Figure 1. First, the experts provided positive feedback indicating that the PSM5 items were connected to fifth-grade SMCs, address the SMPs, could be solved using multiple developmentally-appropriate strategies, were complex enough to be considered problems, and drew upon realistic contexts. Second, response processes results indicated that students were able to use appropriate mathematical strategies while problem solving PSM5 items. Readability of the items was not an issue, as evidence by students' abilities to read and understand what each question asked. Third, evidence about relations to other variables suggested that the PSM5 functioned as desired. Independent samples *t*-tests comparing ability levels, gender, and ethnicity all reported expected results. Higher ability students outperformed average-ability and below average-ability students ($p<.05$). There were no statistically significant differences between white and non-white students ($p>.05$) as well as no differences between performances by gender ($p>.05$). There were also no statistically significant differences between students from different geographic locations (i.e., rural, suburban, and urban; $p>.05$). Some items indicated that females performed better than males whereas other items suggested that males performed better than females, which is normal for an entire measure.

Collectively speaking however, there was no overall difference between male and female performance on the PSM5. Fourth, internal structure evidence was evident that psychometrically the test functioned effectively. Separation and reliability scores of 2.00 and .80 are considered good while 3.00 and .90 are considered excellent (Duncan, Bode, Lai, & Perera, 2003). Person separation (i.e., number of distinct groups that can be classified on the variable) and reliability were trending towards good (i.e., 1.6 and .73 respectively). Item separation and reliability exceeded the threshold for excellent (7.0 and .98 respectively). Finally, the expert panel and students reported that they did not experience or notice any bias in the PSM5. Post-test administration interviews revealed that students felt that the test was similar to a unit test. Students reported feeling satisfied that their results might be used to inform teachers' instruction. Bias analyses from quantitative data revealed that across the test as a whole, bias was not weighted towards one group (e.g., males or females).

Discussion and Next Steps

Taken collectively, the validity evidence indicated that the PSM5 functions as intended. This evidence parallels the quality of validity evidence seen in the PSM6-8 series, which addresses expectations described in the *Standards* (AERA et al., 2014). This new PSM5 also extends the PSM series into elementary grade levels. Work on the PSM3 and PSM4 is running parallel to the

PSM5, which will offer an assessment series that has potential to examine students' progress from elementary school into middle school mathematics content. Teachers and school personnel as well as researchers interested in fifth-grade students' problem-solving outcomes may feel confident that this validity evidence supports results and interpretations linked to the PSM5.

Drawing upon the design-science approach to this work, the development team has revised the PSM5 with the intent to improve the person separation values and to shorten the test. Both features are likely to improve quality and result in better psychometric values. While person separation and reliability are lower than desired, measuring students' problem solving can present issues because problem solving is more difficult than performance on exercises or other routine mathematics items (Bostic & Sondergeld, 2015). Thus, it might be expected to have low person separation scores. Another next step is revising the PSM5 to include fewer items; however, drawing upon high quality items. This may result in higher reliability that meets or exceed recommendations. The results for this validation study stem from data collected May 2019. A revised PSM5 was piloted during September 2019, which will generate new internal structure findings to report and hopefully improved psychometric findings. This manuscript offers validity evidence, which will be taken up for the ensuing PSM validity argument.

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Increasing the Odds for Validity and Assessment

ELEMENTARY MATHEMATICS TEACHER'S KNOWLEDGE AND IMPLEMENTATION OF HIGH LEVERAGE TEACHING PRACTICES

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Over the last two decades, significant attention has been given to mathematics teaching and learning, as demonstrated by national reform models and national curriculum changes. As a result, mathematics teacher educators came to identify a set of practices referred to as High Leverage Teaching Practices (HLTP) that researchers believe improve the teaching and indirectly the learning of mathematics. This study sought to determine if practicing elementary mathematics teachers identified as effective use HLTP and how they describe their use of these practices.

Background

In 1989 the National Council of Teachers of Mathematics (NCTM) released the *Curriculum and Evaluation Standards for School Mathematics,* starting an unprecedented standards-based movement to improve mathematics education systemically in the U.S. (NCTM, 1989, 2000). Concurrent to the adoption of the last NCTM standards, the National Research Council (2001) published two well-received documents providing recommendations about mathematics teaching. Ten years later, a national curriculum emerged in the form of the Common Core State Standards Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) adopted by forty-five states. However, even with the plethora of standards and recommendations for K-12 mathematics education, NCTM acknowledges that the mathematics preparation of students is "far from where we need to be and that much still needs to be accomplished" (NCTM, 2000, p. 6). These curriculum reforms and ongoing improvement of the NCTM standards prompted researchers to begin the development of High Leverage Teaching Practices (HLTP) in mathematics.

High Leverage Teaching Practices

Ball and Forzani (2011), along with the University of Michigan College of Education faculty, developed a common knowledge base for teaching with a thorough analysis of existing research studies and teacher practices. They identified 100 practices that teachers do in their classrooms. As part of this analysis, classroom teachers from all over the United States reviewed the list of practices and made additions and revisions. Using a set of pre-identified criteria, Ball and Forzani began identifying high leverage practices or practices that have the most significant impact on student outcomes. Ball and Forzani (2011) defined high leverage practices as those practices or tasks that are significant to teaching. If carried out skillfully and effectively by the teacher, HLTP will likely result in an increase in student achievement (Ball & Forzani, 2011). HLTP are useful across a wide range of subject areas and grade levels and will likely help to meet the academic needs of all students.

HLTP in Mathematics

Following the work of Ball and Forzani (2011) in the development of High Leverage teaching practices in mathematics, NCTM released Principles to Actions (2014) identifying eight specific high-leverage mathematics teaching practices that offer a framework to improve the teaching and indirectly the learning of mathematics. The following eight practices represent HLTP that were necessary to help students develop deep mathematical understanding: Establish mathematics goals to focus learning, implement tasks that promote reasoning and problem solving, use and connect mathematical representations, facilitate meaningful mathematical discourse, pose purposeful questions, build procedural fluency from conceptual understanding, support productive struggle in learning mathematics, elicit and use evidence of student thinking. For student achievement to increase in elementary mathematics, NCTM believes these practices are "essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2000, p. 21).

Implementing High Leverage Teaching Practices in Mathematics

While elementary mathematics learners would benefit when teachers implement research-informed HLTP, researchers have noted that many pre-service teachers struggle with these practices (Davin, 2013). For example, Davin (2013) studied four elementary pre-service teachers during their field experience to observe their implementation of the specific HLTP of questioning and increased interaction. He found that pre-service teachers consistently had difficulty implementing the specific HLTP that involve meaningful interactions between teacher and student. Davin (2013) suggests that pre-service teachers often struggle to move away from their lesson plans and often miss authentic opportunities for learning. Grossman (2011) echoes the assertions of Davin (2013) concerning pre-service teachers. HLTP may take years of practice to master, and teachers may need to be supported as they struggle with implementation.

Literature Review

In recent years, there has been a renewed interest in the study of practice-based teacher education across many academic areas (Billingsley & Bettini, 2017). In particular, researchers and teacher educators have focused on identifying high-leverage teaching practices (HLTP) or practices that, when implemented correctly by teachers, are believed to support higher levels of student achievement than other teaching practices (Hlas & Hlas, 2012). Currently, many researchers are focusing on identifying effective teaching practices, decomposing them into micro-practices, and using the information gained to inform teacher education (Adnot, Dee, Katz, & Wcykoff, 2017).

Many policymakers are concerned with employing 'better' teachers and developing new approaches to teacher evaluation and accountability rather than advancing the infrastructure and knowledge base required for high-quality instruction (Darling-Hammond, 2004). The teaching profession lacks a standard, widely agreed-upon definition of the characteristics of effective instructional practice. For example, several researchers have identified an essential component of instructional practice as teachers' ability to comprehend, elaborate, respond to, and extend student thinking during classroom discussion (Lampert, Boerst, & Graziani, 2011).

Although effective teaching in mathematics may be similar to productive teaching in different disciplines (Hlas & Hlas, 2012), each subject requires focused attention to those teaching practices that are most effective in supporting student learning that is specific to that discipline (Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005). Research from both mathematics education and cognitive science (Mayer, 2002) support that learning mathematics is an active process, where each student builds his or her understanding based on personal experience, feedback from peers, teachers and themselves. This research has recognized several principles of learning that provide the basis for effective mathematics teaching. Specifically, students should have experiences that allow them to engage with challenging tasks that involve meaning making, connect new learning to prior knowledge, acquire procedural and conceptual understanding, construct knowledge socially, receive descriptive and timely feedback, and develop metacognitive awareness of themselves as learners.

Lampert, Boerst, and Garziani (2011) assert that HLTP aims to not only teach all kinds of students to know mathematics but for students to be able to apply their knowledge to solve authentic, real-world problems. HLTP in mathematics focus on the learning that is co-produced by students and teachers in specific contexts, the practices that are central to teaching mathematics, and address issues of student differences and equity (Ball & Forzani, 2011).

Methodology

The purpose of this study was to describe elementary mathematics teachers' knowledge and implementation of high leverage teaching practices (HLTP). Case study design was used to examine teachers' understanding of high leverage teaching practices in mathematics and subsequent classroom instructional implementation of these practices. Merriam (1998) asserts the appropriateness of selecting a case-study methodology to complete a holistic and intensive analysis of a single, delimited object of study. Also, Yin (2003) concluded that when an investigation takes place in a real-life context and theory development is a goal of the study, a case study is a correct choice.

Participants and Context

This study took place at a public elementary school (grades K-3) located in a metro area in the southeastern U.S. This particular elementary school currently has 452 students and 46 faculty members that are certified teachers. The student body is 71% White, 14% Black, 5% Hispanic, 3% Asian, and 7% Multiracial. The percentage of students eligible for the federally funded free and reduced lunch program is 14%. Additional student demographics include students with disabilities at 10% and students with limited English proficiency at 2%. The participants (3 teachers) in this study were purposefully selected because of their experience with teaching elementary mathematics and being recommended by their principal and instructional coach as effective mathematics teachers. Both the principal and instructional coach were asked to provide their definitions of an effective teacher of mathematics, as well as, the characteristics each expect to observe during classroom instruction. The school principal and instructional coach indicated that an effective mathematics teacher is a teacher that focuses on effective questioning throughout the lesson. Also, effective teachers should encourage and allow students to solve problems in a variety of ways using different solution strategies. They also stated that when observing a lesson, students should be talking and defending their solutions and explaining their thinking in relation to their solutions. Effective teachers foster mathematical dialogue, they do not primarily focus instruction on finding answers and using classic algorithms.

Shannon (pseudonym) is a female teacher holding an Educational Specialist degree. She was near the end of her 14th year of teaching during this study. Shannon expressed that she always had difficulty in math and did not develop a better understanding of concepts until adulthood. Jessica (pseudonym) is a female with 13 years of teaching experience. Jessica holds a bachelor's degree in Early Childhood Education, a master's degree in Reading and Literacy, and an Educational Specialist degree in Educational Leadership. Jessica also reported receiving a K-5 mathematics endorsement that was offered by the school district. Jessica indicated that her motivation to become an elementary teacher stems from her difficulties in mathematics and hoping to provide a different, more positive experience for her students. Sarah (pseudonym) is a female with five years of teaching experience and three years of teaching kindergarten in the school in this study. Sarah holds both bachelor's and master's degrees in Early Childhood Education. She also obtained the K-5 Elementary Mathematics Endorsement. Sarah indicated that she believes that she is very competent in mathematics and also views herself as a strong mathematics teacher.

Data Collection

Data for this study was collected using (a) individual, semi-structured initial interviews of the three participating teachers on their knowledge of HLTP, (b) classroom observations using the Classroom Instructional Observation Protocol (CIOP) instrument, (c) audio recording of the 2 mathematics lessons taught by each participating teacher, and (d) field notes. Data collection occurred in a variety of ways to strengthen the design of the study. Data triangulation involves the cross checking of multiple data sources in order to ensure construct validity. Multiple sources of evidence help to ensure results are valid (Bush, 2012). During the observations of classroom mathematics lessons, teachers were observed in order to document the ways in which teachers implement HLTP. Also, during in-depth interviews teachers were asked to describe their knowledge of HLTP and their perceptions of and experiences with implementing them in their classrooms.

Data Analysis

After each interview and observation, audio recordings were transcribed verbatim and memos written to further ideas or questions that emerged during the transcription process. Constant Comparative data analysis involves the process of making meaning from data, a process involving the consolidation, reduction, and interpretation of what has been said and observed (Merriam, 1998).

All data found from the *a priori* coding of the interviews and field notes was analyzed using the process of content analysis in an attempt to quantify the frequency of themes related to high-leverage teaching practices. For this study, the following *a priori* HLTP were identified as observable behaviors from the full list of HLTP and were used for initial coding: (1) meaningful mathematical discourse (MMD), (2) purposeful questions (PQ), (3) support productive struggle (SPS), and (4) promoting reasoning and problem solving (PRPS).

Findings

Four *a priori* codes were examined for this study. They were: teachers use of purposeful questioning, teachers supporting productive struggle, teachers promoting reasoning and problem solving, and teachers encouraging mathematical discourse. The findings from this study suggest that experienced elementary mathematics teachers do implement high leverage teaching practices in their classrooms with varying frequencies. For the lessons observed with each teacher, Shannon asked 52 purposeful questions, Jessica asked 42 purposeful questions, and Sarah asked 29 purposeful questions. Also, during the lessons, Shannon was observed supporting productive struggle eight times, while Jessica was observed nine times, and Sarah 2 times. Additionally, Shannon was observed promoting reasoning and problem solving 24 times, Jessica was observed developing reasoning and problem solving 31 times, and Sarah was observed 17 times. Finally, during the lesson observations, Shannon was observed encouraging mathematical discourse 52 times, while Jessica was observed 40 times, and Sarah observed 22 times.

Although the use of HLTPs was not identified as a specific criterion for selection, it appears that at least, in this case, elementary mathematics teachers do use HLTPs. All three mathematics teachers were observed implementing HLTP's, and all reported having no formal training in the use of these practices. The primary difference across the three teachers was that the more experienced teacher used high leverage teaching practices more frequently, suggesting that over time, teachers may become more confident in their teaching and become more flexible and extensive in their questioning and use of student-centered practices. Shannon, the most experienced teacher in the study, was observed using HLTP’s a minimum of 10 more times than the other teachers. This is consistent with previous studies that found differences in the amount of use of teaching behaviors exhibited between more novice and experienced teachers (Borko & Livingston, 1989) and Mathematical Knowledge for Teaching (Hill et al., 2005).

During the interviews, all teachers expressed little knowledge of HLTP’s when using the formal label, but when prompted as to what these practices involved they all expressed having a significant amount of understanding about these instructional practices and their use. Sarah, for example indicated, “Yes, I would say that asking questions is just good teaching in math.” Sarah also stated, “I think that using HLTP’s in the classroom provides a framework for teaching. I feel like I can always go back and use the strategies in my planning and instruction.” Shannon agreed with Sarah in stating, “I always try to focus my questioning so my students will have the opportunity to problem solve and communicate their thinking. When I am using these practices, I feel that it helps me to ask the students a lot of how and why questions to further develop their thinking. Jessica, also stated, “Yes, I believe that the HLTP’s help me to focus my instruction on the types of questions and problems I wanted to give to my students. Teaching using these types of practices has helped me to use more open-ended questions with my students to help them understand math better. All three participants in this study were able to describe using HTLP's in their mathematics instruction and exhibited these behaviors in their teaching, providing confirmatory evidence of NCTM's position regarding HLTP.

Table 1

Frequencies of Teacher’s use of HLTP

Purposeful Questions	Shannon (15 years experience)	Jessica (13 years experience)	Sarah (5 years experience)
Lesson 1	27	23	18
Lesson 2	25	17	11
Supporting Productive Struggle			
Lesson 1	5	7	0
Lesson 2	3	2	2
Promoting Reasoning and Problem Solving			
Lesson 1	11	14	12
Lesson 2	13	17	5
Encouraging Mathematical Discourse			
Lesson 1	27	23	15
Lesson 2	25	17	7

Conclusions

This study was a preliminary examination of the use of HLTPs in the elementary classroom. Perhaps its most significant contribution is the exposure of the additional research needed on HLTPs before being implemented on a large-scale basis without clear evidence of impact. There is no doubt that the use of HLTP as a framework for instructional practice in mathematics provides a useful perspective for the professional development of elementary teachers. In the future, it will be necessary for teacher evaluators to assess the rigor and depth of the mathematics content to ensure teacher’s use of HLTP is effectively addressing the mathematics standards.

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EXAMINING PRESERVICE TEACHERS' STEM DISPOSITIONS THROUGH INFORMAL LEARNING

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Preservice teachers' dispositions influence how they teach STEM. This study examined 46 preservice teachers' dispositions about integrated STEM after they participated in an informal STEM learning experience. Data included the Teacher Efficacy and Attitudes Toward STEM Survey. A paired t-test was conducted on the mathematics teaching efficacy and beliefs, elementary STEM instruction, STEM career awareness, and science teaching efficacy and beliefs categories. All categories were significantly different except for 21st Century Skills. This study demonstrates how participating in informal STEM learning environments can be used to positively shape preservice teachers' dispositions towards integrated STEM in mathematics.

Introduction

As society is becoming increasingly more dependent on science, technology, engineering and mathematics (STEM), everyone needs to develop STEM literacy to function in society (Bybee, 2018). Therefore, it is important for preservice teachers to be able to provide students with access to quality STEM education, even if these students choose not to pursue a STEM career (Maiorca & Mohr-Schroeder, in press). All students should have access to relevant and engaging mathematics curriculum that serves as a foundation for STEM (National Council of Teachers of Mathematics, 2018). This can be accomplished through integrated STEM learning experiences (Roberts et al., 2019).

Literature Review

Preservice elementary teachers' dispositions are important as they influence their willingness and ability to use integrated STEM to teach mathematics. Dispositions are defined by the Council for the Accreditation of the Educator Preparation (2015) as, "The habits of professional action and moral commitments that underlie an educator's performance" (Dispositions section, para. 6). Dispositions are influenced by beliefs because beliefs can "be thought of as lenses that affect one's view of some aspect of the world" and are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). Research has shown a connection between dispositions and the mathematical teaching practices implemented in the classroom (Ball & Cohen, 1999; Philipp, 2007; Wilkins, 2008). Often preservice teachers' personal experiences in mathematics remain the default mode of instruction

(Foss & Kleinsasser, 1996; Thomas & Pederson, 2003). The connection between teachers' dispositions and practices is also true in STEM education (Maiorca & Benken, 2019). Thus, if preservice teachers do not have the attitudes, beliefs, and values to teach mathematics through integrated STEM, they may not provide these important experiences to their students. For this study, affective dispositions were defined as the attitudes, beliefs, and values that preservice teachers hold about teaching mathematics through integrated STEM.

One reason preservice teachers may have neutral or negative dispositions about teaching mathematics through integrated STEM is they have little experience with STEM (Bybee, 2018). The dispositions preservice teachers hold about STEM education will affect the choices they make with their teaching, including interactions with students (Mohr-Schroeder, Cavalcanti, & Blyman, 2015). Philipp (2007) found preservice teachers' dispositions about teaching mathematics change when they see children's mathematical thinking; this is also true for their dispositions towards STEM. Jackson et al. (2018) explored the use of non-traditional field placements as ways to shape preservice teachers' dispositions while developing their pedagogical abilities. Our study takes a similar approach by using an informal STEM learning experience as a way to affect the dispositions of preservice teachers towards using integrated STEM to teach mathematics.

Theoretical Framework

STEM situated learning theory was used to examine how the preservice teachers' dispositions were influenced by their participation in an informal STEM learning experience (Kelley & Knowles, 2016). Situated learning theory has been used similarly to explore connections between preservice teachers' dispositions toward STEM in formal learning settings (Maiorca & Benken, 2019). Situated STEM learning occurs when STEM content is integrated within a community of practice where authentic, relevant learning takes place (Kelley & Knowles, 2016). In our study, pedagogical strategies were modeled in an elementary mathematics methods course, which served as a professional learning experience (Bybee, 2018). In this professional learning experience, teachers were active learners and participated in activities in the same manner as their students (Johnson & Sondergeld, 2016). This provided preservice teachers the opportunity to experience the STEM content knowledge and pedagogies they use in their classrooms as learners.

Objectives of the Study

The purpose of this paper is to examine how informal STEM learning environments can be used to positively influence preservice teachers' dispositions. While the literature on preservice teachers' dispositions toward mathematics is broad, there is limited research on how informal STEM experiences affect preservice teacher's dispositions towards STEM. The primary research question for this study is: Does participating in an informal STEM learning experience change preservice teachers' dispositions about implementing integrated STEM?

Methodology

To examine how participating in an informal STEM learning experience affected pre-service teachers' dispositions, including beliefs towards STEM education, data were collected from pre-service teachers, who participated in an informal STEM learning experience as part of their elementary mathematics methods course. This present study took place at a public university in the Western United States and is part of an ongoing study that uses an informal STEM learning experience as a way to positively affect the dispositions of preservice teachers enrolled in an elementary mathematics methods course.

There were 46 participants in this study, 39 identified as females and 7 identified as males. Forty-six percent of the participants identified as Latinx, 42 % as White, 2% as African American, and 9% identified as Asian. In this course preservice teachers completed a collaboratively designed STEM unit (Maiorca & Benken, 2019) and an informal STEM learning experience. Preservice teachers experienced STEM first as learners through a Model-Eliciting Activity (MEA). In this MEA, participants created a scale model to build a shelter (Maiorca & Stohlmann, 2016). Then they experienced STEM as a teacher when they read a STEM lesson for Kindergarten and found the relevant Kindergarten standards. After this unit, they participated in a week-long informal STEM learning experience for 5th through 8th-grade students. During the informal STEM learning experience, the students participated in a half a day of engineering design activities and half a day of robotics. The preservice teachers helped facilitate hands-on, integrated STEM activities with local STEM professionals, and assisted students as they designed and built robots for robotics challenges.

For this study, only the data from the pre and post administration of the *Teacher Efficacy and Attitudes Toward STEM (T-STEM)* Survey were examined (Friday Institute for Educational Innovation, 2012). The T-STEM survey was administered on the first day of the semester, and six weeks later, at the end of the semester, during summer sessions of 2017, 2018, and 2019. As this study reports on initial findings, the composite scores for each participant were calculated by averaging the scores in each category. Then, a paired *t*-test was conducted to compare the pre- and post-scores for the study participants. The following categories were examined: mathematics teaching efficacy and beliefs, elementary STEM instruction, 21st-century learning attitudes, STEM career awareness, and science teaching efficacy and beliefs. These categories were analyzed because they aligned with the work preservice teachers did in the informal learning experience (e.g., working with STEM professionals, teaching integrated STEM units that focused on mathematics and science). For each of the categories examined the Cronbach's Alpha was greater than 0.90 (Friday Institute for Educational Innovation, 2012). The Bonferroni method was used to control for type I error, and the adjusted alpha was 0.01 (Hinkle, Wiersma, & Jurs, 2003).

Results and Discussion

A paired *t*-test was conducted to compare the pre and post scores for the following categories of the T-STEM survey: mathematics teaching efficacy and beliefs, elementary STEM instruction, 21st-century learning attitudes, STEM career awareness, science teaching efficacy and beliefs (Table 1). There was a significant difference between the pre and post scores for all of the categories ($p < 0.001$) except *21st-century learning attitudes* ($p = 0.31$). This could be due to scores that were already high on the pre [$M = 4.49, SD = 0.72$] and post [$M = 4.61, SD = 0.69$] surveys.

For mathematics efficacy and beliefs, participants scored higher on the post-survey [$M = 3.98, SD = 0.63$] than the pre-survey [$M = 3.38, SD = 0.70$], reflecting the participants' higher self-efficacy and beliefs after participating in the integrated STEM learning. Similarly, in science efficacy and beliefs, participants scored higher on the post-survey [$M = 3.86, SD = 0.54$] than the pre-survey [$M =3.21, SD = 0.54$]. After the integrated STEM learning experiences, participants also had higher self-efficacy and beliefs towards teaching both mathematics and science.

Table 1

Results from the T-STEM Survey Collected the First and Last Day of the Semester

Category	**Mean**		**SD**		**P-value**
	Pre	**Post**	**Pre**	**Post**	
Mathematics Teaching Efficacy and Beliefs	3.38	3.98	0.70	0.46	$p < 0.001$
Elementary STEM Instruction	3.28	1.00	4.32	0.79	$p < 0.001$
21st Century Learning Attitudes	4.49	0.72	4.61	0.69	$p < 0.314$
STEM Career Awareness	2.73	0.96	4.21	0.76	$p < 0.001$
Science Teaching Efficacy and Beliefs	3.25	0.51	3.86	0.54	$p < 0.001$

Prior research in this area has found changing beliefs about teaching science and mathematics to be difficult (Foss & Kleinsasser, 1996; Thomas & Pederson, 2003). Thomas and Pederson (2003) found their preservice teachers' initial beliefs "indicated a strong orientation toward an individual who is in charge of classroom knowledge, resources, and the environment" (p. 326). The findings are similar to the findings of this research study. In order to ensure that these positive changes in both beliefs about teaching mathematics and science continue, preservice teachers should be given multiple opportunities to collaborate with their peers and STEM professionals. Additionally, professional development is needed for inservice teachers to provide opportunities for teachers to continue to develop integrated STEM lessons.

For the category elementary STEM instruction, participants also had higher scores on the post-survey [$M = 4.32, SD = 0.61$] than on the pre-survey [$M = 3.28, SD = 0.72$]. Participants reported that they would use more STEM instructional practices in their teaching after the informal STEM learning experience. This suggests interacting within a community of practice of content experts while doing authentic activities (Kelley & Knowles, 2016) positively influenced the preservice teachers' conceptions of STEM.

STEM Career Awareness also showed a significant improvement between the pre and post-survey [$M = 2.73, SD = 0.96; M = 4.21, SD = 0.76$], respectively. The informal STEM learning experience emphasized a variety of STEM careers. This increase in awareness is important because it provides a context for teaching content. If preservice teachers are unaware of different STEM careers, then they will be unable to provide information about them to their students. Another reason this finding is important is that preservice teachers' beliefs about what STEM is influences how they implement STEM activities (Mohr-Schroeder, Cavalcanti, & Blyman, 2015) and the only way for students to learn about different careers is if their teachers can introduce their students to them (Maiorca et al., 2020).

Studies have shown when mathematics is taught using an integrated STEM context, students' performance on assessments improved (Kelley & Knowles, 2016). Moreover, when science is taught using an inquiry approach, such as those in integrated STEM contexts, students take control of their learning (Kelley & Knowles, 2016). Preservice teachers, therefore, need to have positive dispositions towards teaching STEM content. This study showed how the use of an informal STEM learning experience can help preservice teachers develop more positive dispositions towards teaching STEM content which can, as Kelley and Knowles (2016) noted, have impacts on future student learning of STEM content.

Implications

This study demonstrates how participating in informal STEM learning environments can be used to positively shape preservice teachers' beliefs towards integrated STEM in the mathematics classroom. After the preservice teachers participated in the informal STEM learning environment, their dispositions (e.g., attitudes, beliefs, and values) about integrated STEM changed. When their default teaching strategies derive from the way they were taught (Foss & Kleinsasser, 1996), preservice teachers usually do not use more effective teaching methods, such as the effective STEM teaching practices (Steele, 2019). Participating in the informal STEM learning environment allowed the preservice teachers to see the effectiveness of the integrated STEM by first engaging them as students and then having them interact with middle-grade students in integrated STEM contexts.

A second implication is the importance for students to engage in activities that challenge their beliefs. Philipp (2007) noted the importance of changing dispositions to change instructional behaviors. Preservice teachers were immersed in integrated STEM contexts through their participation in a nontraditional field experience. The goal of situating them in a community of practice that was participating in authentic STEM activities (Kelley & Knowles, 2016) was to change the STEM pedagogy strategies preservice teachers enact. The results demonstrate significant changes in mathematics teaching efficacy, elementary STEM instruction, STEM career awareness, and science teaching efficacy. Therefore, situating the preservice teachers in this informal STEM learning environment changed their dispositions.

Limitations and Future Research

One limitation is the small sample size. There have been positive results from year to year and future research will include scaling up the program and including a control group. Future research will focus on growing the program to determine if the findings are consistent with larger numbers of preservice teachers. Another limitation is that the questions in the category, *21-century skills*, on the T-STEM survey don't specifically address robotics, which was the primary technology used throughout the camp. The qualitative data analysis is ongoing, and not included in this study. Preliminary findings support the quantitative results. Future work will provide a more complete picture of the preservice teachers' experiences in the informal STEM learning environment and how those lived experiences shaped their dispositions towards integrated STEM as a context to make mathematics more relevant and engaging for students. In the future, longitudinal studies should follow the preservice teachers as they transition to inservice teachers to determine if there is a lasting change in the STEM pedagogies they enact.

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EXPLORING POSITIVE SHIFTS IN PRESERVICE ELEMENTARY TEACHERS' CONCEPTIONS OF MATHEMATICS

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Preservice elementary teachers have higher mathematics anxiety and lower self-efficacy in teaching mathematics which leads to more negative attitudes toward mathematics. These negative conceptions can adversely influence mathematics instruction (Philipp, 2007). This study qualitatively explored shifts in 84 elementary preservice teachers' conceptions of mathematics after participating in an introductory mathematics education course. Findings show positive shifts in preservice elementary teachers' views about teaching and learning mathematics with an emphasis on the effective mathematics teaching practices (NCTM, 2014). Based on these findings, preservice elementary teachers need intentional, sustained mathematics learning opportunities in order to prepare them to be effective mathematics teachers.

Introduction

Preservice elementary mathematics teachers are more likely to have higher mathematics anxiety (Vinson, 2001), lower mathematics teacher efficacy (Swars, Daane, & Giesen, 2006), and be more likely to implement traditional pedagogical techniques in the classroom (Guillaume & Kirtman, 2010). However, to be effective mathematics teachers, teachers have to understand mathematics content, how students learn mathematics, and effective pedagogical strategies to help scaffold students' learning of mathematics (Ball, Thames, & Phelps, 2008; National Council of Teachers of Mathematics (NCTM), 2014). Prior work (e.g., Roberts, Maiorca, & Roberts, 2019) has demonstrated statistically significant shifts in preservice teachers' attitudes, confidence, and content knowledge after participating in an introductory mathematics education course designed to build content knowledge, and positively influence conceptions of teaching and learning mathematics. However, all changes in conceptions did not occur at the same magnitude and some conceptions (e.g., beliefs) showed changes that were not statistically significant. The purpose of this study is to qualitatively explore how preservice elementary teachers' conceptions changed after participating in the introductory mathematics education course. The research question for this study is: How do preservice elementary teachers describe excellent mathematics teachers and instruction after participating in an introductory mathematics education course?

Conceptual Framework and Related Literature

Teachers not only need content knowledge, but also pedagogical content knowledge. In mathematics, pedagogical content knowledge consists of knowledge of content and students, knowledge of content and teaching, and knowledge of curriculum (Ball et al., 2008). Within pedagogical content knowledge, teachers have to think about students' understanding of content, how to best sequence their teaching, and what strategies would be most effective in teaching the content. The conceptions teachers have about mathematics, therefore, influence the way they teach mathematics (NCTM, 2014; Wilkins, 2008). Thompson (1992) broadly defined conceptions as a "general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences" (p. 130). Related to conceptions, McLeod (1992) defined the affective domain as "a wide range of beliefs, feelings, and moods that are generally regarded as going beyond the domain of cognition" (p. 576). Conceptions and affect include beliefs, attitudes, dispositions, anxiety, and confidence (Philipp, 2007). Dispositions are defined by the Council for the Accreditation of the Educator Preparation (2015) as, "The habits of professional action and moral commitments that underlie an educator's performance." This includes the attitudes, beliefs, and values that teachers hold. These negative conceptions and dispositions will adversely impact preservice teachers' enactment of the effective mathematics teaching practices (see Table 1; NCTM, 2014). The mathematics teaching practices are a "research-informed framework… of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2014, p. 9).

Table 1

Effective Mathematics Teaching Practices

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning
Implement tasks that promote reasoning and problem solving
Use and connect mathematical representations
Facilitate meaningful mathematical discourse
Pose purposeful questions
Builds procedural fluency from conceptual understanding
Support productive struggle in learning mathematics
Elicit and use evidence of student thinking

Elementary and special education teachers usually have lower efficacy in their mathematics teaching abilities and more have negative attitudes toward mathematics (Bursal & Paznokas, 2006; Karp, 1991; Swars et al., 2006; Vinson, 2001). Teachers are less likely to be confident teaching mathematics when they have high mathematics anxiety (Bursal & Paznokas, 2006). Similarly, teachers' negative conceptions and affect can adversely affect their mathematics instruction (Philipp, 2007). A teacher's self-efficacy, and knowledge of both content and pedagogy have been shown to influence how they approach teaching in their classrooms (Brown & Benken, 2009). Given the abundance of literature highlighting these issues, mathematics teacher educators face the challenge of navigating the complex, interrelated relationships between conceptions, content knowledge, and teaching of mathematics (Llinares, 2002) as they prepare preservice teachers to be effective at teaching mathematics.

Methodology

Setting and Participants

This study took place in an introductory mathematics education course taught at a large public research university in the Mid-Western United States. The course modeled and emphasized the importance of engaging in tasks that promote reasoning and problem solving, using and connecting mathematical representations, engaging in meaningful mathematical discourse, supporting productive struggle, and building procedural fluency from conceptual understanding (NCTM, 2014). Thus, the course was designed for preservice teachers to not only gain deeper content knowledge, but also to engage them in the sociomathematical norms critical to implementing effective mathematical teaching practices that they will use as teachers. Preservice teachers (n=84) enrolled in the Spring 2018, Fall 2018, and Spring 2019 semesters of the course participated in this study. Of the 84 preservice teachers, 80 self-identified as female, while four self-identified as male. Eighty-eight percent of the preservice teachers self-identified as White, 7% as Black, 1% as Asian, 1% as Latinx and 3% as other/mixed race.

Data Collection and Analysis

Preservice teachers completed the Mathematics Experiences and Conceptions Surveys – Entry (MECS-E) at the beginning and end of the introductory mathematics education course. The MECS-E is a Likert-type scale with six levels ranging from strongly disagree (1) to strongly agree (6). The MECS-E also has open response questions, including: in what ways do you think students most effectively learn mathematics?; and, imagine you walked into a classroom and saw the ‘best’ teacher teaching mathematics. – what do you see happening in the classroom? (Jong & Hodges, 2015). This paper focuses on participant responses to the open response questions around teaching and learning mathematics. The responses were collected at the beginning and end of the Spring 2018, Fall 2018, and Spring 2019 semesters.

A naturalistic inquiry methodology was used to qualitatively investigate preservice teachers’ conceptions about teaching and learning mathematics. Naturalistic inquiry allowed us to explore multiple realities preservice teachers created during their experience (Lincoln & Guba, 1985). We used a deductive approach with the effective mathematics teaching practices as a lens to frame important issues (Creswell, 2014) relevant to changes in preservice teachers’ conceptions of mathematics after they completed an introductory mathematics education course. One author developed a list of 10 provisional codes (Miles, Huberman, & Saldaña, 2014) and discussed the codes with the research team to make necessary revisions. One author then coded the open responses with the provisional codes for first cycle coding (Saldaña, 2015) to better understand preservice teachers’ conceptions about teaching and learning mathematics through the lens of the effective mathematics teaching practices. After initial coding, the research team conducted second cycle coding through pattern coding (Saldaña, 2015). The groupings identified related to concepts within conceptions and aligned with the effective teaching practices (e.g., productive struggle, using and connecting multiple representations, the importance of discourse). Patterns were used to identify common themes and any divergent cases (Delamont, 1992).

Findings and Discussion

Three primary themes arose from the data analysis: (a) preservice teachers emphasized the importance of productive struggle and a growth mindset in learning mathematics; (b) preservice teachers’ dispositions toward mathematics instruction shifted toward the effective mathematics teaching practices; and, (c) preservice teachers’ expressed more positive beliefs about mathematics.

Preservice teachers consistently identified elements of supporting productive struggle in learning mathematics after participating in the course. Specifically, preservice teachers emphasized the importance of encouraging students to persist in mathematics and to learn from their mistakes. As one preservice teacher explained, students learn mathematics most effectively when they are in “an environment where everyone feels it is okay to make a mistake” (Spring survey, 2018). Similarly, another preservice teacher noted it is important for teachers to allow “students to make corrections and learn from their mistakes, whether the students have a fixed or growth mindset” (Fall survey, 2018). Teachers should also be “encouraging her students to not give up and to keep going even when it is hard” (Spring survey, 2018). These ideas emphasize the importance of supporting productive struggle, one of the effective mathematics teaching practices. Moreover, these statements contrast with their initial responses that focused on “practice” (Spring survey, 2019) and doing “problems to help better their skills” (Spring survey, 2018). The end of course responses emphasize the importance of making mistakes when learning (Boaler, 2016) and show shifts in the preservice teachers’ views on mathematics instruction.

Relatedly, preservice teachers had qualitatively different responses that indicated a positive shift toward the effective mathematics teaching practices. For example, one student noted, “I think students effectively learn math by seeing examples and having hands-on activities. I feel as if doing a lecture for math isn’t the best method” (Spring survey, 2018). Similarly, one student explained, “I think students most effectively learn mathematics when learning multiple ways to solve a problem, over time, with manipulative’s and connecting it to real life situations” (Spring survey, 2018). Another student emphasized the importance of using “hands on activities and visuals… because they [students] can visually see it [the mathematics]” (Spring survey, 2019). These examples demonstrate how students mentioned the importance of using manipulatives, real world scenarios, and other hands-on methods to learn mathematics. All of these statements align with the effective mathematics teaching practices of choosing tasks that promote reasoning and problem solving and using and connecting mathematical representations. While these are broad strategies, the course provided an introduction to teaching mathematics and did not focus on pedagogy to the extent a methods class does. On the pre-survey, responses typically focused on helping “students feel comfortable” (Spring survey, 2019), having “a great teaching philosophy” (Spring survey, 2018), and showing students the “teacher cares about them” (Fall survey, 2018), as examples. One student even noted that students learn mathematics best “how she was taught throughout her years as a student” (Spring survey, 2018). These statements suggest feel good activities and using the methods they experienced as teachers are effective to teach mathematics. Compared to the pre-survey, students’ exposure to teaching that reinforced the effective mathematics teaching practices helped shift their conceptions and dispositions toward to more specific, effective pedagogical practices (NCTM, 2014).

Preservice teachers' beliefs about mathematics also exhibited positive shifts. Prior quantitative analysis showed belief composite scores from the MECS-E increased for students. While not significantly different, this confirms previous literature that demonstrates beliefs change slowly (e.g., Philipp, 2007), and the positive shifts warrant further qualitative exploration. The pre-survey responses emphasize skill based approaches to mathematics. For example, one student suggested students learn mathematics most effectively when "the student practices and does problems to help better their skills" (Spring survey, 2018). This is typical of the pre-survey responses that represents students' beliefs about mathematics as being skill based and detached from the real world. In their post-survey responses, students expressed different beliefs about mathematics. For example, one preservice teacher recommended teachers

> show and explain different ways of solving the same math question, as well as make student show and explain their work. Through both of these techniques, students understand what they are doing a little more and allows them to retain the information instead of just being lectured to (Spring survey, 2018).

This is reflective of the effective mathematics teaching practices that emphasize choosing tasks to promote reasoning, using and connecting mathematical representations, and using and eliciting student responses (NCTM, 2014). Another student identified "relating math to the outside world and multiple ways of solving problems" as critical components of good mathematics teaching (Fall survey, 2018). These responses indicate the importance of using multiple representations, including contextual, visual, verbal, physical, and symbolic representations (Lesh, Post, & Behr, 1987). Although subtle, these shifts are important because they reflect the changes preservice teachers' experience after participating in an introductory mathematics education course. Beliefs change slowly (Philipp, 2007). However, these findings demonstrate that small changes occur in as little as one course. Giving preservice elementary teachers more opportunities to engage in quality mathematics teaching and learning could allow these small changes to accumulate into meaningful changes in their beliefs.

Each of these themes demonstrate how preservice elementary teachers' conceptions changed after one introductory mathematics education course. Pre-survey responses emphasized "a great teaching philosophy" (Spring survey, 2018), "practice" (Spring survey, 2019) and doing "problems to help better their skills" (Spring survey, 2018). These examples are indicative of the generalizations and skill-based responses. Post-survey responses represented a shift in conceptions about mathematics through an emphasis on the effective mathematics teaching practices (NCTM, 2014). Preservice elementary teachers noted the importance of productive struggle, such as one student noting the importance of "encouraging her students to not give up" (Spring survey, 2018). Other students emphasized the importance of reasoning and using multiple representations, such as when one student noted, "I think students most effectively learn mathematics when learning multiple ways to solve a problem, over time, with manipulative's and connecting it to real life situations" (Spring survey, 2018). Together, the qualitative data show how preservice elementary teachers' conceptions about teaching and learning mathematics changed during their first mathematics education course.

Conclusions and Implications

These findings offer confirmation of quantitative results (e.g., Roberts et al., 2019) that demonstrate the significant positive impacts an introductory mathematics education course had on preservice elementary teachers' conceptions of mathematics. Preservice teachers can adopt the importance of growth mindset and instructional dispositions aligned with the effective mathematics teaching practices. Although beliefs do change slowly (Conner & Gomez, 2019), the findings in this paper show that beliefs do begin to positively shift even after one course. This underscores the need for elementary and special education teachers—who, as described above, generally have more negative conceptions of mathematics—to have more experiences in mathematics education courses, such as the minimum of 12 semester-hours focused on elementary mathematics content suggested by the Conference Board of the Mathematical Sciences (2012). More research is needed to determine appropriate sequencing, the best balance of content and pedagogy, and long term implications, especially once preservice teachers graduate and have their own classrooms, of sustained engagement with mathematics.

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FOCUSING ON TEACHING PRACTICES IN SECONDARY MATHEMATICS METHODS COURSES

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The purpose of this multiple case study was to examine mathematics teacher educators' (MTEs') practices by determining which teaching practices MTEs identify as most focused upon with preservice teachers in their secondary mathematics methods courses (SMMCs) and why. A background information survey, Teacher Action Q-Sort (Franz, Wilburne, Polly, & Wagstaff, 2017; Wilburne, Polly, Franz, & Wagstaff, 2018), and interviews were used to collect data. Findings suggest MTEs teaching SMMCs focus on a variety of practices for different reasons with many focusing on MTP 2: Implement tasks that promote reasoning and problem solving (National Council of Teachers of Mathematics [NCTM], 2014).

Education of preservice teachers in teacher educator programs across the United States varies greatly (Arbaugh, Ball, Grossman, Heller, & Monk, 2015). Preparation of mathematics teacher educators (MTEs) also varies (Reys, Glasgow, Teuscher, & Nevels, 2008), thus MTEs' practices may vary as well. Because new faculty members in mathematics education are likely to teach mathematics methods courses (Reys, Reys, & Estapa, 2013), it seems important to explore those practices and how they could be supported. The purpose of this study was to examine MTEs' practices by determining which teaching practices MTEs identify as *most focused upon* with preservice teachers in their secondary mathematics methods courses (SMMCs) and why.

Literature Review

The Mathematics Teaching Practices (MTPs) listed in Table 1 were developed based on 20 years of previous research in the field and can serve as a research-based framework for teaching and learning mathematics (National Council of Teachers of Mathematics [NCTM], 2014).

Table 1

Mathematics Teaching Practices (NCTM, 2014)

Mathematics Teaching Practices (MTPs)
MTP 1: Establish mathematical goals to focus learning.
MTP 2: Implement tasks that promote reasoning and problem solving.
MTP 3: Use and connect mathematical representations.
MTP 4: Facilitate meaningful mathematical discourse.
MTP 5: Pose purposeful questions.
MTP 6: Build procedural fluency from conceptual understanding.
MTP 7: Support productive struggle in learning mathematics.
MTP 8: Elicit and use evidence of student thinking.

Teaching practices, however, may look different for different MTEs. There have been a number of calls for studies related to exploring MTEs and their practices. Arbaugh and Taylor (2008) called for a research program focused specifically on MTEs, which has since been supported by Lee and Mewborn (2009) and Kastberg, Tyminski, and Sanchez (2017). Kastberg and colleagues called for exploration of frameworks and activities to "further allow MTEs to leverage their considerable practical knowledge to build lines of scholarly inquiry supportive of the development of scholarly practices" (2017, p. 1264). In a study of syllabi for mathematics methods courses, Taylor and Ronau (2006) found a great deal of variability in terms of course assignments, goals, and objectives, calling for additional research related to identifying and understanding MTEs' practices as they relate to teaching mathematics methods courses.

One way to examine MTEs' practices is by focusing on a particular dimension of practice, such as their *planning and preparation* (Danielson, 2011). Kastberg et al. (2017) called for research related to MTEs' practices in context, including descriptions and research related to their development:

> Different methods instructors will always rely on different frameworks and have different goals, but a literature base that provides insights into the work of MTEs in context, variation in scholarly practices MTEs develop, and experiences activities afford for [preservice teachers] is needed (p. 27).

The call for this research should be answered to inform MTEs' practices, support their professional development, and support those who are new to the field.

This study can serve as a response to previous calls related to MTEs' teaching practices by examining the practices they focus on with preservice teachers in SMMCs to explore MTEs' instructional decisions. To explore MTEs' practices, the researcher developed the following research question to guide the study: Which teaching practices do mathematics teacher educators identify as *most focused upon* in their secondary mathematics methods courses and why?

Methodology

Participants

The participants in this study were selected using criterion-based purposeful sampling and maximum variation sampling. Criteria required the participants to be (1) a MTE with a terminal degree at a four-year college or university teaching courses in a teacher preparation program and (2) currently teaching or have previously taught a SMMC. A survey was emailed to members of the Association of Mathematics Teacher Educators and the Association of Mathematics Teacher Educators in Texas using *Qualtrics* to locate potential participants and gather data related to their teaching practices. Then, maximum variation sampling was utilized to purposefully select six participants to serve as cases for an in-depth investigation with additional data collection to look for and describe central themes across a great deal of variation (Merriam, 2009; Patton, 2002). The cases in the study are indicated in Table 2. Pseudonyms were used for each of the cases.

Table 2

The Cases

Case	*Terminal Degree*	*Current Role*	*Years of Teaching Experience by Type*
Tammy	Ph.D. in Mathematics Education	Professor, Mathematics Department, Public Research University	8 Years – High School 27 Years – University*
Natalie	Ed.D. in Mathematics Education	Associate Professor, Mathematics Department, Public University	14 Years – High School 10 Years – University*
Amber	Ph.D. in Curriculum and Instruction	Assistant Professor, Education Department, Private Liberal Arts College	4 Years – High School 9 Years – University*
Ted	Ph.D. in Curriculum and Instruction	Associate Professor, College of Education, Private University	4 Years – Middle Grades 11 Years – High School 9 Years – University*
Nick	Ph.D. in Mathematics	Associate Professor, Mathematics Department, Public Liberal Arts University	18 Years – University*
Nora	Ph.D. in Mathematics Education	Assistant Professor, Mathematics Department, Public Research University	3 Years – Middle Grades 9 Years – University*

*May include time as a doctoral student and teacher assistant

Data Collection and Analysis

Utilizing multiple sources of data supported an in-depth understanding in this multiple case study (Creswell, 2018; Yin, 2018). The primary measures in this study included the (a) MTE Background Information Survey, (b) Teacher Action Q-Sort (Franz, Wilburne, Polly, & Wagstaff, 2017; Wilburne, Polly, Franz, & Wagstaff, 2018), and (c) MTE interviews. Artifacts including MTEs' instructional notes and activities related to the *most focused upon* MTPs were also collected to provide context. The MTE Background Information Survey was utilized to collect descriptive data related to participants' experiences and teaching practices. The survey required MTEs to rank the MTPs (NCTM, 2014) from the practice *most focused upon* in their SMMCs to the practice *least focused upon*. From the participants who completed the survey, six cases were selected for additional in-depth data collection. The Teacher Action Q-Sort was then utilized to collect information from the cases related to the MTPs (NCTM, 2014) focused upon in their SMMCs. The Teacher Action Q-Sort was initially designed such that teachers place 37 Teacher Actions Statements related to the MTPs on a forced Q-Sort grid, ranking the actions from those least characteristic of a teacher's classroom to most characteristic of a teacher's classroom (Franz et al., 2017; Wilburne et al., 2018). For the current study, the Teacher Action Q-Sort was adapted so that MTEs ranked the 37 statements from those that were *least focused upon* with preservice teachers in their SMMC to those that were *most focused upon*. The MTEs sorted the statements to determine which ones were areas of focus for preservice teachers' learning as future teachers. This provided additional information to examine which MTPs (NCTM, 2014) were *most focused upon* in the SMMCs. Lastly, the researcher conducted an interview of each case and collected artifacts to learn more about the MTEs' experiences and practices related to focusing on these MTPs with preservice teachers. The interview was made up of 14 questions; it was designed to explore instructional decisions related to focusing on particular MTPs, including reasons for MTEs' selections and designing and planning for teaching content in SMMCs. The researcher examined responses from both the MTE Background Survey and the analysis of the Teacher Action Q-Sort (Franz et al., 2017; Wilburne et al., 2018). Further, the researcher sought clarification in the interview when the *most focused upon* MTPs (NCTM, 2014) indicated by the measures did not seem to be aligned for an MTE.

Data was transcribed as needed and read multiple times by the researcher. The researcher utilized the MTPs (NCTM, 2014) to conduct initial coding of the data. A coding team including the researcher and two doctoral students analyzed the interview transcripts to enhance the validity of the study. One interview transcript was randomly selected and independently coded by each member of the coding team. The researcher then met with the team to agree upon codes. The researcher independently coded each of the remaining interviews, and the two remaining team members were each assigned four of the eight MTPs to code the remaining interviews. The researcher then met with the team again to compare findings. Constant comparative analysis of the remaining measures was used to compare the analysis to any emerging categories (Creswell, 2018). Axial coding was utilized to allow for consideration of divergent codes so that the researcher could identify any additional themes. A cross-case analysis, in which the researcher sought to build an explanation that fit each case, was conducted to lead the researcher to themes that conceptualized the data (Merriam, 2009). Member checking was utilized for each case.

Findings

The researcher reviewed the *most focused upon* MTPs (NCTM, 2014) in each case's SMMC according to the MTE Background Information Survey, the Teacher Action Q-Sort (Franz et al., 2017; Wilburne et al., 2018), and the interviews. Taken together, and confirmed by the member check, the measures revealed which MTPs (NCTM, 2014) were *most focused upon* with preservice teachers in SMMCs (see Table 3).

Table 3

Most Focused Upon Mathematics Teaching Practices

Cases	MTP 1	MTP 2	MTP 3	MTP 4	MTP 5	MTP 6	MTP 7	MTP 8
Tammy			X					
Natalie					X			X
Amber	X	X						
Ted	X				X			X
Nick		X		X				
Nora		X						

The analysis of the *most focused upon* MTPs revealed three key themes from across the data.

First, it is noticeable that when the cases are organized by the MTPs, MTEs seem to value a variety of these practices when making decisions related to their SMMCs. While MTP 2: Implement tasks that promote reasoning and problem solving is one of the *most focused upon* practices self-reported by the cases, it was selected only by three of the six cases, namely Amber, Nick, and Nora. Data analysis revealed that while this practice was selected by these three MTEs, they focused on this practice for different reasons. Amber focuses on MTP 2 because it is not addressed in any other courses in her teacher preparation program. Further, she explained that many preservice teachers in her course are already familiar with direct instruction in mathematics but need exposure to project-based learning, real world problems, rigorous mathematical content, and critical thinking. Nick utilizes rich tasks as a vehicle for supporting many other practices, such as MTP 4: Facilitate meaningful mathematical discourse, and explained that doing mathematics is about reasoning and problem solving, looking for connections, and explaining and justifying the work. Similar to Nick, Nora believes she should develop preservice teachers' understanding of teaching mathematics in a methods course as they learn to pose better tasks so that they have a chance of "actually glimpsing something cool" that opens the door for addressing other practices such as multiple representations.

It seems important to consider that MTP 2: Implement tasks that promote reasoning and problem solving was shown to be one of the *most focused upon* teaching practices for three of the six cases. While Amber indicated that focusing on this practice was important because it was not addressed in other courses, both Nick and Nora saw it as a way to address other teaching practices. Nick and Nora were the only two cases who had limited experience teaching in the secondary grades. While Nick taught mathematics education content courses at the university level, Nora taught mathematics in the middle grades. Further, Amber had the next least amount of high school teaching experience. The focus on MTP 2 by these MTEs seems to highlight its importance. Further, it is possible that MTEs with limited experiences teaching at the secondary level consider it to be particularly important to focus on with preservice teachers in SMMCs because it allows the MTEs to address other practices through the lens of rich tasks.

The second key point highlighted by Table 3 is that MTP 6: Build procedural fluency from conceptual understanding and MTP 7: Support productive struggle in learning mathematics were not identified as the *most focused upon* practice by any of the cases in this study. Many of the MTEs indicated that these practices are addressed in preservice teachers' other courses or field experiences, not the SMMC, and some MTEs explained that they address these practices in SMMCs through the lens of other practices. It is interesting that even though a variety of practices were selected to be the focus in SMMCs, these particular practices were not selected.

Third, two MTEs saw a connection between MTP 5: Pose purposeful questions and MTP 8: Elicit and use evidence of student thinking. While other connections among the MTPs were highlighted by the cases, it is of note that both Natalie and Ted selected these two particular practices to serve as focuses in their SMMCs while highlighting slightly different reasons for the connections between them. Natalie shared that "the way to do a good job of eliciting what your students are thinking is to pose questions that do that for you." Ted explained that as preservice teachers move from focusing on the mathematical goals to focusing on student mathematical thinking, this impacts the questions they ask to elicit and use that thinking. Further, Natalie believes it is crucial to begin with students' prior knowledge, and she works with preservice teachers to be able to elicit and use evidence of student thinking so that they can help their own students to move forward in their learning. Ted indicated that MTP 8 is a high-leverage teaching practice that supports the other practices. He explained that focusing on eliciting and using evidence of student thinking requires the use of high cognitive demand tasks and for preservice teachers to attend to students' progress toward meeting goals. Ted sees this as a way to support preservice teachers in becoming new teachers and so that they will continue to learn from their own teaching as they reflect on and modify lessons to support student understanding.

Discussion and Implications

Findings suggest that MTEs who teach SMMCs focus on a variety of MTPs (NCTM, 2014) with many electing to focus on MTP 2: Implement tasks that promote reasoning and problem solving. Three of the six cases focused on MTP 2 and address it for a variety of reasons, including that it is not addressed in any other courses in their teacher preparation program, the belief that preservice teachers are familiar with direct instruction but need exposure to rich tasks, that it is fundamental to mathematics, and that this practice is a vehicle for supporting other practices. Yee, Otten, and Taylor (2018) conducted a survey sent to all active members of AMTE to determine what topics were broadly valued across the field according to teachers of SMMCs. They found that *adapting, choosing, and generating mathematical tasks* was in the top five of 41 valued items, following *understanding of practices/process standards, multiple representations of mathematical ideas, attending to student thinking and using student ideas to push understandings forward,* and *mathematical knowledge for teaching.* It seems the current study can provide evidence to support Yee and colleagues' (2018) findings related to the importance of mathematical tasks. However, this study provides evidence that MTP 2 may be focused upon more than multiple representations or attending to student thinking and using student ideas.

During the interviews, some cases indicated that they were surprised at discoveries based on their reflections, such as when a MTE's initial ranking of the MTPs on the MTE Background Information Survey did not seem to align with the Teacher Action Q-Sort (Franz et al., 2017; Wilburne et al., 2018). Perhaps the most pressing recommendation related to the current study is that MTEs should analyze and reflect on the MTPs (NCTM, 2014) that are *most focused upon* in their own courses and why. This could result in making adjustments to align course syllabi and tasks to MTEs' desired focus for preservice teachers' learning. Future research related to SMMCs could build on the work of this study. Additional exploration related to the MTPs *most* and *least focused upon* based on factors such as faculty rank, years of experience as a MTE, or teaching experience with various grade bands should be conducted. Observations of MTEs teaching SMMCs or data related to preservice teachers, such as classwork, could provide an additional lens to further examine MTEs' practices. It may also be important to conduct similar studies with MTEs and elementary or middle grades mathematics methods courses.

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LESSON STUDY AND TEACHER'S DIALOGUE ABOUT SMP 5

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The purpose of this paper is to share research on the dialogue of teachers related to the Standard for Mathematical Practice 5 during the post-lesson debrief of Lesson Study. Lesson Study debriefs were recorded and transcribed for teacher teams conducting Lesson Study to improve students' mathematical problem solving. Inductive analysis was used to find similarities and differences between teacher dialogues about the SMPs. Conclusions and implications about teachers' dialogue are shared.

Introduction

In 1999, Stigler and Hiebert's book titled, *The Teaching Gap*, called for lesson study to be tried and tested in the United States (p. 131). Since that time, several researchers have shown that when it is implemented well and for sufficient duration, similar positive results to Japanese lesson studies are found (Lewis & Hurd, 2011; Lewis, Perry, & Hurd, 2009). Teachers who enact lesson studies provide an authentic window through which researchers can understand teacher professional decisions and thinking. Of particular interest in this study are the conversations teachers have during the debriefing stage of a lesson that incorporated the promotion of at least one Standard for Mathematical Practice (SMP; CCSSI, 2010). Through the lens of teachers' authentic dialogue about students' learning and the lessons that develop students' learning abilities, we consider what is understood about SMP 5 and how teachers use those understandings to improve instruction. For the purpose of this proceeding, we focus on teachers' dialogue about SMP 5, "Use appropriate tools strategically" (CCSSI, 2010).

Related Literature

Lesson Study

Lesson Study is a "comprehensive and well-articulated process for examining practice" (Fernandez, Cannon, & Chokshi, 2003, p. 171) and a method of professional development that encourages teachers to reflect on their teaching practice through a cyclical process of collaborative lesson planning, lesson observation, and examination of student learning (Lenski, Caskey, & Anfara, 2009). Lesson study allows teachers to view teaching and learning as they occur in the classroom. There is not a singular approach to all lesson studies. For the research conducted here, the approach to lesson study is based on Lewis and Hurd's (2011, p. 2) cycle of studying curriculum and formulating goals, planning the research lesson, conducting the research lesson, and reflecting. The first stage, studying curriculum and formulating goals, stems from the Japanese term kyozaikenkyu, in which teachers should take the time to study curriculum, materials, and standards to help develop the research lesson. Also in this stage, teachers should formulate short-term and long-term goals for what they hope students learn. The second stage, planning the research lesson, requires teachers to use a critical lens to determine what aspects should be incorporated into the research lesson and why those aspects are the best ways for students to reach the goal of the lesson. The third stage, conducting the research lesson, consists of one team member from the lesson study group teaching the research lesson to a group of students while the other team members observe the lesson. These team members typically follow an agreed upon observation protocol and do not interact directly with the students. The final stage of the lesson study cycle, the debriefing stage, is a time for teachers to reflect and discuss their observations, considering improvements that could be made to better student learning. Following the debriefing stage, the lesson study team has the option to re-teach the research lesson using the observations and modifications to learn from a new set of students.

Debriefing Phase of Lesson Study

During the debriefing phase of lesson study, also known as the post-lesson discussion, the team of teachers and any other outside experts who observed the lesson will engage in reflection about the lesson. While debriefing, the teachers are assessing student learning, what aspects of the lesson promote student learning, and what can be done better in the future to improve their practice (Lewis & Hurd, 2011; Takahashi & McDougal, 2016). Before beginning to debrief as a group, the lesson study group members should take time to reflect individually and gather their thoughts to ensure their discussion stays organized and focused; this debrief should not become a retelling of the lesson. Prior to starting, Lewis and Hurd (2011) recommend that the team create a set of norms to ensure the discussion is respectful of all participants and no one teacher feels singled out. All members should understand it is a group effort and they have shared responsibility for this lesson. Lastly, it is crucial that this phase focuses on student learning and data recorded, not personal feelings and judgements on the teacher (Lewis & Hurd, 2011).

Standards for Mathematical Practice (SMPs)

The SMPs were incorporated as part of the Common Core State Standards Initiative (CCSSI) in an attempt to promote consistent learning goals across states (CCSSI, 2010). These standards were enacted to promote students' mathematical proficiency for readiness and preparation as they advanced toward their college and career paths. The SMPs provide specific processes and proficiencies that mathematics educators should be integrating throughout their instruction and facilitation of student learning (CCSSI, 2010). They place the focus of the learning on the students, and key components of mathematical knowledge are interwoven throughout these practices, which include problem solving, reasoning, representation, productive mathematical discourse, conceptual understanding, and procedural fluency. There are a total of eight SMPs. For the purposes of this proceeding the focus will be on SMP 5.

SMP 5: Use appropriate tools strategically. SMP 5 focuses on the importance of giving students opportunities to consider available tools and decide which tools would be helpful in various problem solving situations (Bostic, Matney & Sondergeld, 2019; CCSSI, 2010). Allowing students the space to select a tool that is appropriate and demonstrate the tool's use strategically is a fundamental component of this SMP. It would be a misconception of the intent of SMP 5 to assume that if students are using tools, for any reason, then they are necessarily engaging in SMP 5. Mathematically proficient students should become accustomed to making sound decisions about the situations in which specific tools might be most beneficial while also recognizing the limitations of each (CCSSI, 2010).

Methodology

The research here considers the authentic context of teacher dialogue that develops after teams of teachers plan and enact lessons together. The research question is: What does teacher dialogue in the debriefing phase of lesson study reveal about teachers' understanding of SMP 5?

Context

Participants and lesson study design. There were 52 teachers involved in the research, all residents of the same state in the mid-west. All teachers taught in K-5 classrooms and ranged in professional experience from two years to thirty-one years. The participants were solicited to join a one-year project focused on developing students' mathematical problem solving (Inprasitha, 2015). The 52 participants constituted 12 lesson study teams that elected to join the project. The teams consisted of teachers from across the grade levels of K-2 and 3-5. There were six K-2 teams and six 3-5 teams. All but one of the teams consisted of teachers from the same school. The participants had two days of professional learning about the process of lesson study and problem solving (Changsri, 2015; Isoda, 2015; Kadroon & Inprasitha, 2013).

Additionally, participants read and discussed Lewis and Hurd's (2011) *Lesson Study: Step by Step*. Participants developed norms to keep their lesson study focused and professionally respectful throughout the planning and debriefing phase discussions (Lewis & Hurd, 2011). Next, the 12 teams of participants enacted two full lesson study cycles during the one-year project; one lesson study took place in the fall semester and one in the spring semester for a total of 24 lesson studies. Each team spent between six and eight hours researching ideas involving student learning in the content domain Operations and Algebraic Thinking and consisting of at least one SMP (CCSSI, 2010). The teams constructed their collaborative research lesson toward the end of this research and development time. Each team taught and observed students' mathematical thinking and problem solving during the lesson and then met immediately afterward to reflect and revise. Then, the team taught, observed, reflected, and revised a second time with a new group of students in the same day. At the end of these two teaching cycles the teams reflected on their own professional learning.

Background and incorporation of the SMPs in the project. The SMPs had been part of the state standards for seven years prior to the start of the research, and all participants indicated that they were aware of the state's expectations for promoting the SMPs during instruction. The participants were asked to use their knowledge of the SMPs in two ways: focus on the promotion of at least one SMP during the collaborative research lesson planning phase and discuss, during the debriefing stage, whether or not there was any evidence of the SMPs being enacted by students. In preparation for the post-lesson discussion, a debriefing protocol was developed during the project based on teachers' reading of Lewis and Hurd (2011, p. 57-64). Though questions and focuses varied by team during these discussions, each debriefing protocol had the question "What evidence is there that the lesson provided an opportunity for student engagement in mathematical proficiencies (SMP's)?" for each participant to reflect on and discuss with their team.

Data and Analysis

Each of the written and revised collaborative research lesson plans were collected, as well as videos of each post-lesson debrief session. The videos totaled more than 19 hours of teacher discussion about the enacted lesson and revisions that needed to be made to improve students' mathematical learning. To answer the research question, videos were transcribed and an inductive analysis (Hatch, 2002) was performed on the resulting text. Figure 1 shows the step-by-step process taken when analyzing the debrief sessions.

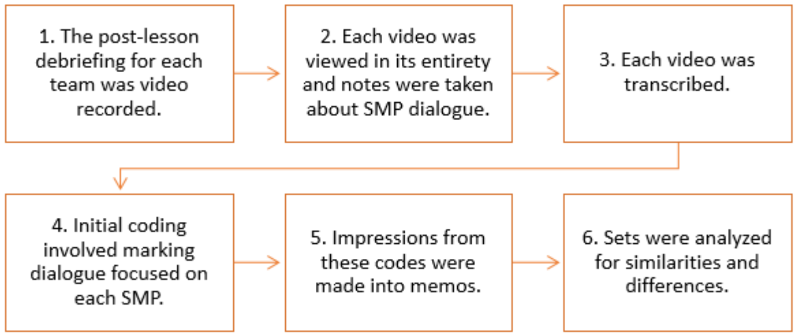

Figure 1. Steps taken in the analysis of debriefing sessions.

Findings

The inductive analysis reveals three themes from the dialogue of participants. Teams made very few connections to SMP 5 despite the structural opportunity to discuss such things through the Lesson Study process. Teams lacked an SMP 5 mindset when thinking about the use of tools in their lessons. Teams who focused directly on SMP 5 revealed misunderstandings about its expectations. These themes are explicated below.

Few Connections to SMP 5

Overall, data revealed that the majority of teams missed opportunities to discuss and implement direct connections from SMP 5 into their lessons to foster student sense making during problem solving. Throughout each debriefing session, the teams suggested changes to their research lesson. Each team was asked to have a focus SMP for their research lesson and had a debriefing protocol which included *discussing evidence of student engagement in any of the SMPs*. Seven of the 24 lessons stated SMP 5 as the focus. However, tools could have been used by students to makes sense of the problems in each of the 24 lessons. Dialogue occurring among 21 of the 24 lessons missed connections about how using tools could have strategically benefited students in solving the problems. During the 24 debriefing sessions only one group used language from SMP 5 in their discussions about tools. Although seven teams directed their focus toward SMP 5, less than half had dialogue that discussed tools in ways related to SMP 5, including: the need to provide manipulatives for students to select and use (Team 4; Team 8) and give students time to learn the manipulatives before the research lesson is conducted (Team 1;

Team 8). These two changes relate to the appropriate use of tools and opportunity to understand a tool as a strategic choice.

Lack of SMP 5 Mindset

Teams also made several suggested changes involving tools that did not necessarily promote SMP 5 but held potential if the teams shifted their mindset toward improving students' mathematical proficiencies. Five out of 12 teams suggested changes that were potentially related to SMP 5. These changes include: take away manipulatives that were offered in the first teaching (Team 1; Team 10), change the manipulative being offered (Team 5; Team 10), and give a specific manipulative directly to each group (Team 6; Team 7). In each of these cases, discussion by the teams were focused on classroom management and not on students' strategic and appropriate use of tools to improve their mathematics proficiency.

Participants struggled to engage their students in SMP 5. More often than not, participants gave students specific tools to use for specific purposes, and this caused the students to focus on what should be done with the tool instead of making sense of mathematics. Additionally, some students made sense of the mathematics but became confused at how the tool was supposed to help. One example of this comes from Team 8 in speaking about observation of a student who "knew that there was supposed to be eight bears, she didn't understand" how to use them to help her. Other students tended to use the tools inappropriately, as a game for making patterns, which participants said inhibited learning. This was challenging for participants because they had not thought to look into this issue during research and planning. Having given the tools to students for a specific purpose, without enough time for students to make sense of the tool itself and ideas it may be connected to, the participants observed, "what it [tools] became was a game to make designs" (Team 2).

Misunderstanding SMP 5

Analysis of dialogue from the debriefing sessions revealed that the participants focusing on SMP 5 held the belief that if the students were using manipulatives, this represented a convincing argument for evidence of SMP 5 during the lesson. For example, the only group who used SMP 5 language in their dialogue began the discussion by stating, "And you [T1] provided opportunity…for appropriate tools," which another teacher responded to by stating, "Right. They might not have necessarily all used them correctly, but you used them" (Team 10). Conversations like this one revealed that participants may not view the appropriate and strategic use of tools as an imperative aspect of the lesson. In the statements made by the participants, the perceived importance of the lesson giving students opportunities to use tools, whether or not they were using them appropriately and strategically, demonstrates a lack of participant understanding of the expectations of SMP 5.

Discussion

Teachers need time to more fully understand and implement pedagogical strategies that promote SMP 5. This would include giving students time to become familiar with new manipulatives before using them in the classroom to understand how they can be used appropriately and strategically. Teaching professionals should think carefully about the introduction of manipulatives, as tools for problem solving and making sense of mathematics, and not simple objects of manipulation. The understanding of tools in the former sense opens students up to thinking about the tool as appropriate or not and for what strategic purpose they are using the tool. In our findings, teachers did not discuss the idea of giving the students the opportunity to decide which tools they felt would be of most benefit to them. The teams demonstrated the belief that as long as tools were provided for the students, they were encouraging student engagement in SMP 5. However, we contend that evidence of a students' mathematical proficiency, in the essence of SMP 5, means that students astutely consider the tools' appropriateness for themselves and choose the tools in connection with their own strategic trajectory for solving the problem. This must be done with tools appropriate to the context, such as using a protractor to measure a needed angle rather than a ruler.

Conclusion

In this study we consider teachers' dialogue about lessons in which at least one SMP was their focus and all teams could have provided access to tools for their students' problem solving. Although seven teams directly focused on SMP 5 and the SMPs were part of the teachers' standards for seven years, our findings revealed that implementing and promoting SMP 5 during instruction was professionally challenging. The one team that used language directly from SMP 5 showed a lack of understanding of what constitutes evidence of student engagement in SMP 5, in that, the teachers believed simply using the given tools as directed by the teacher constituted evidence for SMP 5. Furthermore, the teams who used tools encouraged the students to use the tools as a form of manipulation or representation but not as a means to make mathematical sense of the problem. Teachers need time to reflect more deeply upon SMP 5 and acquire a thorough understanding of how to best promote it among students. Following these lessons, we recommended to the participants that they consider conducting lesson studies focused on researching and understanding SMP 5. The implementation of tools in the classroom should supplement effective pedagogy, and to accomplish this, teachers need to work together to find effective ways to utilize tools to promote understanding and success for students' problem solving.

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USING STUDENT THINKING IN INSTRUCTION: LEVERAGING VERSUS ENDORSING

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There is emerging evidence that responsive teaching, characterized by instructional practices that elicit and connect student thinking to disciplinary ways of reasoning, has many affordances. As such, it is important to better understand this type of teaching. We contribute to this literature by investigating a case where a teacher used students' ideas to advance the mathematical agenda, yet her students failed to develop a meaningful understanding of the content. This examination resulted in the identification of an instructional practice critical to responsive teaching, leveraging, which we characterize and contrast with a similar, less productive practice of endorsing.

Spurred by the 1989 standards (National Council of Teachers of Mathematics, 1989), researchers (e.g., Ball, 1993) began presenting educators with images of a new form of teaching in which teachers elicit and connect student thinking to disciplinary ways of reasoning, what is often referred to now as responsive teaching. In the intervening decades, researchers have provided evidence of the benefits of responsive teaching, namely that it can improve student learning outcomes (Pierson, 2008), develop conceptual understanding (Fennema et al., 1996), and promote more equitable participation (Empson, 2003). More recently, as the impact of responsive teaching has become more established, scholars have worked to better characterize and conceptualize such thinking (see Robertson, Scherr, & Hammer, 2016). Nonetheless, it continues to be important for researchers to gain insights into the processes by which responsive teaching leads to the desirable outcomes previously identified.

One such effort, providing a potential model for better understanding these processes was offered by Singer-Gabella, Stengel, Shahan, and Kim (2016). They studied three novice teachers who varied considerably in their implementation of teaching practices consistent with responsive teaching. Studying teachers engaging in a range of such practices created contrasts that highlighted sources of tension in learning to teach responsively. By studying teachers with a range of abilities to implement responsive practices, as opposed to just teachers who do it well, the contrasts highlighted critical components of responsive teaching that help bring about the desired outcomes. Similarly, we examined the practices of a teacher who had an orientation and goals consistent with responsive teaching yet was still grappling with its implementation. In particular, we answered the following research question: In what ways does a teacher elevate and connect different examples of student thinking in her instruction and how do these choices relate to students' subsequent reasoning?

Methods

Our goal was to study the practices of a teacher whose teaching was grounded in student thinking, but who was somewhat new to responsive teaching and consequently still grappling with how to productively coordinate a wide range of ideas to support all students in developing a thorough understanding of the content. While we anticipated seeing many practices associated with the images of responsive teaching put forth in the literature, we also wondered in what ways her instruction would differ given her lack of experience with such teaching. We conjectured that investigating connections between her practice and students' subsequent reasoning would reveal specific ways in which she had effectively leveraged and connected students' varying conceptual resources, but also ways she might have missed opportunities to do so. We hoped this would help us understand the critical features of responsive teaching. To do this, we first examined the students' resulting reasoning and then used this to guide our analysis of the teacher's practice.

Data Collection

We first identified a teacher, Ms. White[1], from participants in a week long professional development (PD) session we held during the summer. The session focused on developing teachers' capacity to teach figural patterns (see Figure 1). Specifically, we supported teachers in using these tasks by introducing them to an instructional trajectory aimed at engendering in students a quantitative understanding of algebraic symbols through generalization. In the PD session, we suggested specific activities the teachers might use while teaching this unit and had the teachers rehearse (Lampert et al., 2013) these activities. From these rehearsals we could see that Ms. White possessed a strong understanding of the content and was comfortable orchestrating a whole class discussion. She had 9 years of experience and was teaching in a Title 1 middle school at the time of the study.

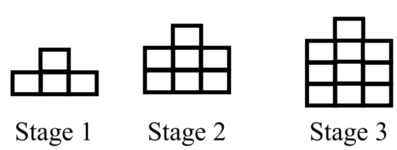

Figure 1. An example figural pattern.

[1] All names in this report are pseudonyms.

According to our instructional trajectory, students explored figural patterns by first drawing future stages of the pattern, then writing and sharing verbal descriptions of future stages, then writing numeric expressions (e.g., 3+3+3+3+3+1 or 5+6+5 for Figure 1), and finally a general algebraic expression (e.g., $3n+1$ or $n+(n+1)+n$). These activities were meant to help students impose structure on the figural patterns (Hawthorne & Druken, 2019), make that structure explicit through whole class discussions, and then represent the quantities in the structure with numbers and variables.

Ms. White then implemented the activities we had rehearsed in the PD session over three days of instruction in her class. During this time, the students explored three different figural patterns. The first pattern is shown in Figure 1; the second was a growing H (see Figure 4); and the third was a growing W. We videotaped her instruction over the three days and used this video as the main source of data to analyze her instruction.

On the last day of instruction, we interviewed 7 students after class to better understand how they could reason about figural pattern tasks. In the interview, we presented students a new pattern (Figure 2) and asked them to write numerical expressions for how they were seeing it as well as a general algebraic expression. We followed-up with probing questions to investigate the meaning each student had of the symbols in their expressions (Ginsburg, 1997).

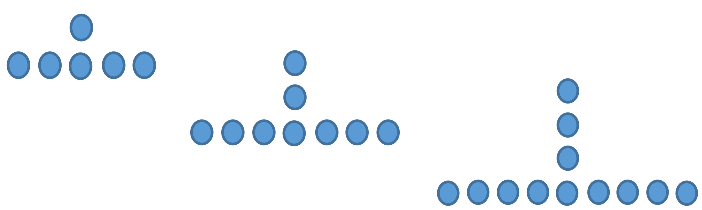

Figure 2. The pattern for the interview task.

Data Analysis

We started by analyzing the student interview data to understand the nature of student reasoning. This understanding informed our analysis of the classroom data, as we looked for attributes of the instruction that helped explain why students were reasoning in the ways they were in the interviews. We used a constant comparison method looking between subjects to identify trends and differences in reasoning (Strauss & Corbin, 1994). This gave rise to an initial set of codes, which was then refined through multiple iterations analysis and discussion. In our analysis we identified both common conceptual resources and common difficulties.

The results from the interview data guided our analysis of the classroom data. In particular, the interview data revealed that students seemed to have unproductive implicit rules that guided how they broke down the figure so they could create the algebraic expression. To explain these trends in unproductive ways to decompose the figure, we analyzed the classroom discourse. We first created descriptive accounts of the three days of instruction (Miles & Huberman, 1994), which we split into episodes. We marked any episode where a decomposition of a figure was presented in whole class. We then noted how each decomposition was treated in the discourse (e.g., if it was ignored, compared with other decompositions, used to develop an algebraic expression, etc.). We also marked episodes where explicit reflective talk about decompositions occurred. This allowed us to see both the explicit and implicit messages communicated to students about how to strategically decompose a figure to write an algebraic expression and how Ms. White used student contributions to communicate these messages.

Results

We intended for the interview task to necessitate adaptive reasoning to generalize and communicate the perceived structure of the figural pattern. While the task was closely related to the work the students had done over the previous three days, we saw it as slightly more difficult because a connection between differing decompositions and the stage number was less obvious. Despite this challenge, students demonstrated a variety of productive ways to think about the task and emergent understanding about meaningful interpretations of the symbols in the numerical and algebraic expressions. However, six of the seven students struggled to use their understanding to ultimately provide a correct algebraic expression. Their difficulty seemed to stem from their inability to integrate their own productive ways of reasoning with the ideas and approaches expressed during instruction of this unit. In this report, we focus on their inability to use their initial productive decompositions to create an algebraic expression, relying instead on unproductive implicit rules they gleaned from participation in class discussions.

Adopted Norms for Decomposing

Students' norms for decomposing seemed connected to their understanding of algebraic symbols. In general, it seemed students saw the $mn+b$ form as a template to be filled in. Thus, they created rules for the meaning of these symbols (e.g., m was the number of n sized groups) and would decompose the figure accordingly to extract these values. Most notably they believed that the group size must equal the stage number and sought to impose this structure on the pattern. The coefficient then would represent the number of groups and the constant term was interpreted as the remaining pieces by some students and the change between stages by others. Only one student was able to use these rules successfully on the task, grouping the figure into three groups of size n, with three blocks leftover to create the expression $3x+3$. For the other students, these inferred rules seemed to interfere with their productive reasoning as they altered their initial ideas to comply.

For example, while Madelyn had identified an explicit relationship between her decomposition of the figure and the stage number, her attempt to fill in the template resulted in an incorrect algebraic expression. Initially, she had created groups whose size was one more than the stage number and was able to generalize this relationship when asked to write numeric expressions. She correctly wrote the expression 7 + 7 + 7 for stage 6 and 33 + 33 + 33 for stage 32. However, when asked for an algebraic expression, she said it should be $3x + 2$, explaining,

> The three is the number of sections that you would divide it into; the x is the number of, like the number of dots in the section in the stage; and the 2 is the number of dots you are adding on.

After demonstrating such a rich understanding of the figure, such an abrupt and disconnected interpretation was quite striking. Two other students' initial decompositions were also not of the form m groups n, but then slowly reinterpreted the figures to align with these rules.

In addition, students' responses indicated they were not clear on the purpose of decomposing. For example, Leanne seemed to arbitrarily circle groups. After previously correctly generalizing the pattern recursively, she circled the figure in two completely different ways when asked for numerical expressions for stage 4 and 6 (see Figure 3). Moreover, neither of her expressions related to her previous way of thinking nor the stage number. For stage 4, she circled 3 groups of 4, writing the expression 4(3) + 3 and for stage 6, she circled 4 groups of 5, writing 5(4) + 1. In both cases, she identified the first number as representing the dots per group, the second number as the number of groups, and the last number as the extra dots. When asked about her method of grouping, she explained, "I was just grouping it around to make it easier." She seemed to circle not to impose structure that could be used to find explicit patterns, but to arbitrarily identify groups to satisfy the rules for creating an expression she had inferred. In general, while the students demonstrated a strong potential for generalizing figural patterns and used various representations to communicate their understanding, their perceived rules for the meaning of symbols and lack of clarity around the role of decomposing seemed to stymie their productive thinking.

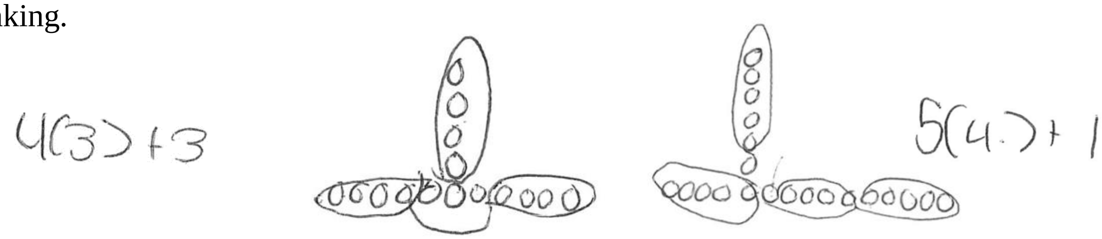

Figure 3. Leanne's decomposing and numerical expressions for stage 4 and 6.

Instructional Foci and Missed Opportunities

Students' unproductive norms for decomposing can be partially explained by the types of decompositions that were explored in class. In particular, while Ms. White allowed all students to share their thinking, she only engaged mathematically with decompositions that mapped onto the form $mn+b$, failing to support students struggling to decompose or confront less strategic ways of decomposing. For example, when exploring the second pattern, Ms. White had 4 volunteers share their view of the pattern: one where the student saw the middle piece as a groups of size n and the four other pieces growing recursively (Figure 4A), one where the student saw five groups of size n (Figure 4B), one with the columns viewed as a single unit (Figure 4C), and one in which the groupings did not appear to be strategic (Figure 4D). Ms. White showed appreciation for the diversity of views, but did not make comparisons or connections between them. Instead, she erased each of these decompositions, choosing only to analyze the second one (of the form $mx+b$, Figure 3B). To emphasize the rationale behind such a decomposition, she turned to the two boys who had come up with this view and asked them why they circled these five groups. When the students did not reply, Ms. White answered her own inquiry saying, "because they all have the same amount." Similarly, after the next student drew the H pattern with the columns interpreted as whole pieces (similar to 4C), Ms. White went back to the previous decomposition and asked students who used this method to clarify why they separated the two dots on the sides. After one student replied indistinctly, Ms. White revoiced his answer, "So you are keeping the top and bottom equal…and the middle equal. And so those were just on the ends extra." Again, this implied that students should decompose the figure by always creating groups of size n. This occurred as students explored the last pattern as well, with the teacher encouraging students to create groups of size n, even when they initially saw the pattern in other productive ways.

A B C D

Figure 4A, B, C, and D. Decompositions presented in whole class for H pattern.

Discussion

While previous research shows that responsive teaching has great potential, researcher still lack detailed understanding to processes by which responsive teaching leads to desirable outcomes. By analyzing a teacher whose goals and intentions were consistent with responsive teaching, but was still learning to master the practice, we gained insights into some features of responsive teaching we posit are critical. In responsive teaching, the teacher centers instruction on student thinking and uses that thinking to move the mathematical agenda forward. In many ways, Ms. White did this. She created an environment where students were allowed to explore the growth in the figural patterns freely. She then had many students share out and celebrated the diversity of ideas. In interactions with students, she often said they did not have to follow a particular approach. Furthermore, the advancement of the mathematical agenda at the classroom level was always rooted in strategies that came from students. However, rather than drawing connections between the students' various approaches, she selectively focused on certain strategies that she felt were productive while acknowledging, but not developing, others. We call this type of orchestration, where only a certain type of response is developed, *endorsing* since only specific strategies are endorsed, albeit implicitly, by the teacher. Such an instructional structure seemed to result in students making sense of the strategies that were being endorsed in unproductive ways (e.g., thinking that groups had to be of size n or interpreting the coefficient as always meaning number of groups).

We contrast this type of orchestration with images existent in the literature where a variety of contributions, more representative of the range of ideas in the class, are presented with connections drawn between them. We call this type of orchestration *leveraging*. We hypothesize that students would have been more successful with the interview task if they had seen how their personal ways of reasoning were connected to the methods being presented in class. In this instance, if the purposes for decomposing had been made more clear by highlighting how different decompositions could be represented using different algebraic expressions, students may have been able to more strategically decompose the figure and use algebraic expressions to capture their understanding of the relationships they saw between quantities.

This means that to effectively teach "responsively" to the whole class, teachers must go beyond simply using student strategies to advance the mathematical agenda of the classroom. Rather, they must ensure that as the lesson moves forward, all students see connections between their personal ways of reasoning and the more sophisticated ways of reasoning that are being developed in class. By contributing to the identification of this critical feature of responsive teaching, we hope to help shape how teachers think about orchestrating discourse. In particular, we hope that as teachers anticipate various student strategies, they can also think about how a variety of ways of reasoning can be leveraged to develop more sophisticated understanding.

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